THE STONE-TAMMELO DEONTIC LOGIC

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I

Legal systems are sometimes distinguished as closed or open according as they do or do not endorse the maxim, what is not forbidden is permitted. Prof. J. Stone has recently argued ([1]) that if we accept this distinction then von Wright's 1951 deontic logic (DL) ([2]) has certain shortcomings, as follows.

1. It is adequate only for the «logical apprehension» of closed legal systems.
2. The modality, indifferent, of DL is not adequate for expressing the absence of law or legal neutrality because it really means, under a legal interpretation, permissory by virtue of a licensory norm and not obligatory.
3. For closed legal systems we need only the modalities obligatory and permissory, distinguishing within the latter the «sub-modality», licensory, to cover those permissory cases arising from a licensory norm.
4. For open legal systems we need a further modality, the allowable, covering both the permissory and the deontically neutral.
5. For closed legal systems the licensory is a «sub-modality» of the permissory; for open legal systems the neutral and the permissory, and therefore the licensory, are submodalities of the allowable.

I shall argue that any distinction between open and closed legal systems is an extra-logical matter, and that the modalities licensory, allowable and neutral can safely be dispensed with.

As the basis for my arguments I shall use a propositional

version of DL and the Hohfeldian theory of legal concepts. ([3]) Since the latter is an informal legal precursor of DL, any shortcomings of DL are implicit in Hohfeld, and any justification of DL applies equally to the Hohfeldian theory. Most theories about law, including that of Hohfeld and also that part of Stone's work considered here, can be regarded as theories dealing with what follows from legal rules. Deontic logic can also be regarded as the logic of what follows from social rules (legal, moral and otherwise). Hence the variables of a nonquantificational DL can be thought of as ranging over propositions ([4]) (Fregean sense of indicative sentences). This makes Anderson's versions of DL particularly appropriate; it also makes available the semantic techniques of Kripke ([5]) and Hintikka ([6]) for proving the consistency and completeness of DL.

II

A propositional version of DL is obtained by adjoining to a standard version of PC:

i) an onperator «O» such that «Oα» means it is obligatory that α, where α ranges over wffs of DL;
ii) an operator «P» such that «Pα» means it is permissory (Stone's preference in place of the more commonly used «permitted») that α; ([7])
iii) abbreviative definitions:
Fα = df. ONα, where «Fα» means it is prohibitory (Stone's preference in place of the more usual «forbidden») that α; ([8])
Iα = df. KPαPNα, where «Iα» means it is indifferent that α.

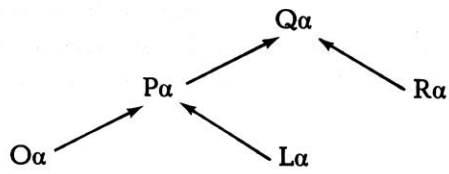
The following will be theses: ([9])

1) EPαNONα
2) COαPα
3) EOαNPNα
4) COαNONα

5) CNFαPα

The latter thesis, 5), is the maxim of closed legal systems. For open legal systems, Stone argues, we need another deontic logic, DL', based on DL plus two additional modalities Q and R, the former signifying the allowable and the latter the neutral. The implication relations between the modalities of DL' are as follows (where '→' signifies implication and α ranges over wffs):

Diagram 1.



From what Stone says, the theses of DL' must include

i) all theses of DL except that whenever P occurs in a thesis of DL it must be replaced by Q to get a thesis of DL';

ii) CPαQα (but not conversely), and
CRαQα (but not conversely).

On the basis of i) we have ⊣ CNFαPα in DL'. ([10])
But we also have in DL', by i),

2') COαQα
5') CNFαQα

so that notwithstanding ii) we have a «closure maxim», 5') for DL' analogous to 5) for DL: open legal systems are «closed» also.

It should also be noted that since DL' and the propositional version of DL have PC as their underlying logic, CKNFαFαRα is provable in DL' and CKNFαFαIα is provable in DL, both being instances of CKNppq: anything is a logical consequence of a contradiction. This is the inevitable result of employing material implication [Cpq = df.ANpq] for the «if-then-» connective. Hence, the fact that legal neutrality can be shown in

DL, to follow from a contradictory legal state-of-affairs is not a defect peculiar to DL'. In any event, as long as Stone is satisfied with a DL' built on DL, he has no basis for a logical distinction between open and closed legal systems .And the fact that he feels it necessary to *stipulate* that the legally allowable is broader than he legally permissory indicates that he may have serious doubts about a legal distinction between open and closed systems. ([11]) No criteria are given for distinguishing either legally or logically between P and Q: the differentiating factor, R, appears to be purely arbitrary. But even though the distinction between open and closed legal systems cannot, on the basis of Stone's argument, be supported legally or logically, this is not to say that the distinction may not be meaningful from a sociological standpoint; but that is another matter.

III

Stone's theory has been formalized by this colleague, Dr. Ilmar Tammelo in the latter's recent book; ([12]) and an axiomatic version of Tammelo's DL' (hereafter TDL') has been constructed by Mr. Ron Klinger, ([13]) as follows.

Symbols:
a ranges over acts,
o ranges over omissions;
: deontic «adjectives» (operators):
O (obligatory),
P (permissory),
L (licensory),
R (neutral),
Q (allowable);
: primitive variables (deontic modalities):
O*a*, O*o*, P*a*, P*o*, . . . ;
: PC Connectives N, C, A, K, E. ([14]).

Abbreviative Dfs: $Pa = NOo$
$Po = NOa$
$La = KPaPo.$
$Lo = La.$

Formation Rules:

i) A primitive variable is wf
ii) If α is wf, so is $N\alpha$
iii) If α and β are wffs, so are $C\alpha\beta$, $A\alpha\beta$, $K\alpha\beta$, $E\alpha\beta$.

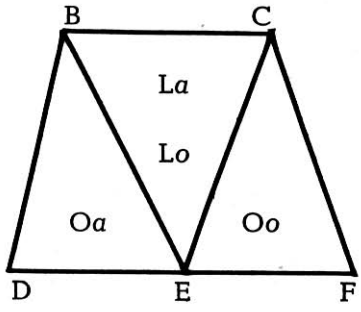
Underlying logic: PC.
DL' Axiom: $NKOaOo$

It will be noted

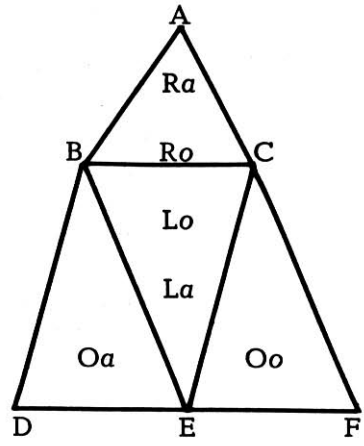
a) NPa is wf, but PNa is not wf: negation cannot operate «internally» within deontic modalities. ([15]).
b) The structures of TDL and TDL' are depicted diagramatically as follows.

Diagram 2.

TDL (closed)



TDL' (open)

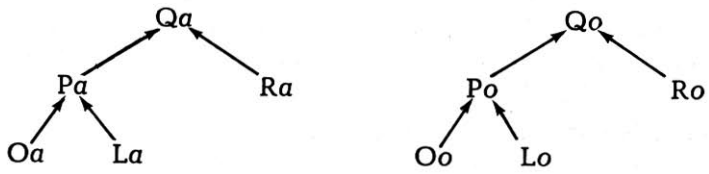


In TDL',

$$Ra = ABC \qquad Ro = ABC$$
$$Pa = BCED \qquad Po = BCFE$$
$$TDL' = TDL + ABC$$

c) The implication relations holding between deontic modalities in TDL' are as follows:

Diagram 3.



These follow the pattern in Liagram 1.

d) No implication relation can hold between P*a* and R*a*, nor between P*o* and R*o*.

The variables of TDL' range over acts and omissions. ([16]) But this means that any formula will require some extralogical qualification if absurd interpretations are to be avoided. Thus, the axiom NKO*a*O*o* could be read:

> It is not both obligatory to file an income tax return and obligatory to omit to steal.

Similarly the abbreviative definition P*a* = NO*o* could be read:

> It is permitted to file an income tax return = df. It is not obligatory to omit to put the cat out at night.

It will be recalled that von Wright's 1951 DL was «act»-based rather than «proposition»-based. Let us suppose that the primitive variables of such a DL are a_1, a_2, a_3, ...

We are then faced with the immediate problem; ([17]) how does one *assert* «*a*» or «N*a*» ? If we juxtapose say, «P», to «*a*» or «N*a*», we get «P*a*» or «PN*a*». These latter formulae *can* be asserted because they stand for propositions rather than acts or omissions. And since «act-statements» can always be rendered propositionally, a propositional-based DL avoids the difficulty of distinguishing between «positive» and «negative». Consider «entering» and «staying off»; which is the «act» and which is the «omission ?» To answer this question one must invoke extra-logical considerations. Propositions, on the other hand, though they may be denied, are neither positive nor negative. It is noteworthy in this connection, that von Wright has now abandoned «act»-based DL in favor of «proposition»-based DL. ([18])

Although Klinger gives a decision procedure for TDL and TDL', there are no semantics and completeness theorem; Tammelo feels that it is not yet sufficiently clear what values are carried by the wffs of a logic of norms. To avoid this problem, he treats norms as «propositions having normative import» ([19]) and analyses them in terms of norm-subject, norm-object and norm-nexus: modality is determined by the norm nexus, and «normative import» is reflected in modal operators. Norm-subject and norm-object are then analogous to grammatical subject and object. So all we really have is a grammatical analysis of «propositions having normative import»: no light is shed on the «nature» of norms. But propositions are either true or false whereas norms are neither true nor false, so what values can be given to «propositions having normative import» ? Tammelo's answer is that «[t]hey do not mean truth or falsity by virtue of actual facts but by virtue of legal fiats», ([20]) which does not really explain anything. In fact it compounds the difficulties caused by the «act» basis of TDL and TDL': wffs built out of «deontic modalities» consisting of modal «adjectives» operating on «act(omission)» variables are to be interpreted as «propositions having normative import» which are true or false «by legal fiat».

Until we have a plausible analysis of the concept of a norm-which Hart has warned is even more problematic than the con-

cept of law ([21]) — it will be better to treat it as primitive and address ourselves to the logic of two types of discourse: ([22])

i) Normative discourse, involving statements of the form: It is obligatory (permissory) that α.
The logic of this discourse is *not* truth-functional.

ii) Assertoric normative discourse, involving statemens of the form:
It is a fact that it is obligatory (permissory) that α.
The logic of this discourse *is* truth-functional. However, in this case, if paradoxes are to be avoided, we need a non-extensional operator for the «if. . .then» relation. ([23])

So far as the semantics of DL are concerned we have the Kripke-Hintikka notion of truth in deontically possible (permitted ideal) worlds, based on the Leibnizian notion of (necessary) truth in all possible worlds, which permits the construciton of a completeness theorem for DL. ([24]) It is, however, essentially a truth-functional semantics: the semantic tableau used as a decision procedure are simply sophisticated truth-tables. Philosophically, there are objections in that the Leibnizian-based semantics takes a «chess-board» view of the world: when a piece is moved, nothing else moves or changes. But, of course, when anything is moved all sorts of things and relations change besides just the position of the thing moved.

TDL' is regarded by Tammelo and Klinger as a logic *of* norms, an object language. How can the *absence* of norms be expressed in such a language ? How does one «represent» a normatively open system in a logically closed calculus ? Consider the natural numbers, N. The system $<N, +, ., \dot{-}>$ is closed under the operation $\dot{-}$; but $<N, +, ., —>$ is not closed under the operation —. To close the latter, we must add to N the negative integers; one cannot close $<N, +, ., —>$ by adding another *operator*. Similarly, to close an open system of norms, we should add more *norms*. Now Stone does not add any norms to the open system; he adds another *operator* R, which yields the «non-normative» modalities R*a* and R*o*, s.t. R*a*=R*o*=the «normatively neutral.» R*a* and R*o*, being deontic

modalities in TDL', are part of the primitive variables of TDL'. So if TDL' is a logic of norms, R*a* and R*o* must represent *norms*, more precisely «non-normative norms.» Of course ,it is not Stone's intention to close TDL' by adding the operator R; rather R is a device used to give a logical representation of the openness of a normatively open system. But it means, as Stone recognizes, ([25]) that TDL' contains symbols representing norms which are non-normative. And by the further introduction of Q to cover both P and R, we get the «closure maxim» 5') CNFαQα. Here we have a paradox which is not caused by the use of material implication. ([26]) No such problem arises in DL; we have 5) CNFαPα for closed systems and Iα=KPαPNα for open systems.

More generally, the distinction between open and closed legal systems could rest on the dubious assumption that law should constitute the whole of social control and ignores such things as rules of morals, religion, etiquette, social games etc. ([27]) What is to be served by calling such areas «legally neutral» simply because they are not dealt with in *legal* rules ? They are «mathematically neutral» also. To assert CNFαPα is to say that the absence of a norm regarding α is sufficient for α to be permitted; it is not to say that the absence of a norm is necessary in order for α to be permitted. In fact, if there is no norm regarding α then Iα: KPαPNα. Consider two legal systems, L_1 and L_2: in L_1 we have CNFαPα, i.e. L_1 is «closed», and in L_2 we do not have CNFαPα, i.e. L_2 is «open». Suppose neither L_1 nor L_2 has a specific norm dealing with *q*. Then in both systems it is true that P*q*: in L_1 by virtue of *modus ponens* (CNF*q*P*q*, NF*q* ⊢ P*q*) and in L_2 by a direct inference from the absence of a norm (CNF*q* ⊢ P*q*). And notwithstanding CNFαPα in L_1, it is true in L_1 that I*q*: KP*q*PN*q*. It would be absurd to suggest that CNFαPα constitutes a norm making *q* permitted: CNFαPα does not fill any normative gap because it is not a norm. In fact, by Fα=df. ONα, we have CNON*q*P*q*: the absence of a norm regarding *q* is sufficient to make *q* permitted. From a logical standpoint, the distinction between «open» and «closed» legal systems reflects no more than that in the absence of a norm regarding *q* we may deduce P*q* either

directly or by *modus ponens;* and normatively (legally) speaking the distinction rests not on whether the system recognizes CNFαPα but on whether there is a norm regarding *q*: to say that L_1 recognizes CNFαPα is not to say that L_1 contains a *norm* regarding *q*. Logically the distinction is trivial; legally, if we are to use the expressions «open» and «closed» at all it should be only by way of reference to whether or not there is a norm specifically dealing with certain conduct, *q*.

IV

There still remains Stone's criticism of the deontic indifferent of DL defined by Iα = df. KPαPNα.
This, according to Stone,

i) captures the «indeterminacy aspect» of the situation expressible in Hohfeldian terminology by «Privilege α and Privilege-not α»;
ii) does not mean deontically neutral but relates rather to the situation in which one is permitted to do something without being obligated to do it, i.e., KPαNOα, i.e., permissory by virtue of a licensory norm and not obligatory: the licensory implies the permissory, but not conversely;
iii) does not signify the absence of law or legal neutrality: for that situation we need the further modalities, allowable and neutral.

von Wright therefore fails to distinguish, says Stone, between
a) the case of legal norms permitting the omission of something, and
b) the case of absence of law and legal neutrality.

von Wright's indifferent covers only a) which is really the licensory situation; and additional modalities, the allowable and the neutral, are required for b).

We have already dealt with b). So far as a) is concerned we must distinguish three situations:

i) Pα undera licensory norm but NOα;

ii) KPαPNα, the legally indifferent without the absence of law; and
iii) KPαPNα, the legally indifferent and the absence of law.

The above discussion of open and closed legal systems justifies treating any distinction between ii) and iii) as extra-logical; there is no formal basis for such a distinction. As for i), we must note it involves two things: the permission under a licensory norm, and the absence of an obligation or duty. It is not entirely clear what Stone means by a licensory norm, and so it is not entirely clear what he means by the licensory as a deontic category necessary for the «logical apprehension» of legal systems. One legal possibility is the Hohfeldian privilege. Since in this case, i), the privilege would arise by virtue of a licensory norm, the norm, or perhaps the official acts giving rise to the norm, must constitute what Hohfeld calls positive operative facts ([28]) which extinguish a duty-not (ONα) and create an opposite privilege (Pα): privilege α is the negation of duty-not α (EPαNONα). ([29]). The other legal possibility is the Hohfeldian power, ([30]) which would mean that the licensory norm itself corresponds to what Hart calls a secondary power-conferring rule. ([31]) In both cases, of course, the official acts creating the licensory norm would require the authorization of public power-conferring rules; and the licensory norm reflecting the official acts could have the effect either of extinguishing a duty and creating a privilege, or of creating a private, as contrasted with a public, power. We shall now consider these two possibilities in the languages of DL and of Hohfeld. ([32])

Hohfeld's aim was to distil out of the mass of legal materials the several distinct fundamental senses in which the word «right» is used in the law. As his starting point he took the notion of «right» in the simple (and largely undefined) sense of a «claim recognized or secured by law,» ([33]) developing his analysis, so far as presently relevant, in terms of two or possibly more (legal) persons and one factual issue or state of affairs. If we let x and y stand for individuals (the parties in interest) and let *p* signify a state of affairs (what is in issue

between the parties), Hohfeld's theory may be summarized in the following and subsequent tables.

Table 1

a) Claim-right (y, p, x) b) Duty (y, p, x)
c) No-right (y, p, x) d) Privilege-not (y, p, x)

a) is read: x has a claim-right over y that p.
b) is read: y has a duty toward x that p.
c) is read: x has a no right over y that p.
d) is read: y has a privilege toward x not that p.

The relationships between these «fundamental legal conceptions» are expressed by Hohfeld in the following terms:

a) and b) } are pairs of jural correlatives; and
c) and d) }

a) and c) } are pairs of jural opposites.
b) and d) }

By «jural correlative,» Hohfeld means either logical equivalent ([34]) or logical converse ([35]) (for present purposes I adopt the former interpretation); and by «jural opposite,» Hohfeld means simply (external) negation in the weak sense of contradiction. ([36]) Given these interpretations of «correlative» and «opposite», the four conceptions in Table 1 can be explicated as follows:

i) a) «x has a claim-right over y that p» means
   b) «y has a duty toward x that p» is true, and
   c) «x has a no-right over y that p» is false.
ii) b) «y has a duty toward x that p» means
   a) «x has a claim-right over y that p» is true, and
   d) «y has a privilege toward x not that p» is false.

And similarly for no-right and privilege-not. Moreover, when duty b) is the case it is also true that y has a privilege toward x that p. Hence

iii) d') «y has a privilege toward x that p» means
c') «x has a no-right over y not that p» is true, and
b') «y has a duty toward x notthat p» is false.

In general, if α is a jural relation and it is true that α, then we can point to its correlative which is also true, and to its opposite which is false. A square of opposition following the traditional pattern may then be obtained as follows (correlatives in parenthesis).

Table 2

b) Duty (Claim-right)	b') Duty-not (Claim-right-not)
d') Privilege (No-right-not)	d) Privilege-not (No-right).

b) and b') are contraries;
d') and d) are sub-contraries;
the diagonals are contradictories (Hohfeldian opposites); and b) implies d'), and b') implies d).

This suggests the following pairs of alternative «abbreviative» definitions:
Duty = df. no privilege-not (i.e. no exemption). ([37])
Privilege = df. no duty-not (i.e. no prohibition). ([38])
No-right = df. no claim right.
Claim-right = df. no no-right.

Table 2 can therefore be treated as a legal version of the square of opposition for deontic modalities; whence Hohfeld's theory is an informal and legal precursor of von Wright's 1951 deontic logic. ([39]) Moreover, since Table 1 deals with legal duties or obligations,. the general rules under which the above four legal conceptions and their interrelations arise will be of the kind that Hart calls primary (duty-imposing) rules, ([40]) which correspond to what Stone calls «ordaining norms.» ([41])

The function of the other Hartian category of general legal rules, secondary power-conferring rules, is to enable individuals having the requisite legal capacity to do certain things,

e.g. make wills, contracts, get married, etc., in ways that reasons of policy dictate are necessary if legal effect is to be given to what is done. The «legal conceptions,» and their interrelations, which arise under these rules can be expressed as follows:

Table 3

e) Power (x, Dp, y)	f) Liability (x, Dp, y)
g) Disability (x, Dp, y)	h) Immunity (x, Dp, y)

e) and f) } are pairs of jural correlatives in the
g) and h) } sense explained re Table 1; and

e) and g) } are pairs of jural opposites in the
f) and h) } sense explained re Table 1.

These concepts are explicated in the same manner as those in Table 1.

The symbol, D, juxtaposed to p in Table 3 serves to emphasize the important distinction between privilege d') and power e). In both cases it would be correct to say (it followed from the relevant legal rules) that it was in some sense permitted to do something. In the case of privilege d'), the law looks upon «doing something» merely as the natural or social acts involved. In the case of power e), however, «doing something» is constituted what Ross calls an «act-in-law,» ([42]) meaning that when the «something» is done it will, under the relevant rules, have the effect of changing the legal relations of the parties involved, e.g. by creating further rights and duties. Thus, it might be said

d') Privilege (x, p, y)

means that it follows from certain general rules that x is in some sense permitted to do p to or for y; and

e) Power (x, Dp, y)

means that it follows from certain general legal rules that x

is in some sense permitted to change the legal status of p to or for y.

Since power e) differs from privilege d') only in having the modal variable D juxtaposed to p, whatever sense of permission is used to interpret the P of d') will also be used to interpret the P of e): but more of this later. The distinction between power and privilege can be made clearer by considering an Andersonian adaptation of DL (herein DL»). ([43])

Just as Hohfeld's theory is essentially a theory of what follows from general legal rules, so Anderson's system is precisely a logic of what follows from social rules (including *inter alia* legal rules). DL» is obtained by adjoining to DL

i) individual variables, x, y, z;
ii) a deontic variable, D, ranging over the operators O and P (negated and non-negated);
iii) a 3-ary predicate constant, M, such that «M(x, p, y)» means x brings it about that (does) p to or for y, and «M(x, Dp, y)» means x brings about a change in the deontic status of p to or for y.

Atomic formulae are now of the form M (x, p, y) and M (x, Dp, y). Quantification, though allowable given the addition of suitable axioms, may be ignored for present purposes. Tables 1, 2, and 3 may now be rendered in DL» as follows:

Table 1'

a) OM (y, p, x)	b) OM (y, p, x)
c) NOM (y, p, x)	d) PNM (y, p, x)

Table 2'

b) OM (y, p, x) $\begin{bmatrix}\text{Duty}\\ \text{Claim-right}\end{bmatrix}$	b') ONM (y, p ,x) $\begin{bmatrix}\text{Duty-not}\\ \text{Claim-right-not}\end{bmatrix}$
d') PM (y, p, x) $\begin{bmatrix}\text{Privilege}\\ \text{No-right-not}\end{bmatrix}$	d) PNM (y, p, x) $\begin{bmatrix}\text{Privilege-not}\\ \text{No-right}\end{bmatrix}$

In each parenthetical pair in Table 2', each member is the correlative of the other. The five theses of DL given earlier can be rendered in Hohfeldian terminology as follows:

1'') Privilege is the correlative of no-right-not: d') iff c').
2'') Duty implies privilege : b) implies d').
3'') Duty is the opposite of privilege-not : b) iff not d).
4'') Claim-right implies no-right-not : a) implies c').
5'') No duty-not implies privilege : not b') implies d').

Table 3'

e) PM (x, Dp, y)	f) PM (x, Dp, y)
g) NPM (x, Dp, y)	h) NPM (x, Dp, y)

It will be noted that there is no change in modal operator in Table 3' so that the equivalences between e) and f) and between g) and h) are trivial, and the opposition between e) and g) and between f) and h) is clearly that of negation in the (weak) sense of logical contradiction.

Since the operator P does service for both privilege d') and power e), we must now attend more closely to the distinction between these two concepts, noting in particular that Hohfeld cautions us «... to distinguish carefully between the legal power, the physical power to do the things necessary for the 'exercise' of the legal power, and, finally, the *privilege* of so doing these things ...» (44) The atomic formula «M(x, Dp, y)» can be read

x changes the legal status of p and
y is affected by the change, i.e.
x changes his legal relations with y, i.e.
x exercises a legal power, i.e.
x performs an «act-in-law.»

On the other hand, there is nothing juristically special about «M(x, p, y)» which is read simply as «x brings it about that (does) p to or for y,» which could signify either the exercise of a claim-right or privilege, or the fulfillment of a duty; it is

not an «act-in-law.» However, when it is the case that

7) M(x, Dp, y), e.g. M(x, OM (y, p, x), y)

then it must also be the case that

8) M(x,q,y), i.e.

x does those natural or social acts, q, which under the relevant secondary rules change the legal status of p in relation to y. Thus when, e.g.

8) x says the words «I accept your offer» to y, i.e. x does q, this speech-act, under the rules of contract law, is constituted

7) the exercise of a power to accept y's offer, and so to create new rights and duties in x and y with respect to the state-of-affairs p.

Provided that the speech-act, 8), is not, as such, covered by any general rules and x has made no promises to any one that he will not do that speech-act, 8), then

9) PM(x, q, y): x has a privilege toward y of doing q,

which, in turn, must be distinguished from

10) PM(x,OM(y, p, x), y): x has a power over y that Dp,

being the power to bring about 7) by doing 8):

8) is the exercise of privilege 9), and 7) is the exercise of power 10). In other words, if y has made an offer to x (itself an «act-in-law», the exercise of a power) then

10) x has a power over y to create a duty in y toward x that p; and 9) x has a privilege toward y of doing q.

And if x says «I accept your offer ...» then

8) x exercises his privilege toward y of doing the speech-act, q, and thereby under the relevant rules

7) x exercises his power of acceptance and so changes his legal relations with y with respect to p.

Put quite simply, the distinction between 8) and 7) is that whereas 8) is descriptive of facts, 7) is descriptive of those facts in relation to rules. ([45])

Regarding the sense in which the operator «P» means it is

permitted that, we may note a distinction drawn by Hare between

A) it is alright to, and
B) you may (= I (hereby) permit you to). ([46])

Hare limits the deontic operator P to permissory in sense A), for in that sense only does it imply that something has changed: sense B) merely expresses a personal permission such that reasons for the permission need not arise. Since the P of DL and the Hohfeldian privilege or power must follow from certain operative facts subsumed under general rules it is clear that to interpret them in terms of permission is to adopt meaning A) rather than meaning B). In sense A), permission implies that something has changed; and this is precisely the sense in which Hohfeld employs the concept of privilege. Suppose y has a duty not to hit x. If x then hits y, this positive operative fact under the relevant general rules extinguishes y's duty and creates a privilege in y of self-defense: it is now false that y has a duty not to hit x, and it is now alright for y to hit x back: y is permitted (now) in sense A) to hit x. This theory of Hohfeld, of «operative facts» under general rules extinguishing a duty imposed by those rules and creating an opposite privilege, ([47]) corresponds to Hart's theory of the defeasibility of duty-imposing rules: ([48]) the «operative facts» correspond to the «unless factors.» It is clear from the foregoing that DL» is an adequate formalization of Hohfeld; once again we may note that Hohfeld's theory is an informal deontic logic.

In Hohfeldian theory a duty may be extinguished and a privilege created by both

a) positive operative facts, as, e.g. when x has a duty not to hit y, and y hits x thereby extinguishing x's duty-not and creating a privilege in x «having a content or tenor exactly opposite to that of the duty[-not]», and
b) negative operative facts, as, e.g. when y asserts that x has a duty toward y that p, and x shows he made no promises to y regarding p, and thereby «extinguishes» the

duty asserted by y, «creating» a privilege (in x) «having a content or tenor exactly opposite to that of the duty».

As we would expect, from 7) to 10) supra ,the important operative fact regarding x's *power* over y that p is not that y have done anything ([49]) but rather that x have the legal capacity required by the secondary rules.

To say that something is a matter of legal indifference is to say in DL: KPαPNα; and in Hohfeld: Privilege α and Privilege-not α. Whether or not Iα reflects Stone's allowable plus the absence of law cannot be detected from a mere inspection of logical form; it is a matter of fact, the semantics of DL, rather than of form, the logic and syntax of DL. In order for Iα to be interpreted as allowable and legally neutral it would have to be the case that

i) neither Pα nor PNα arose by virtue of positive operative facts under general legal rules;
ii) no enabling rules are necessary for α to have legal effect; and
iii) no promises have been given regarding α or Nα.

If one or the other of Pα and PNα arose by virtue of positive operative facts so that we cannot justify both Pα and PNα simply by negative operative facts, then although it is the case that Iα, we do not have a case of absence of law. My «right» to wear a bow tie is a case of Iα and the absence of law. On the other hand, suppose x has a duty and therefore a privilege to stay off y's land. If y now conveys the land to x, this extinguishes x's duty (but not necessarily his privilege) to stay off and creates an opposite privilege of not staying off (i.e. of entering). And if x makes no promises to not stay off (i.e. to enter) then he also has a privilege of staying off. Hence x now has both a privilege of not staying off and a privilege of staying off and we have another example of Iα. But only the bow tie case is an example of the absence of law; in the second example one of the two sub-contrary privileges, the privilege not to stay off, arose by virtue of positive operative

facts subsumed under general legal rules. However, we cannot distinguish the two cases from a formal standpoint; logically, both are examples of Iα. Stone's allowable and neutral are extra-logical concepts and do not pertain to the logic or syntax of DL.

There are two possible interpretations in DL and Hohfeld of Stone's licensory defined as permissory under a licensory norm but not obligatory, viz. privilege and power. If «Pα under a licensory norm» means «privilege α» then, as already noted, the official acts giving rise to the norm must be regarded as positive operative facts extinguishing a pre-existing duty-not, ONα, so that NONα, thus creating an opposite privilege, Pα. But since, by definition, it is also the case that NOα, hence PNα, we have KPαPNα, i.e. Iα: the licensory is just a special case of the indifferent, Iα, in which one of the two sub-contrary privileges was created by *official* acts rather than unofficial acts as was the case when y conveyed his land to x and so created a privilege in x of not staying off the land. This factor, the official nature of the positive operative facts creating the privilege, the distinguishing feature of the licensory, obviously does not pertain to the logic or syntax of DL.

The other possibility, that «Pα under a licensory norm» means a Hohfeldian power, is less likely. In this case, Pα will arise by virtue of secondary power conferring rules, so that in the language of DL», the licensory reflects

11) KPM(x, Dp, y) NOM(x, q, y) i.e.
12) KPM(x, Dp, y) PNM(x, q, y):

x has a power over y to change the legal status of p, but he has a privilege-not to do those social acts, q, which under the relevant rules, when done, would constitute the exercise of the power. On this basis, the licensory is a hybrid composed of a power e) and a privilege-not d); but this hardly justifies treating it as another formal category. In fact 12) is true of every case in which a power exists: we have the power and also we are privileged-not (though not empowered-not) to do those social acts which under the relevant secondary rules

are constituted the exercise of the power .

We may conclude, therefore, that Stone's licensory is no more than a special case of the deontic indifferent in which the two defining sub-contrary permissions are interpreted as two sub-contrary privileges one of which was created by positive operative facts of an official rather than unofficial nature; and this does not justify the creation of another formal category, another modality, the licensory.

V

A final point to consider is whether there is anything in von Wright's writings to suggest that by the deontic indifferent he really means no more than Stone's licensory (permissory under a norm but not obligatory). The one example von Wright uses to illustrate a situation in which the deontic indifferent would apply is, «... in a smoking compartment we may smoke but we may also not smoke.» ([50]) As our previous discussion of the licensory indicates, this could not mean that smoking is permitted in a certain compartment(s) under a secondary power-conferring rule. For then it would follow that only in the designated compartment(s) would smoking have the effect of changing our legal relations, and this is nonsense.

In terms of privilege, there are two plausible explanations.

A) There are secondary rules which enable officials to designate both

i) no smoking compartments: in these one has a duty-not to smoke; and
ii) smoking permitted compartments: in these one has no duty to smoke and no duty-not to smoke.

In the case of i) we can say ONα by reason of a primary rule enacted by an official pursuant to a secondary (public) power-conferring rule. In the case of ii) we can say Iα by reason, *inter alia,* of a (positive) act of an official pursuant to a second-

ary (public) power-conferring rule. Clearly there would be no point in designating some compartments as ii) «smoking permitted» compartments unless there were also other compartments designated «no smoking,» just as there would be no point in designating some compartments as «breathing permitted» compartments if there were no «no breathing» compartments also.

B) There is a general primary rule (possibly enacted by an official pursuant to a secondary (public) power-conferring rule) by virtue of which smoking is prohibited in compartments; but secondary rules enable officials to waive the duty-not to smoke imposed by the primary rule. Again, there would be no point in designating some compartments «smoking permitted» unless there was a general «no smoking»-in-compartments rule in the first place. Treating the licensory norm as being manifested by the official acts extinguishing the duty-not to smoke and creating an opposite privilege to smoke, again such a norm could hardly be said to come into existence unless there was a pre-existing prohibitory norm. Given the prior prohibitory norm, the licensory norm will have the effect of extinguishing the prohibition and creating an opposite privilege, viz. a privilege to smoke. And if an occupant of a «smoking permitted» compartment has made no promises that he will smoke ($NO\alpha$), this negative operative fact will establish the opposite privilege-not to smoke ($PN\alpha$) which is the sub-contrary of the privilege to smoke ($P\alpha$) so that $KP\alpha PN\alpha$, i.e. $I\alpha$. von Wright's illustration yields a case of $I\alpha$ without the absence of law; the bow tie example is a case of $I\alpha$ and the absence of law. The former Stone calls the licensory and the latter the allowable and neutral. Both are extra-logical matters so that neither requires the addition of further modal operators to DL.

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FOOTNOTES

(1) STONE [15], pp. 186-197.
(2) VON WRIGHT [17].
(3) HOHFELD [11].
(4) In VON WRIGHT [17], the variables ranged over act-types rather than over propositions.
(5) KRIPKE [13].
(6) HINTIKKA [10].
(7) STONE [15], .p 194.
(8) *Ibid.*

(9) All five theses are recognized in Stone [15], pp. 196-197.

(10) The symbol '⊢' signifies non-provability.

(11) Stone [15], p. 194.

(12) Tammelo [16].

(13) Klinger [12].

(14) N (negation), C (material implication), A (inclusive disjuction), K (conjunction), E (material equivalence). Thus, corresponding to any proposition *p* there is a denial N*p* which is true iff *p* is false, and for any two propositions *p* and *q* there is a conjunction Kpq (which is true iff both *p* and *q* are true), a disjunction Apq (which is true iff at least one of *p* and *q* is true), an implication Cpq (which is true iff it is not the case that *p* is true and *q* is false), and an equivalence relation Epq (which is true iff p and q are both true or both false).

(15). c.f. Hare [17a], pp. 20-21.

(16) This rather like having a calculus of the natural numbers with just two variables, *e* for even numbers and *o* for odd numbers.

(17) Suggested by A.R. Anderson.

(18) von Wright [18].

(19) Tammelo [16], p. 87.

(20) *Ibid.*

(21) Hart [9].

(22) Suggested by R. Goodwin.

(23) Anderson [18a].

(24) Hanson [4].

(25) Stone [15], p. 192.

(26) In correspondence, Tammelo has stated he regards this paradox as apparent only, and does not subscribe to the notion of a «logically open legal system».

(27) In correspondence, Tammelo has stated he does not hold such a view.

(28) Hohfeld [11], pp. 32-35.

(29) See *infra.*

(30) See *infra.*

(31) Hart [9].

(32) Hohfeld [11].

(33) Hohfeld [11], p. 38, note 32a.

(34) Anderson [1].

(35) Fitch [3].

(36) Anderson [1] argues that Hohfeld is confused about negation. For a contrary view see Mullock [14].

(37) Fitch [3].

(38) *Ibid.*

(39) von Wright [17].

(40) Hart [9].

(41) Stone [15], pp. 187, 201.

(42) Ross [19].

(43) ANDERSON [1].
(44) HOHFELD [11], p. 58.
(45) HART [7], [8].
(46) HARE [5].
(47) HOHFELD [11], pp. 32-35.
(48) HART [6], AUSTIN [2].
(49) Although, of course, a power in x could be created, *inter alia*, by either y's act-in-law or y's breach of a legal duty.
(50) VON WRIGHT [17], p. 3.