NOTES TOWARD AN AXIOMATIZATION OF RECKLESSNESS

State University of New York at Buffalo
Dpt. of Philosophy
4244 Ridge Lea Road Buffalo New York 14226

Recently the concept of recklessness has been attracting increased attention in the philosophical literature. As a result of investigations by this author, it seems possible to adequately represent the logic of recklessness in axiomatized form. The resultant clarity and sense of manageability seem to make such efforts worthwhile. The business of this paper is to present initial efforts toward such an axiomatization. At the end of the paper, the symbolic apparatus developed will be applied in a resolution of a current disagreement concerning recklessness, as an example of the benefits to be derived.

Recklessness is normally predicated of acts or actions. Let 'R' be a monadic sentential operator which when applied to sentential well-formed formulas yields in turn a sentential well-formed formula. 'R' thus represents, roughly, «... is (or was) reckless.» Let 'p', 'q', 'r', ... be sentential variables which range over statements of acts or actions. Then «Rp» states that the action of p is reckless.

As an example, suppose that «p» is «John weaves in and out of traffic». Then «Rp» would normally be translated by the nominalization of «p» and predicating recklessness of it, i.e. «John's weaving in and out of traffic was reckless.» Equivalent translations might be «It is (or was) reckless of John to weave in and out of traffic», and so on.

Reckless acts invariably have a condition of recklessness, that is, some specifiable state of affairs which is the ground or reason of their recklessness. For example, the reason that John's weaving in and out of traffic is reckless may be that it unnecessarily risks injury of death. A condition of recklessness of an act occurs when the risk involved in that condition is not justified by anything which might be achieved through that act. Let 'C' be a dyadic sentential operator representing the relation of the condition of recklessness to the reckless act. «Crp» thus means that r is a condition of recklessness of p, that is, if «r» is true, then p is (or would be) reckless to perform. More precisely, «Crp» means that r is a sufficient condition of the recklessness of p.

It should be noted that a condition of recklessness need not itself be an act, though in some cases it could be. For the sake of uniformity of the interpretation of sentential variables, let the range of sentential variables accordingly be extended so as to include all sentences, whether a statement of action or not. Such an extension has the odd (but tolerable) result that 'Rp' is well-formed even when «p» is not an action. We need to stipulate only that if 'p' stands for anything but an action, then «Rp» is false.

We may now set forth initial axioms governing the operators 'R' and 'C'.

Ax. 1. $\sim(p)(Rp \vee R\sim p)$
Ax. 2. $Rp \equiv (\exists r)(Crp \,\&\, r)$
Ax. 3. (Addition) $Rp \supset R(r \,\&\, p)$
Ax. 4. (Simplification) $(R(p \,\&\, q) \,\&\, \sim Rp) \supset Rq$
Ax. 5. (Distribution I.) $[R(p \,\&\, q) \,\&\, \sim(Cpq \vee Cqp)] \supset (Rp \vee Rq)$
Ax. 6. (Distribution II). $R(p \vee q) \supset (Rp \,\&\, Rq)$
Ax. 7. (Negation) $R\sim p \supset \sim Rp$

Discussion and examples will help clarify the sense of these axioms. Ax. 1. is a denial of the law of excluded middle with respect to recklessness. For example, neither driving down the street nor not driving down the street need be reckless. Ax. 2. states that a necessary and sufficient condition for the recklessness of an act is that there is a condition of recklessness of that act and that condition obtains. ('Crp & r' is to be read that r is a condition of recklessness of p, and «r» is true). The axiom of simplification makes it possible to infer from the recklessness of John's drawing his pistol and firing and from the fact that drawing his pistol is not reckless, that his firing is reckless. According to the first axiom of distribution, if John's driving down the highway and weaving in and out of traffic is reckless, this entails either the recklessness of driving down the highway or the recklessness of weaving in and out of traffic. Here it is necessary to add the qualification that one of the conjuncts is not a condition of recklessness of the other, for omitting it leads to incorrect inferences, e.g., as from the recklessness of Mary turning up the stove heat under a pan of grease and leaving the kitchen to infer that either turning up the heat is reckless or leaving the kitchen is reckless. Neither may *itself* be reckless, but rather the recklessness of her leaving the kitchen may be *due* to her turning up the heat. The second axiom of distribution states that if it is reckless to perform either of two actions, then both are independently reckless. Finally, the law of negation with respect to recklessness states that if it is reckless not to perform p, it cannot be reckless to perform p. Note that the conrary does not hold. It may not be reckless to drive down a particular street, for example, but also not reckless not to drive down that street.

Certain theorems may now be proven without much difficulty, employing in addition to our axioms the usual axioms and rules of protothetics.

Theorem 1. $\sim(Rp \,\&\, R{\sim}p)$

Proof: Suppose (1) $Rp \,\&\, R{\sim}p$.

Then from the second conjunct of (1) and Ax. 7,

(2) $\sim Rp$.

But then from the first conjunct of (1) and (2)

(3) $Rp \,\&\, {\sim}Rp$

which is impossible.

Theorem 2. $\sim R(p \vee {\sim}p)$

Proof: Suppose (1) $R(p \vee {\sim}p)$.

Then by Ax. 6,

(2) $Rp \,\&\, R{\sim}p$.

But this contradicts Thm. 1.

Theorem 3. $Cp{\sim}p \equiv C{\sim}pp$

Proof: Suppose (1) $Cp{\sim}p$

Then substituting '$\sim$p' for 'p'

(2) C$\sim$p$\sim\sim$p.

And by Double Negation

(3) C$\sim$pp.

So Cp$\sim$p$\supset$C$\sim$pp, by (1) through (3).

'Cp$\sim$p' similarly follows from 'C$\sim$pp'.

Theorem 4: $\sim$Cp$\sim$p (No action can be a condition of recklessness for not performing that same action.)

Proof: Suppose (1) Cp$\sim$p.

Then if (2) p,

then (3) Cp$\sim$p & p.

So by E.G. (4) ($\exists$r) (Cr$\sim$p & r).

So by Ax. 2 (5) R$\sim$p.

And if (6) $\sim$p

then from (1) and Thm. 3,

(7) C$\sim$pp,

and from (6) and (7), (8) C$\sim$pp & $\sim$p.

So by E.G.

(9) ($\exists$r) (Crp & r)

i.e. by Ax. 2,

(10) Rp.

But

(11) pv$\sim$p.

So from (2) through (5), (6) through (10), (11), and constructive dilemma,

(12) RpvR$\sim$p

By U.G. on (12)

(13) (p) (RpvR$\sim$p).

But this contradicts Ax. 1,

So (14) $\sim$Cp$\sim$p.

Theorem 5: $\sim$($\exists$r) (Cr(pv$\sim$p) & r)

Proof: Follows directly from Thm. 2. and Ax. 2.

Theorem 6: $\sim$R(p & $\sim$p)

Proof: Suppose (1) R(p & $\sim$p).

Now from theorems 3 and 4,

(2) $\sim Cp \sim p \ \& \ \sim C \sim pp$.

From (2) by DeMorgan's law,

(3) $\sim (Cp \sim p v C \sim pp)$.

So by (1), (3) and Ax. 5,

(4) $Rp v R \sim p$.

By U.G. on (4),

(5) $(p) (Rp v R \sim p)$

which contradicts Ax. 1.

So (6) $\sim R(p \ \& \ \sim p)$.

Theorem 7: $\sim (\exists r) (Cr(p \ \& \ \sim p) \ \& \ r)$

Proof: Follows directly from Thm. 6 and Ax. 2.

Theorem 8: $[R(p \ \& \ q) \ \& \ (r) (Crp \supset \sim r)] \supset Rq$

Proof: Suppose (1) $R(p \ \& \ q) \ \& \ (r) (Crp \supset \sim r)$.

Also suppose (2) $(\exists r) (Crp \ \& \ r)$.

Then from (2) by E.I.

(3) $Crp \ \& \ r$.

From (1) by (ordinary) simplification

(4) $(r) (Crp \supset \sim r)$.

And by U.I. on (4),

(5) $Crp \supset \sim r$

which contradicts (3). So

(6) $\sim (\exists r) (Crp \ \& \ r)$

which by Ax. 2 gives

(7) $\sim Rp$.

Now by simplification on (1), and (7)

(8) $R(p \ \& \ q) \ \& \ \sim Rp$.

By (8) and Ax. 4,

(9) Rq. Q.E.D.

In some instances it may not be an act but recklessness of the act which is a condition of recklessness of another. For example, if Mary's turning up the stove heat to high under a pan of grease is reckless, then it is the fact of that recklessness which *a fortiori* makes it further reckless to leave the kitchen. Let us symbolize this relation of recklessness entailment as

'$Rp \rightarrow Rq$'. The following axiom seems sufficient to govern recklessness entailment:

Ax. 8. (Recklessness Entailment)

$$(Rp \rightarrow Rq) \supset (r)\,[(Crp \,\&\, r) \supset Crq]$$

Certain further theorems now follow immediately, e.g.

Theorem 9: $(Rp \rightarrow Rq) \supset (Rp \supset Rq)$ [1]

Proof: Suppose (1) $Rp \rightarrow Rq$
and (2) Rp.
Then from (2) by Ax. 2,
(3) $(\exists r)\,(Crp \,\&\, r)$.
By E.I. on (3),
(4) $Crp \,\&\, r$.
From (1) and Ax. 8,
(5) $(r)\,[(Crp \,\&\, r) \supset Crq]$.
By U.I. on (5),
(6) $(Crp \,\&\, r) \supset Crq$.
By modus ponens on (4) and (6)
(7) Crq.
So by simplification on (4), and (7)
(8) $Crq \,\&\, r$.
E.G. on (8) gives
(9) $(\exists r)\,(Crq \,\&\, r)$
which with Ax. 2 gives
(10) Rq. Q.E.D.

Should the logic of recklessness be extended beyond protothetics to the predicate calculus ? The following considerations make it clear that it should be. As an example, suppose «p» means «The policeman fired at the jaywalker.» Then there is a danger of ambiguity in «Rp». This may be intended, in accord with the custom up to now, to be a symbolization of «It was reckless of the policeman to fire at the jaywalker,» but also possibly «The policeman fired recklessly at the jaywalker.» The sense of these two must be distinguished. It might be reckless of him to fire because of the unjustifiable risk of injuring an innocent bystander. But firing recklessly has a different sense, shooting from the hip in a wild or haphazard fashion, that is, wihout taking careful aim.

It is necessary that these differences be reflected in a difference in symbolization. Let 'Rp', as before, stand for the first sense. Now let 'R' also be applicable to *predicates* in the following way: if 'P' is a predicate (of action [2]) then 'RP' represents the reckless performance of that action (e.g. in the sense of «firing recklessly»). Then '(RP)*a*' is to be translated, to continue the example, as «*a* fired recklessly», the second of the senses above.

In order to bring Ax. 8 to bear on these two senses, let us say that «x fires recklessly» («(RP)x») is itself a reckless act, in the appropriate sense, i.e. symbolizable as 'Rq.' [3] Clearly, «Rp ≡ Rq» is false. Yet we may further ask, is either «Rp → Rq» or «Rq → Rp» true ? According to Ax. 8 Rp → Rq only if all conditions of recklessness of p are conditions of recklessness of q. But this is not so; for example, a condition of recklessness of firing may be the proximtiy of innocent bystanders, but this is not a condition of firing recklessly. Conversely, firing from the hip is a condition of firing recklessly but not of the recklessness of firing at all. Consequently, neither «Rp → Rq» nor «Rq → Rp» is true.

Nevertheless, firing recklessly is a firing, so if firing is reckless, then firing recklessly is reckless, i.e. «R(Px) → R(RP)x». The converse does not hold. For example, suppose that there are no bystanders nearby so there is no danger of injury to them. Thus it may not be reckless to fire. But still it might be reckless to fire recklessly if, e.g., this would unjustifiably risk the escape of the person fired at. So «R(RP)x → R(Px)» is false. It would be reckless of the policeman to fire recklessly, but not reckless to fire at all.

***

One last axiom will now be added. This will be called the axiom of indifference. [4] In order to express this axiom, three new primitive symbols will have to be introduced. Let 'I' be a dyadic relation symbolizing indifference on the part of a person to a risk. Thus «Ixp» is to be read «x is indifferent to the risk involved in p» (where «p» will normally be a condition of recklessness, involving a risk.) Let 'A' be a second dyadic relation symbolizing awareness, such that «Axp» is read «x is aware of the nature and gravity of the risk involved in p». Finally let 'U' be a triadic relation such that «UxpPx» is read «x thinks (or judges) that his performing P would not justify the risk involved in p.» The axiom of indifference is then formulable as:

Ax. 9. $(CpPx \,\&\, p) \supset [(Axp \,\&\, UxpPx) \supset (Px \supset Ixp)]$

The axiom states that where p is a condition of recklessness of Px which obtains, then if x is aware of the nature and gravity of the risk involved in p and thinks that performing P would not justiy that risk, if he performs P then he is indifferent to the risk.

The following theorem follows trivially:

Theorem 10: $(CpPx \,\&\, p) \supset \{ {\sim} Ixp \supset ({\sim} Px \vee {\sim} UxpPx)] \}$

***

An example which apparently (but only apparently) contradicts the axiom of indifference can be found in a recent article by Winslade: [5]

> Suppose a policeman sees a man who has just jaywalked. The policeman calls to the man to stop; he does. When the policeman approaches the jaywalker and begins to question him, the jaywalker attacks the policeman, throws him to the ground, and runs away. The policeman realizes that it is risky to fire at the jaywalker because there is a crowd in the street. It is reasonable to presume that the policeman cares about injuring innocent bystanders, but he also does not want to let the jaywalker escape-partly as a matter of pride. The policeman fervently hopes and desires not to hit an innocent bystander so he takes careful aim; he fires, misses the jaywalker but hits an innocent bystander.

According to the example, there is a risk in firing which the policeman is aware of, but which he performs anyway although not indiferent to the risk. Does this contradict Ax. 9 ? I think there is a subtle confusion lurking in the example, and that careful adherence to our symbolic analysis would bring that confusion to the surface. The point is that we must distinguish between the policeman's shooting («Pa») and his shooting recklessly («$(RP)a$»). It happens that the proximity of bystanders («p») is a condition of the recklessness of both of these acts, i.e. $CpP\alpha$ and $Cp(RP)\alpha$. Since according to the example, Aap and $\sim Iap$, then from two applications of theorem 10,

$$[A] \quad \sim Pa v \sim UapPa$$

and

$$[B] \quad \sim (RP)a v \sim Uap(RP)a$$

Now again according to the example, $\sim(RP)a$ («he takes careful aim»), so [B] is true. However, Pa, so from [A] we must conclude that $\sim UapPa$.

Consequently, Winslade's example does not contradict Ax. 9. To defend the correctness of the axiom we need only maintain that the policeman could not have thought that shooting (at all) could not justify the risk involved in p, although he may well have realized that shooting recklessly could not justify the risk in p. (In fact, presumably he did since he was careful not to tshoot recklessly.)

In short, the policeman realized the recklessness of his shooting recklessly, but not the recklessness of his shooting at all. This is consistent with the analysis above which leads to the conclusion that «$R(RP)a \rightarrow R(Pa)$» does not hold. If it did, then when the policeman realized the recklessness of his firing recklessly, then in his judgment of the situation he may well have been expected to realize also the recklessness of his firing at all, but not otherwise.

It seems clear that symbolic analysis can continue to help minimize unclarity in future investigations. Aside from application, as matters of intrinsic interest the questions of indenendence, consistency, and completeness of the axioms of recklessness appear to be worthy of further attention.

Charles H. Lambros
SUNY at Buffalo

FOOTNOTES

(1) The converse does not hold. If «$(Rp \supset Rq) \supset (Rp \rightarrow Rq)$» were true, then for example, if q were reckless then the recklessness of *any* act p would entail its recklessness. And then also, by Ax. 8, the condition of recklessness of any reckless act *whatsoever* would be a condition of recklessness of q.

(2) Ordinarily an act is expressed through a verb phrase (e.g. «weaves in and out of traffic»), not a predicate. However, such verb phrases can be uniformly transcribed into predicates (e.g. «one who weaves in and out of traffic») so that the predicate calculus is sufficient to handle them.

As before, if 'P' is a predicate which is not a rewriting of an action verb, then «(RP)x» is simply false for all values of 'x'.

(3) In one sense it is consistent with the above analysis and axioms to rewrite '(RP)x' as 'Rq'. For example, there are specifiable conditions (firing haphazardly, etc.) for reckless firing. This is consistent with Ax. 2. But it is not consistent in another sense in that up to this point in the text it is assumed that there is, invariably, a risk connected with that condition. But there does not seem to be any such risk in firing haphazardly, (unless reckless firing is itself reckless for another reason). It does not seem advisable, therefore, to uniformly construe '(RP)x' as symbolizable also as 'Rq'. Nevertheless, the point made in the text through such symbolization does not turn on this inconsistent sense.

(4) For the content of this axiom and for several other related matters, I am indebted to Professor James B. Brady.

(5) William J. Winslade, «Recklessness», *Analysis*, Vol. 30, March, 1970, pp. 135-140.