NATURAL DEDUCTION «PUZZLE»

Jason Xenakis

Deree College
P.O. Box 472 ATHENS Greece

ND (Natural Deduction) turns a nontautologous biconditional into a tautology:

$$O \equiv \square$$
$$(O \supset \square) \,.\, (\square \supset O)$$
$$O \supset \square$$
$$\square \supset O$$
$$O \supset O$$

You get the same result whether one side of the biconditional is the contradictory of or merely different from the other side (whether the biconditional is a contradiction or a contingency), while if the sides are the same (the biconditional a tautology), the last-line tautology appears earlier (3rd line).

Of course the 1st line materially implies the last: that's not the problem. The *transformation* is. Another example:

$$\Delta$$
$$\Delta \vee -\Delta$$

The paradox picks up momentum if «reduction» is used: any nontautologous biconditional — or any nontautology period — *reduces* to a tautology by ND; and ND is an approved procedure of *logic.*

One way out is in terms of MI (Material Implication), in which case the so-called paradozes of MI (one anyway) infect ND as well; which is as it should be: both belong to ex-

tensionalism. Anyway, while «reduction» fits ND it doesn't MI.

And if you (arbitrarily ?) reject «reduction», how about «transformation» ? Can't escape *that* word here.

Of course you can always say that «transformation» — or it *too* — is not used in extensional logic in the ordinary sense, but means only — or instead — truth-retention (hypothetically). I.e. all logical (extensional) inference permits («transformations») are so fashioned that you never go from T to F — and moreover, that that's all extensional logic cares about.

Deree College, Athens Jason Xenakis