PRIOR ON EXTENSIONALITY

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There are many principles of extensionality and versions of extensionalism, but one of the first statements of the doctrine of extensionality was expressed by the principle that all functions of sentences or propositions are truth functions. Russell once asked

> ... can all propositions which do not contain apparent variables be built up from atomic propositions by means of the stroke ? If this were the case, we should have, if fp is any function of propositions,

$$p \equiv q . \supset . fp \equiv fq$$

> Consequently, according to the definition
> * 13.01

$$p \equiv q . \supset . p = q$$

> There will thus be only two propositions, one true and one false. ([1])

Prior suggests that the extensionalist view arose from a parallel suggested on the basis of Frege's semantics. The parallel is this: truth and falsehood are objects denoted by sentences just as numbers are objects denoted by numerical expressions. Which out of the values 'true' or 'false' is denoted by 'Fp' depends on which is denoted by 'p' just as which number is denoted by 'fx' depends on which number is denoted by 'x'. Prior says:

> [many extensionalists contend that] the only feature of 'p' on which the truth-value of any function of it can depend is its own truth-value. For the list of possible func-

tions does not include any which are, say, true with some true arguments and false with other true arguments. If 'X thinks that p', for example, were a function of 'p', it would have precisely this character. Why on earth should not the truth-value of a function of 'p' depend on some other feature of 'p' than truth-value ? To say that this is impossible is like saying that for any genuine function fx of a number x, whether fx is greater than 0 *must* depend on whether x is greater than 0 — an assumption which is plainly false, for example, if the function is x—1; since in some cases when x is greater than 0, e.g., when x = 2, x — 1 is also greater than 0, whereas in other such cases, e.g., x = 1, x — 1 is *not* greater than 0. Whether this function of x is greater than 0 clearly depends not on whether x itself is greater than 0, but on whether it is greater than 1. Similarly, whether *it is or is not the case* that X believes that p does not depend on whether *it is or is not the case* that p, but on whether *it is or is not* believed by X that p. Why on earth not ? (²)

Prior contends that 'X believes that', for example, is a genuine function of p for it is a one-place connective which when affixed to a sentence p yields a new sentence of the form 'X believes that p.' (³) In *this* respect 'X believes that' is like 'it is the case that'. But all this shows is that 'X believes that' is a *syntactic* function of p. This has no edge against the extensionalist for his complaint is not that 'X believes that', for example, is not a syntactic function; his complaint is rather that it is not a *semantic* function of p. Ajdukiewicz, for example, says:

> Every extensional sentential formula establishes a functional (many-one) relation between the values of its variable and the logical values (i.e., truth or falsehood) of the sentences obtained from that formula by the substitution of the symbol of those values for the variable. Further, every extensional sentential formula determines a class of those values of the variable, which satisfy that formula...

> This cannot be said of the intensional formulae ... e.g., the sentential formula 'Newton knew that x = 8' does not establish a many-one relation between the values of the variable 'x' and the logical value of the sentences obtained from that formula. Thus, both truth and falsehood as the logical values of a sentence obtained from that formula by the substitution for 'x' of a symbol of the number 8 can correspond to that number 8 which belongs to the values of the variable 'x'. The logical value of the sentence 'Newton knew that 2 + 6 = 8' is truth, and the logical value of the sentence 'Newton knew that the atomic number of oxygen = 8' is falsehood, in spite of the fact that both sentences are obtained from the sentential formula 'Newton knew that x = 8' by the substitution for 'x' of symbols of the same number ...
> Consequently, an intensional formula does not determine unequivocally any class of objects which would satisfy that formula. (⁴)

It is clear that Prior's remark that 'X thinks that' is a syntactic function of p does not threaten Ajdukiewicz's argument.

Let us now turn to Prior's criticism of the extensionalist position mentioned above. Such an extensionalist holds that

(1) The value of a function Fp of p depends solely on the value of p

not

(2) For any property G, the value of a function Fp of p is such that G(Fp) if Gp

If G = 'is false' then an instance of (2) is

(3) The value of a function Fp of p is false if p is false

Another instance of (2) is

(4) The value of a function Fp of p is the same as the value of p.

Whoever thought the extensionalist was committed to (2) and/

or to (4) ? Prior never explicitly says that the extensionalist is committed to these though he does pour scorn on an instance of a numerical analogue of (2), viz.,

(5) The value of a function fx of x is > 0 if $x > 0$.

But this doesn't show anything wrong with (1) whose numerical analogue is the eminently acceptable

   (6) The value of a numerical function fx of x depends solely on the value of x.

It is true that whether the value of some function fx of a number x is greater than 0 may not depend solely on whether x itself is greater than 0, as Prior has shown. Accordingly, (5) is false and so is the general principle (2). But this does not establish that the value of a numerical function fx does not depend on the value of x *whatever that value may be,* i.e., it is not an argument against (6) which is clearly true. Prior seems to recognise this as well, but then it is hard to see why he thinks that (5) is analogous to anything asserted by the extensionalist or that because (5) is false this demonstrates the falsity of the extensionalist view. All our extensionalist is claiming is (1), namely, that if Fp is a genuine function of a sentence p its value depends on the truth-value of p *whatever it may be.* He is not asserting (2), and in particular, he is not asserting (4), which is roughly analogous to (5) and is, like (5), false for let Fp be 'it is false that p'; now if p is false the value of Fp is *true.* So if this is the point of Prior's analogy, then the extensionalist is made to say something he need not, and, indeed, does not, and which would be false if he did.

   Perhaps, however, the point of the numerical analogy is this: just as it is not true to assert that

   (7) Any genuine function fx of a number x is such that whether the value of fx > 0 must depend on whether $x > 0$

for there are functions which do not have this feature; so it is not true to assert that

(8) Any genuine function Fp of a sentence p is such that
    the value of Fp depends solely on the truth-value of p.
But (7) does not itself serve to establish (8) which must be ar-
gued for independently. And this Prior attempts to do. He
suggests that the truth or falsity of 'X believes that p', for
example, depends not on the truth or falsity of p, but on whe-
ther it is true or false that X believes that p. This is true but
trivial and consequently is not a compelling argument. For it
says no more than X believes that p if and only if X believes
that p.

Any attempt to restate Prior's analogy reveals its idle char-
acter. Our extensionalist's claim that (A) the only genuine
functions of sentences are truth functions is analogous to the
contention that (B) the only genuine functions of numbers are
numerical functions. But (B) is a banality and so can shed no
light on the acceptability of (A). Whatever the demerits of (A),
it is not impugned by Prior's criticism.

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