ON INCOHERENT QUANTIFICATION IN LANGUAGES WITHOUT CONSTANTS

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Quine's favorite argument that quatification is incoherent in quantified modal logic turns on the failure of substitutivity of identity for names and descriptions in certain modal contexts. But, as is well known, Quine holds that constants and descriptions are theoretically eliminable. So if the incoherence of quantification is a problem worth worrying about, it ought to appear in a language devoid of singular terms except variables; in other words, if incoherent quantification depends *essentially* on the presence of names and descriptions, then it should cease to be a problem when these terms are eliminated. Quine, in fact, maintains that incoherent quantification in modal logic cannot be avoided in this way; quantification is objectionable even if no singular terms except variables are available. In his reply to Sellars in *Words and Objections,* he writes:

> If a position of quantification can be objected to on the score of failures of substitutivity involving descriptions, then it remains equally objectionable when no singular terms except variables are available.
>
> (Quine (8), p. 339)

I shall examine Quine's argument for the passage just cited. I present his argument uncritically and then offer three interpretations of it. None of the formulations succeeds entirely in overcoming the difficulties that arise from a closer scrutiny of the original argument. The first two rest on dubious premises. The third, while somewhat more promising, does not fit very well with other views held by Quine, and it raises perplexing questions about the locus and character of Quine's criticism of quantified modal logic.

I

In his reply to Sellars, Quine questions the coherence of statements like

(1) (Ex)N(x is odd),

in languages containing no singular terms except variables. (Also see Quine (5), p. 149) Thus he asks: of which object is the open sentence

(2) N(x is odd),

true ? A seemingly plausible answer is that (2) is true of the object uniquely specified by

(3) (Ey) $(y \neq x = yy = y + y + y)$.

Why ? Because (3) entails

(4) x is odd,

and so, in Quine's words, "evidently sustains" (2). But if (2) is true of the object uniquely specified by (3), it is also true of the object uniquely specified by

(5) there are x planets,

because (3) and (5) uniquely specify the same object. Unlike (3), however, (5) does not entail (4), and so does not sustain (2). Since (3) and (5) specify the same object, we are forced to the paradoxical conclusion that (2) is both true and not true of one and the same object. Presumably, the same kind of question can be raised about any object of which (2) is supposed to be true. Therefore, the semantical intelligibility of (2), hence (1), is questionable even in languages in which the *only* singular terms are variables.

II

The word "sustains" is a key one in Quine's argument. What does it mean ? The logically relevant definition of "to susain" is "to support as true". Since the relation of support intended presumably is a logical one and since (2) and (3) are open sentences, to say that (3) sustains (2) is to say that (2) is true of whatever (3) is true of. Formally, this claim may be rendered as

(6) $(x)\ (Rx \supset NOx)$.

Similarly, the claim that (5) does not sustain (2) may be rendered as

(7) $-(x)\ (Px \supset NOx)$.

(Here "Rx", "Ox", and "Px" are used in place of (3), (4), and (5) for the sake of greater prespicuity.) The claim that (3) and (5) uniquely specify the same object may be represented by

(8) $(x)\ ((Px \equiv Rx)\ \&\ (y)\ (Py \equiv y = x))$.

By classical principles, (7) implies

(9) $(Ex)\ (Px\ \&\ -NOx)$.

By simplification and transitivity, (6) and (8) yield

(10) $(x)\ (Px \supset NOx)$.

And, by classical principles again, (9) and (10) imply

(11) $(Ex)\ (NOx\ \&\ -NOx)$.

Thus, it follows, from assumptions (6) - (8), that there is some object of which the sentence "NOx" is both true and not true; and this seems to be the sort of quandary that Quine has in mind.

Unfortunately for the present formulation of the argument, premise (6) is open to question. Though it may be granted that "(x) (Rx ⊃ Ox)" is true — even necessarily true — it does not follow that (6) is true. On this treatment of "sustains", Quine's inference from "(3) entails (4)" to "(3) sustains (2)" — that is, premise (6) — is suspect.

In reply to this criticism, it may be urge that the apparent invalidity of the inference merely reveals its enthymematic form. Indeed there are plausible additional premises which make the argumet valid. One such premise is

(12) (x) (Rx ⊃ NRx).

The plausibility of (12) rests on the fact the "Rx" expresses a numerical property of numbers. That a given number satisfies "Rx" follows from its having the place in the number series that it does. Further, it can reasonably be argued that the place occupied by a number in the number series is an essential, hence necessary, property of that number. (Kaplan (3), p. 223) Since having the property R follows from this necessary property of the number, it follows that if a number has R, then it has it necessarily. (1)

Employing (12), the inference from "(3) entails (4)" to "(3) sustains (2)" can be justified. The entailment claim is rendered as

(13) N(x) (Rx ⊃ Ox).

By the converse Barcan principle, (13) implies

(14) (x) N (Rx ⊃ Ox).

and, using modal distribution, (14) yields

(15) (x) (NRx ⊃ NOx).

By transitivity, (12) and (15) imply premise (6) — that is,

(16) (x) (Rx ⊃ NOx).

But this strategem is suspect for several reasons. In the first place, the need for additional assumptions — in the present case, the distribution law and the converse Barcan principle — is nowhere even suggested by Quine. Second, this proposal makes the argument turn on a very special property of the predicate "R"; namely, that "R" is the sort of predicate such that anything in its extension is necessarily in its extension. Quine gives us no reason to believe that his argument turns on any such feature of the uniquely specifying predicate "R".

Finally, Quine no doubt intends his argument to be telling against quantified modal formulas generally; for example, against

(16) (Ex) N (x is a rational being).

Is the proposed response plausible in this case ? For it to be satisfactory, there must be open sentences "Gx" and "Hx" which uniquely specify a given object and such that "Gx" entails "x is a rational being" while "Hx" does not. Moreover, to parallel the crucial assumption (12), "G" must be such that if some unique individual has G, then it necessarily has G. But in the case of concrete particulars, there appear to be no such conditions — a being's name, time and place or origin, thoughts, desires might well have been different from what they are. (²) So, it seems to me that this response is in the end unsatisfactory and that this formulation of Quine's argument is untenable.

III

A second way of interpreting Quine's argument is suggested by a reconsideration of its context. Recall that his main claim is in the form of a conditional: "If a position of quantification can be objected to on the score of failures of substitutivity involving descriptions, it remains equally objectionable when no singular terms except variables are available." Quine envisions first a language in which quantification is objectionable because of failures of substitutivity involving descriptions; for example (using our earlier abbrevations), a language in which

(17) (Ix)Rx = (Ix) Px,

(18) NO(Ix)Rx,

are true, but

(19) NO(Ix)Px,

is false. (Here "I" is the description operator) Since (18) is true, it is alleged that "NOx" is true of the number denoted by "(Ix)Rx", and since (19) is false, "NOx" is not true of the number denoted by "(Ix)Px". Here we have a fairly clear sense in which the number denoted by "(Ix)Rx" may be said to sustain "NOx"; namely, that number is such that "NOx" is true of it.

How might these considerations be used to show that quantification is unintelligible in modal languages containing only variables? The truth of (18) assures that "NOx" is true of (Ix)Rx. If the procedure for eliminating definite descriptions is a good one, as Quine thinks Russell's is, then the sentence replacing (18) on elimination should assure that "NOx" is true of the object uniquely specified by "Rx". Similarly, the falsehood of (19) assures that "NOx' 'is not true of (Ix)Px. By parallel reasoning, the sentence replacing (19) on elimination should assure that "NOx" is not true of the number uniquely specified by "Px". However, since "Px" and "Rx" uniquely specify the same number, it follows that there is some number such that "NOx" is both true and not true of it — a paradoxical consequence of the sort Quine sems to have in mind.

Although such an interpretation is trongly suggested by Quine's way of phrasing his main claim, it, too, is unsatisfactory. Even if the argument presented is sound, it raises difficult questions about the scope and nature of Quine's criticism of modal logic. First, if the conditional character of his claim about quantification is taken seriously, then it is not clear that his criticisms apply to quantified modal logics in general. Suppose a modal logic is constructed which, *ab initio,* contains no singular terms except variables. Is quantification in this language objectionable ? Certainly not because of failures of substitutivity involving descriptions. So, either quantification is intelligible in some modal logics, or some other grounds for objection must be given.

Second, it would be somewhat anomalous for Quine to base his objections to quantification on what happens in languages containing descriptions. From his point of view, these terms are theoretically superfluous. Hence, objections based on same would be objections based on what is, for him, a dispensable part of the formal system.

More serious than these internal criticisms, however, are questions that can be raised about the soundness of the argument. Two crucial claims required by the present formulation of the argument are these:

(a) Prior to using the elimination procedure, (17) - (19) constitutes a failure of substitutivity.

(b) The statement that replaces (18) assures that "NOx" is true of the object uniquely specified by "Rx".

Now, Smullyan imitating Russell, argues that the definite descriptions in (18) and (19) are ambiguous as to scope. (Smullyan (10)) On the one hand, (18) and (19) may be rendered as having wide scope:

(18') (Ix)Rx NO(Ix)Rx,

(19') (Ix)Px NO(Ix)Px.

Or they may be rendered as having narrow scopes:

(18") N (Ix)Rx O(Ix)Rx,

(19") N (Ix)Px O(Ix)Px.

Whether the present formulation is sound can thus be determined by deciding if either rendering of scopes makes both (a) and (b) above true.

The wide scope readings do not satisfy the needs of Quine's argument because the statements (17), (18'), and (19') do not constitute a failure of substitutivity. Smullyan has shown that eliminating the descriptions from these statements by Russell's method yields an argument that is quantificationally valid. So, if the elimination method is a good one, the wide scope readings fail to satisfy point (a).

Do the narrow scope readings satisfy (a) ? They seem to because elimination of the descriptions by Russell's method yields an argument which is not valid in any standard system of modal logic. So, if the elimination method is a good one, it is reasonable to suppose that the statements (17), (18''), and (19'') represent a failure of substitivity in the language which contains definite descriptions.

But do the narrow scope readings satisfy point (b) ? They do only if the statement replacing (18'') on elimination assure the "NOx" is true of the object uniquely specified by "Rx". For (18''), the elimination statement is

(20) $N((Ex)((y)(Ry \equiv y = x)) \& (z)(Rz \supset Oz))$.

Suppose that M is the number which is in fact uniquely specified by "Rx". Statement (20) implies

(21) $N(z)(Rz \supset Oz)$;

so necessarily if "Rx" is true of something, then so is "Ox". Thus, "Ox" is true of M. But, of course, this fact does not imply that "NOx" is true of M. Prima facie then, the narrow scope readings fail to satisfy point (b).

The temptation to suppose that they do satisfy (b) may well arise from the fact that the wide scope readings do satisfy it. The elimination statement for (18') is

(22) $(Ex)(y)(Ry \equiv y = x) \& (z)(Rz \supset NOz)$

Since (22) implies

(23) (z) (Rz ⊃ NOz),

it follows that "NOx" is true of M, the number uniquely specified by "Rx." So if one supposed that (20) implies (22), one would suppose that the narrow scope readings satisfied both points (a) and (b). But (20) implies (22) only if (21) implies (23). And this, of course, is the very problem which led to the demise of the first formulation of Quine's argument; the difficulties raised there would also apply in the present case.

In summary, the second formulation is inadequate as soon as the scope distinctions involving definite descriptions are brought to light. If this formulation accurately represents Quine's intentions, then he might simply have confused the logical powers of (20) (or (21)) with (22) (or (23)). Although prima facie unlikely, such an oversight does seem to occur in at least one similar example discussed by Quine, namely, the case of the cycling mathematician. ([3]) (Quine (9), p. 199) Alternatively, he may be clear about the distinction but sup pose that (21) implies (23). But then the present formulation of the argument reduces to the immediately preceding formulation and is subject to the defects noted there.

IV

The final formulation of the argument concentrates on Quine's contention that "Rx" entails "Ox". Formally rendered, this claim is

(24) N(x) (Rx ⊃ Ox).

The claim that "Px" does not entail "Ox" may be rendered as

(25) — N(x) (Px ⊃ Ox).

Again "Rx" and "Px" uniquely specify the same object;

$$(26)\quad (x)((Px \equiv Rx) \;\&\; (y)(Py \equiv y = x).$$

This statement in turn implies

$$(27)\quad (x)(Px \equiv Rx).$$

I assum the availability of the principle of predicate substitutituvity, a principle whose adoption Quine has urged (Quine (8), pp. 333-4):

$$(28)\quad (x_1) \ldots (x_n)(A \equiv B) \supset (C \equiv D).$$

Here D is like C, except for containing B at some places where C contains A, and $x_1,\ldots,x_n$ exhaust the variables with respect to which the occurrences of A and B are bound in C and D respectively. (Quine (6), p. 98).

An instance of (28) is

$$(29)\quad (x)(Px \equiv Rx) \supset .\ N(x)(Rx \supset Ox) \equiv N(x)(Px \supset Ox).$$

Statements (24), (27), and (29) yield

$$(30)\quad N(x)(Px \supset Ox),$$

and (30) contradicts (25). Perhaps, it is a conflict of this sort that Quine has in mind when he talks about the incoherence of quantification in quantified modal logic.

Such an interpretation raises three questions. First, is it at all plausible to place this construction on Quine's argument in *Words and Objections*? Second, does this formulation of the argument fit with other views held by Quine? Third, is the argument so construed sound? My subsequent remarks will be limited mainly to the first two questions.

The present formulation does not turn on, and in fact ignores, Quine's claim that "Rx" sustains "NOx". Does it thereby leave out an indispensable part of Quine's argument? There are grounds — though not incontrovertible ones — for saying that it does not. In my first formulations of the argument, I construed the word "sustains" in a straightforward way, as meaning, in effect, "makes true". But that interpretation led to insoluble difficuliets about the soundness of Quine's argument. Perhaps a reconsideration of this straight forward treatment is in order. For exemple, might not Quine's claim that "Rx" sustains "NOx" merely be another way of saying that "Rx" entails "Ox" ?

Some support for this contention comes from the way Quine states a similar argument in "Reference and Modality". There he writes:

> Whatever is greater than seven is a number, and any given number x greater than seven can be uniquely determined by any of various conditions, some of which have 'x is greater than seven' as a *necessary consequence*, and some of which do not. One and the same number x is uniquely determined by the condition:
>
> (32) $x = \sqrt{x} + \sqrt{x} + \sqrt{x} \neq \sqrt{x}$,
>
> and by the condition:
>
> (33) There are exactly x planets,
>
> but (32) has 'x is greater than seven' as a *necessary consequence* while (33) does not.
>
> (Quine (5), p. 149, emphasis mine)

In this argument, no mention is made of conditions sustaining, or failing to sustain, "N(x is greater than seven)". All he says is that "x is greater than seven" is a necessary consequence of (32) but not of (33). While the phrase "necessary consequence" is not entirely clear, surely the most natural interpretation of Quine's claim is that "x is greater than seven" is entailed by (32) but not by (33). Given the similarity of this argument to the one in *Words and Objections*, it seems not unreasonable to regard "'Rx' sustains 'Nox'" as just another way of saying the "Rx" entails "Ox".

Let us consider whether this formulation of the argument harmonizes with other Quinean views. In "Three Grades of Modal Involvement", Quine distinguishes between a second grade of modal involvement, in which "necessarily" is accepted as an operator on closed sentences, and a third grade, in which it is accepted as an operator on sentences, both open and closed. Quine is not particularly pleased with the prospects of even second-degree modal languages. For example, they make for idle speculation about iterated modality; they tempt us to condemn the material conditional by wrongly associating it with "implies"; and, most importantly, they tempt us to go a step further and accept a third-degree operator, a step which Quine considers disastrous. But it is important to notice what Quine does not say about such languages. He does not claim that quantification in such languages is incoherent or that such languages are committed to essentialism. These complaints are reserved exclusively for languages containing third-degree modal operators.

Of course, second-grade languages may contain quantifiers. A language of this kind differs from ordinary quantified modal logic by requiring that the modal operator attach only to statements. Consequently,

(31) Necessarily (Ex) (x is greater than seven),

is well-formed, whereas

(32) (Ex) necessarily (x is greater than seven),

is not. How does Quine view second-degree languages of this kind ? His remarks conform to the views expressed in "Three Grades of Modal Involvement". For example, in "Reference and Modality", he writes:

Note that

(30) (Ex) necessarily (x is greater than seven),

and

(31) (Ex) necessarily (if there is life on the Evening Star then there is life on x),

are not to be confused with:

Necessarily (Ex) (x is greater than seven),

Necessarily (Ex) (if there is life on the Evening Star then there is life on x),

which present no problems of interpretation comparable to (30) and (31).

Quine (5), p. 148)

This again suggests that Quine sees a crucial difference between second degree modal languages — even with quantifiers — and third degree ones. His harshest criticisms — unintelligible quantification, committment to essentialism — are reserved exclusively for the latter.

But if Quine's argument is accurately represented by the present formulation, then it is difficult to see why he should attach so much importance to the distinction between second and third degree languages. For the present formulation does not involve the use of third-grade modal statements; throughout the argument, the modal operator attaches only to closed sentences. Hence, this sort of argument could be carried through in a second grade modal language. So if Quine's argument, as presently formulated, did show that quantification was incoherent, it would follow that quantification was incoherent in some second grade modal logics. And this would imply that the incoherence of quantification in modal logic was not attributable merely to the attempt to quantify into referentially opaque contexts from outside them. Of course, these conclusions are flagrantly opposed to most of Quine's published work on the subject and would, if true, require a thorough reappraisal of the nature of Quine's criticism of modal logic.

Finally, is the third formulation of Quine's argument sound ? I have little to say about this matter, except to locate the main point of contention. That point is the insistence on the principle of predicate substitutivity. Faced with the present argument, proponents of modal logic are sure to ask: "What is so sacrosanct about predicate substitutivity ? Why should we insist that it be preserved ? Give it up — what becomes of your argument then ?" Since Quine views failures of predicate substitutivity as failures of extensionality, his answer to these questions typically takes the form of a defense of extensionality or extensional languages. He was suggested a number of defenses. The latest is that a Tarskian definition of truth cannot be given for intensional languages. (Quine (8), pp. 333-4) Although I do not find his argument on this matter convincing, it does involve difficult issues, ones which lie beyond the scope of the present paper. (4)

NOTES

(*) Thanks are due to Karel Lambert and Peter Woodruff for help in the preparation of this paper.

(1) Although this argument involves appeal to essential properties of numbers, it may still be acceptable to Quine; witness his remarks on numbers and essentialism in his reply to Kaplan in Quine (8), p. 343.

(2) Chisolm argues this point convincingly in (1), and Quine seems to agree in his remarks on essentialism and concrete particulars in Quine (8), p. 343.

(3) These confusions are sorted out and criticized in Marcus (4).

(4) For discussion and criticism of this contention, see Grandy (2).

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