ECTHESIS

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It seems that Aristotle may have intended to base the convertibility of universal negatives on something more fundamental:

> If A belongs to no B, B in turn will belong to no A. If indeed B did belong to some A, to Γ for example, it would not be true that A belonged to no B, since Γ is a certain B. (*An. Pr.* A2, 25a15-17).

This process of selecting an A («Γ for example») is called *ecthesis*.

But are we to take Γ as a term-variable or as an individual-variable ? If, with Waitz (*Aristotelis Organum* Vol. I p. 374), we take it to be a term-variable, the ecthetic proof of conversion seems to go as follows:

> If A belongs to no B then B belongs to no A; for, if B does not belong to no A then it belongs to some A, and so, for some Γ, B belongs to all Γ and A belongs to all Γ, in which case (by a Darapti with permuted premisses) A belongs to some B and does not belong to no B.

It has been objected to this that Darapti is proved by the convertibility of the universal affirmative, which in turn depends on that of the universal negative, so that there is a circular demonstration here. But, while it is true that Darapti can be proved that way, it is not true that it must be, since it can also be reduced indirectly to Celarent:

$$\frac{\text{A belongs to no B} \quad \text{B belongs to all } \Gamma}{\text{A belongs to no } \Gamma}\ (\text{Celarent}) \rightarrow \frac{\text{A belongs to all } \Gamma \quad \text{B belongs to all } \Gamma}{\text{A belongs to some B}}\ (\text{Darapti})$$

Waitz's proof, then, is not circular; but it does assume that if A belongs to some B then, for some Γ, A belongs to all Γ and B belongs to all Γ. This assumption is embodied in the rule *R*:

$$\frac{Q \ \text{A belongs to all } \Gamma \quad \text{B belongs to all } \Gamma}{q} \rightarrow \frac{Q \ \text{A belongs to some B}}{q}$$

where Q is a sequence of categoricals (possibly null) and q is a single categorical. The proof of the convertibility of the universal negative from Darapti is then:

$$\frac{\text{A belongs to all } \Gamma \quad \text{B belongs to all } \Gamma}{\text{A belongs to some B}}$$

$$\therefore \frac{\text{B belongs to all } \Gamma \quad \text{A belongs to all } \Gamma}{\text{A belongs to some B}} \ \text{(by Permutation)}$$

$$\therefore \frac{\text{B belongs to some A}}{\text{A belongs to some B}} \ \text{(by the above rule)}$$

$$\therefore \frac{\text{A belongs to no B}}{\text{B belongs to no A}} \ \text{(by Transposition)}$$

(The horizontal lines here are to be read «If ... then».) All syllogisms can still be reduced to Barbara and Celarent, since Darapti reduces to Celarent.

Notice, however, that rule *R* is reversible, so that we also have:

$$\frac{Q \ \text{A belongs to some B}}{q} \rightarrow \frac{Q \ \text{A belongs to all } \Gamma \quad \text{B belongs to all } \Gamma}{q}$$

And this rule can be used to derive Darapti from the thesis:

$$\frac{\Pi \text{ belongs to some } P}{\Pi \text{ belongs to some } P}$$

which in turn follows from the convertibility of the particular affirmative. The resulting system is not circular if we have an alternative method of establishing Darapti (e.g., by reducing it to Celarent).

Alternatively we could follow Łukasiewicz (*Aristotle's syllogistic* § 19) in having not rules but theses of ecthesis. These are:

B belongs to some A ≡ $(\exists \Gamma)$ (B belongs to all Γ and A belongs to all Γ)
B does not belong to some A ≡ $(\exists \Gamma)$ (B belongs to no Γ and A belongs to all Γ).

The convertibility of the universal negative can then be proved via the commutativity of conjunction; it follows fairly easily from the first of these theses and does not presuppose Darapti. Similarly Darapti can be proved if we have the thesis:

(Π belongs to all Σ and P belongs to all Σ) $\supset$ $(\exists \Gamma)$ (Π belongs to all Γ and P belongs to all Γ).

The principal advantage of this interpretation is that it does not make the ecthetic proof of conversion depend on the validity of Darapti. For, even if such a dependence would not involve circularity, it would be rather odd of Aristotle simply to assume the validity of a syllogism he was later going to prove. On the other hand the interpretation requires syllogistic to be subjoined not only to propositional logic but to second-order predicate logic. All criticisms applicable to syllogistics subjointed to propositional logic will therefore apply with renewed force to such a system. (See my «Aristotle's syllogsitic», forthcoming *Notre Dame journal of formal logic*, for these criticisms.)

These criticisms can be avoided by adopting rules instead of theses of ecthesis. The two rules governing particular affirmatives have been mentioned already, and there are two similar ones for particular negatives.

Darapti reduces to Celarent; and the laws of conversion follow from Darapti using our ecthetic rules; and given these, all syllogisms are provable.

Darapti itself follows from:

$$\frac{\Pi \text{ belongs to some } P}{\Pi \text{ belongs to some } P} \quad \text{(which follows from conversion)}$$

by means of the rule:

$$\frac{Q \text{ A belongs to some B}}{q} \rightarrow \frac{Q \text{ A belongs to all } \Gamma \quad \text{B belongs to all } \Gamma}{q}$$

and this is what the ecthetic derivation of Darapti looks like in this system. But this derivation is not an autonomous *proof* of Darapti, since the rule used depends for its proof on Darapti. It could, however, be turned into a proof by making the rule primitive.

Even so, this is hardly convincing as an interpretation of Aristotle's ecthetic proof of Darapti; for it contains no «selected» variable different from the variables in the thesis of be proved. Such a variable (surely the characteristic feature of ecthetic proof) does occur in the following proof of Datisi (*cf. An.Pr.* A6, 28 b 14):

$$\frac{\Pi \text{ belongs to all } \Sigma \quad \Sigma \text{ belongs to all N} \quad \text{P belongs to all N}}{\Pi \text{ belongs to some P}}$$

[from Barbara and Darapti]

$$\therefore \frac{\Pi \text{ belongs to all } \Sigma \quad \text{P belongs to some } \Sigma}{\Pi \text{ belongs to some P}}$$

[by Permutation and R]

Despite a certain elegance in this presentation of syllogistic, then, it cannot stand as an interpretation of ecthesis: it makes the ecthetic proof of convertibility of the universal negative depend on Darapti, and it does not give a plausible representation of the ecthetic proof of Darapti.

On an alternative view ecthesis selects individual-not term-variables, and ecthetic proofs depend on rules relating categorical to singular propositional forms and on a system of syllogisms containing individual-variables. This view was common in the Middle Ages, when ecthetic proofs were explained via a system of 'expository' syllogisms (from «expositio», the Latin version of «ecthesis»).

We shall stipulate that individual-variables cannot occur in predicate-position, and that they be thought of as *restricted* (i.e. tied to specific term-variables). Thus we introduce a range of individual-variables formed from the lower-case Greek letter corresponding to the term-variables. We take «A belongs to β» and «A does not belong to β» as contradictories. Each of the three figures now takes three forms (putting the predicate first and subject second):

	Figure I	Figure II	Figure III
(a)	AB Bγ	AB Aγ	AB Γβ
(b)	Aβ BΓ	Aβ AΓ	Aβ ΓB
(c)	Aβ Bγ	Aβ Aγ	Aβ Γβ

But there will be no well-formed conclusions in the second and third varieties of Figure II since individual-variables may not occur as predicates. It also turns out that there are no syllokisms in the second and third varieties of Figure I.

In fact the only syllogisms are the following:

Figure I (a)

(1) $\dfrac{\text{A belongs to all B} \quad \text{B belongs to } \gamma}{\text{A belongs to } \gamma}$

(2) $\dfrac{\text{A belongs to no B} \quad \text{B belongs to } \gamma}{\text{A does not belong to } \gamma}$

Figure II (a)

$$(3)\quad \frac{\text{A belongs to no B} \quad \text{A belongs to } \gamma}{\text{B does not belong to } \gamma}$$

$$(4)\quad \frac{\text{A belongs to all B} \quad \text{A does not belong to } \gamma}{\text{B does not belong to } \gamma}$$

Figure III (a)

$$(5)\quad \frac{\text{A belongs to all B} \quad \Gamma \text{ belongs to } \beta}{\text{A belongs to some } \Gamma}$$

$$(6)\quad \frac{\text{A belongs to no B} \quad \Gamma \text{ belongs to } \beta}{\text{A does not belong to some } \Gamma}$$

Figure III (b)

$$(7)\quad \frac{\text{A belongs to } \beta \quad \Gamma \text{ belongs to all B}}{\text{A belongs to some } \Gamma}$$

$$(8)\quad \frac{\text{A does not belong to } \beta \quad \Gamma \text{ belongs to all B}}{\text{A does not belong to some } \Gamma}$$

Figure III (c)

$$(9)\quad \frac{\text{A belongs to } \beta \quad \Gamma \text{ belongs to } \beta}{\text{A belongs to some } \Gamma}$$

$$(10)\quad \frac{\text{A does not belong to } \beta \quad \Gamma \text{ belongs to } \beta}{\text{A does not belong to some } \Gamma}$$

The remaining premiss-moods can be shown to be non-concludent by Aristotle's method of contrasted instances.

Syllogisms (4) and (10) reduce indirectly to (1); (3) and (9) similarly to (2). The rest reduce to (1) or (2) given that a proposition like «A belongs to α» (which expresses the restricted nature of the individual-variables) is always a logical truth, and so may be dropped from any sequence of premisses.

Proof of (5)

$$\frac{\text{A belongs to all B} \quad \text{B belongs to } \beta}{\text{A belongs to } \beta}\ (1)$$

$$\therefore \frac{\text{A belongs to all B}}{\text{A belongs to } \beta}\ [\text{dropping «B belongs to } \beta\text{»}]$$

$$\frac{\text{A belongs to } \beta \quad \Gamma \text{ belongs to } \beta}{\text{A belongs to some } \Gamma}\ (9)$$

$\therefore$ (5) [combining the last two steps]

Proof of (6)

$$\frac{\text{A belongs to no B} \quad \text{B belongs to } \beta}{\text{A does not belong to } \beta}\ (2)$$

$$\therefore \frac{\text{A belongs to no B}}{\text{A does not belong to } \beta}\ [\text{dropping «B belongs to } \beta\text{»}]$$

$$\frac{\text{A does not belong to } \beta \quad \Gamma \text{ belongs to } \beta}{\text{A does not belong to some } \Gamma}\ (10)$$

$\therefore$ (6) [combining the last two steps]

Similarly for (7) and (8).

In establishing this we have proved:

$$\frac{\text{A belongs to all B}}{\text{A belongs to } \beta} \qquad \frac{\text{A belongs to no B}}{\text{A does not belong to } \beta}$$

whence follow:

$$\frac{\text{A does not belong to } \beta}{\text{A does not belong to some B}} \qquad \frac{\text{A belongs to } \beta}{\text{A belongs to some B}}$$

by Transposition. Thus we may generate subaltern syllogisms by replacing singular conclusions with particulars, or singular premisses with universals.

As for ecthesis, the Jesuits of Coimbra, in their commentary on *An. Pr.* A2, Q.1, art. 1 and A6, Q.1, art. 2, thought it depended on the invalid principles:

$$\frac{\text{A belongs to some B}}{\text{A belongs to } \beta} \qquad \frac{\text{A does not belong to some B}}{\text{A does not belong to } \beta}$$

and accordingly denied it was a formal process. In fact, however, it depends on the formally valid rules E:

$$\frac{Q}{\text{A belongs to } \beta} \rightarrow \frac{Q}{\text{A belongs to all B}}, \text{ where } \beta \text{ is not in } Q$$

$$\frac{Q}{\text{A does not belong to } \beta} \rightarrow \frac{Q}{\text{A belongs to no B}}, \text{ where } \beta \text{ is not in } Q$$

and their consequences E':

$$\frac{Q \text{ A belongs to } \beta}{q} \rightarrow \frac{Q \text{ A belongs to some B}}{q},$$

where β is not in Q or q

$$\frac{Q \text{ A does not belong to } \beta}{q} \rightarrow \frac{Q \text{ A does not belong to some B}}{q},$$

where β not in Q or q

We can now represent Aristotle's ecthetic proofs.

Conversion

$$\frac{\text{A belongs to } \alpha \quad \text{B belongs to } \alpha}{\text{A belongs to some B}} \quad (9)$$

$$\therefore \frac{\text{B belongs to } \alpha}{\text{A belongs to some B}} \text{[dropping «A belongs to } \alpha\text{»]}$$

$$\therefore \frac{\text{B belongs to some A}}{\text{A belongs to some B}} \quad [\text{by } E']$$

$$\therefore \frac{\text{A belongs to no B}}{\text{B belongs to no A}} \quad [\text{by Transposition}]$$

Darapti

$$\frac{\Pi \text{ belongs to all } \Sigma}{\Pi \text{ belongs to } \sigma} \qquad \frac{\text{P belongs to all } \Sigma}{\text{P belongs to } \sigma}$$

$$\frac{\Pi \text{ belongs to } \sigma \quad P \text{ belongs to } \sigma}{\Pi \text{ belongs to some } P} \; (9)$$

∴ Darapti [combining the two steps]

Datisi

$$\frac{\Pi \text{ belongs to all } \Sigma}{\Pi \text{ belongs to } \sigma}$$

$$\frac{\Pi \text{ belongs to } \sigma \quad P \text{ belongs to } \sigma}{\Pi \text{ belongs to some } P} \; (9)$$

$$\therefore \frac{\Pi \text{ belongs to all } \Sigma \quad P \text{ belongs to } \sigma}{\Pi \text{ belongs to some } P}$$

[combining the two steps]

∴ Datisi [by *E'*]

Bocardo (An. Pr. A6, 28b21)

$$\frac{P \text{ belongs to all } \Sigma}{P \text{ belongs to } \sigma}$$

$$\frac{\Pi \text{ does not belong to } \sigma \quad P \text{ belongs to } \sigma}{\Pi \text{ does not belong to some } P}$$

$$\therefore \frac{\Pi \text{ does not belong to } \sigma \quad P \text{ belongs to all } \Sigma}{\Pi \text{ does not belong to some } P} \; (10)$$

[combining the two steps]

∴ Bocardo [by *E'*]

This account surpasses our previous one in making the ecthetic proof of conversion independent of Darapti, and in the elegance of its ecthetic proofs. Moreover, the ecthetic proof of Darapti is closer to Aristotle's own, in selecting a variable different from those in the syllogism to be proved. It is also preferable to the suggestion of Ivo Thomas (*Dominican Studies* 1950, pp. 183-4) according to which the ecthetic proof of Darapti relies on the thesis

$$\frac{\Pi \text{ belongs to all } \Sigma \quad P \text{ belongs to all } \Sigma}{(\Pi \text{ and } P) \text{ belongs to all } \Sigma}$$

whose consequent has a conjunctive predicate, as well as on syllogisms with individual variables. There is in the *Analytics* no logic of complex terms.

The whole of Aristotle's syllogistic can be based on this ecthetic system. Conversion is provable as above, Barbara reduces indirectly to Bocardo, Celarent to Datisi, and the law of Identity is derivable from «A belongs to α» by *E*, if we represent logical truths as implied by zero premisses. Finally, since «A belongs to all B» implies «A belongs to β», which implies «A belongs to some B», we have the laws of subalternation.

This gives rise to a problem: if, in effect, *all* of Aristotle's syllogisms are provable via ecthesis, why did he not adopt such an ecthetic system instead of mentioning just a handful of ecthetic proofs by way of footnote to a system which in no way relies essentially on ecthesis ?

Łukasiewicz (*op. cit.* § 3) claims that the *Analytics* simply contain no syllogisms with terms or variables for individuals. Patzig (*Die Aristotelische Syllogistik* § 3) has shown this to be an exaggeration. Still, it remains true that Aristotle on the whole excludes such syllogisms from consideration: even if the interpretation of ecthesis just suggested is correct, and all ecthetic variables are individual-variables, ecthesis itself occupies only a peripheral place in Aristotle's thought, and in his official division of the kinds of protasis at *An. Pr.* A1, 24a 16-22, singular forms are not mentioned at all.

In attempting to explain this neglect on Aristotle's part, Łukasiewicz refers to a passage (A27, 43a25-43) in which all beings are divided into those which like 'sensible particulars' are not predicated of anything universally, those predicable of others but having nothing more general predicated of them, and those that are both subjectible to more general and predicable of less general beings; scientific questions, Aristotle says, mostly concern members of the third class. Łukasiewicz understands Aristotle to be restricting the range of the variables in syllogistic protases to members of the third class, because in every figure some variable occurs n both subject- and predicate-positions. This is true, but shows nothing.

First, as Patzig says, it is also true that in every figure some variable occurs in only one of these positions, so by parity of reasoning one should interpret the above passage as including terms of all three classes in the range of the variables. Second, (as our ecthetic system shows) even if individual-variables are not allowed in predicate-position the doctrine of the figures is compatible with the introduction of individual-variables: and since, for the purpose of reckoning figures, we can count a restricted individual-variable as the same variable as its associated term-variable, it is even possible to have the same variable now in subject-position (*qua* term- or individual-variable) and now in predicate-position (*qua* term-variable). (This happens with the *middle* in Figures III (a) and III (b)). Third, Łukasiewicz assumes (correctly) that every syllogistic protasis is capable of having *true* propositions substituted for it, but he also assumes that there is something in the nature of the syllogism which restricts the substituends for the variables to terms that are truly subjectible to *more general* terms and truly predicable of *less general* ones. And this is entirely unwarranted. Every such substituend must be truly subjectible and truly predicable in a proposition of the universal affirmative form, because it must satisfy the law of Identity that A belongs to all A; but there is nothing in Aristotle's syllogistic to prevent the term 'being' (than which nothing is more general) from being substituted for some term-variable.

Patzig too thinks Aristotle wants to confine substituends for the variables to terms truly subjectible to more general ones and truly predicable of less general ones. In support of this he cites A27-28 in which Aristotle says that to prove a proposition of the form «A belongs to all B» we should try to find a term common to the subjects of A and the predicates of B — thus simply *assuming* that A is truly predicable and B truly subjectible in a universal affirmative proposition. Ackrill ably criticized this view in his review of Patzig (*Mind*, 1962 p. 109). The essential point is that the text does not clearly require that A truly apply to a *less general* subject and B be truly subjectible to a *more general* predicate. Nor can Aristotle have intended this; for he is perfectly willing to countenance syllogisms in which all three terms are truly predicable of one another in universal affirmative propositions. (Cf. B5; B22, 67b27-32; *An. Post.* A3, 73a6-20).

Aristotle's neglect of singular forms and propositions should probably be explained as follows. He saw that some of his syllogisms could be proved by ecthesis; but he did not want to make this process an integral part of his system because he saw that it used individual-variables. For, he held that there is no demonstrative knowledge of singulars, (apart from the passage mentioned earlier cf. A23, 40b23-25 and *An. Post.* A31.) and he also constructed the syllogistic system as a prolegomenon to the theory of demonstrative knowledge in the *Posterior Analytics,* intending the syllogism as the instrument of demonstration. (The first sentence of the *Analytics* states that «our subject is demonstration». A1, 24a10-11.) Given that this was his purpose, there would simply be no need for him to pursue the investigation of syllogisms containing individual-variables.

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