THE UNTENABILITY OF GENERA

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By *'genera'* I mean the putative referents of *generic descriptions* like 'the lion', 'the scythe', 'man' as used in

The lion is tawny.

The scythe is useful for cutting tall grass.

Man is a curious animal.

I have argued elsewhere [GDD; GCU parts I, II, V] that such descriptions do not in their natural habitat denote. That conclusion extends not only to generic descriptions, but also to mass nouns and plural-noun phrases.

Now I wish to explore whether generic descriptions (and by implication these related types of noun phrase) can be *made to* denote. I.e., can a formal system be constructed in which the counterparts of generic descriptions function as genuine singular terms ? The failure of two promising attempts is exhibited below. That does not, of course, preclude the possibility that further ingenuity might succeed. But such failures strengthen the doubts already raised in [GDD] as to the tenability of gennera.

I. Genera as representative objects

10. Leśniewski contra Twardowski. When we say

The lion is a mammal

we seem to be saying that

All lions are mammals.

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In general, it seems plausible to hold that the lion has just those properties that all lions have; i.e.,

G1. $G\text{þ}xFx \equiv \forall x(Fx \supset Gx)$

where 'þ' is the generic descriptor. Genera in this sense were recognized by Twardowski; he called them «general objects», and held that they «represented» the particulars with which they were correlated. Already in 1913, Leśniewski demonstrated the untenability of Twardowski's conception of general objects [WS] (cf. Luschei [LSL 22f, 26, 308ff n. 21]). For presumably, like everything else, genera are self-identical:

1. $\text{þ}xFx = \text{þ}xFx$

Hence, by the principle G1,

2. $\forall x(Fx \supset x = \text{þ}xFx)$,

i.e., each F thing is identical with its own genus, and hence with every other F thing: there is at most one F thing. In fact, since F could be any property, there is at most one thing. Putting self-identity for F, we get

3.	$x = x \supset x = \text{þ}x(x = x)$	$y = y \supset y = \text{þ}x(x = x)$	2, UI
4.	$x = x$	$y = y$	reflexivity of =
5.	$x = \text{þ}x(x = x)$	$y = \text{þ}x(x = x)$	4, 3, MP
6.	$x = y$		5, transitivity of =
7.	$\forall x \forall y(x = y)$		6, UG (twice)

Such extreme monism, while not logically self-contradictory, is surely false.

11. Destruction of quantification. Notice that our reconstruction of Leśniewski's argument makes use only of the left-to-right half of G1:

G2. $G\text{þ}xFx \supset \forall x(Fx \supset Gx)$

A related anomaly, «converse subalternation», can be derived from G2 as follows:

1.	$\sim G\text{þ}xFx \supset \forall x(Fx \supset \sim Gx)$	G2
2.	$\sim \forall x(Fx \supset \sim Gx) \supset G\text{þ}xFx$	1, transposition
3.	$\exists x(Fx \,\&\, Gx) \supset G\text{þ}xFx$	2, familiar equivalence
4.	$G\text{þ}xFx \supset \forall x(Fx \supset Gx)$	G2
5.	$\exists x(Fx \,\&\, Gx) \supset \forall x(Fx \supset Gx)$	3, 4, transitivity of $\supset$

Again putting 'x = x' for '*F*x', we get

6. $\exists xGx \supset \forall xGx$

G2 thus leads to the conflation of particular and universal quantification. If at most one thing has any property, quantification is indeed superfluous.

12. Self-predication. The failure of G2 already largely undercuts the intuitive conception of genera held by Twardowski and formulated in G1. But there is still the possibility that the other half of G1,

G3. $\forall x(Fx \supset Gx) \supset G\text{þ}xFx$,

could be retained as a principle in the logic of genera. G3 will not do as it stands, however. With '*F*' substituted for 'G', it implies.

$F\text{þ}xFx$

which, adapting Vlastos' term, I call «self-predication». If we admit generic descriptions as unrestricted substituends for bound variables, self-predication conversely implies G3, so that the two principles become equivalent. The trouble with self-predication is that it leads straight to contradiction:

$$F\text{þ}x(Fx \,\&\, {\sim}Fx) \,\&\, {\sim}F\text{þ}x(Fx \,\&\, {\sim}Fx)$$

13. Monism again. Thus G3, or *mutatis mutandis* self-predication, must somehow be restricted so as to block such contradictions. The obvious restriction is the following:

G3' $\forall x(Fx \supset Gx) \,\&\, \exists xFx \supset G\text{þ}xFx$

Alternatively, following Hilbert and Bernays' treatment of their co-operator [GLM 10f], we could refrain from introducing '$\text{þ}xFx$' at all until '$\exists xFx$' had been established, and then use G3 in its original form. The restriction '$\exists xFx$' may seem too strict, in that '$\Diamond \exists xFx$' would suffice to block the contradiction. Be that as it may, even under the stronger restriction G3' leads to the same absurdity as G2 in the presence of two additional plausible assumptions about genera. Using 'e' for embodiment, the relation of a thing to its genus, we may state these assumptions as follows:

G4. $Fy \supset ye\text{þ}xFx$ (generic abstraction)

G5. $ye\text{þ}xFx \supset Fy$ (generic concretion).

We may then argue—

1. $\forall x[x = x \supset \exists z(z = x)]$ *familiar theorem*

2. $\exists x(x = x)$ "

3. $\exists z[z = \text{þ}x(x = x)]$ (1) 1, 2, G3'

4. $\forall z[z = þx(x = x) \supset z = þx(x = x)]$ familiar theorem

5. $þz[z = þx(x = x)] = þx(x = x)$ 4, 3, G3′

6. $y = y$ reflexivity of =

7. $y\epsilon þx(x = x)$ 6, G4

8. $y\epsilon þz[z = þx(x = x)]$ 7, 5, substitutivity of =

9. $y = þx(x = x)$ 8, G5

The argument concludes as in §10, steps 5-7: there is at most one thing.

14. Collapse of representative objects. Of G3′-5, G4-5 seem uncontroversial, particularly since their analogues already hold for classes. Thus G3′ must be the culprit. Once G3′ along with G2 has been called into question, however, the intuitively appealing Twardowskian conception G1 of genera falls altogether.

II. Genera as abstractions

20. Restriction on predication. When Russell encountered an antinomy in his attempt to formalize class theory, he found it expedient to question the assumption that whatever could be said of the members of a class could just as meaningfully be said of the class itself. The result was his theory of types. A wise move for us at this point would seem to be to raise the same question about genera. Perhaps not everything that can be said of the embodiments can meaningfully be said of their genus. Leo was brought to the Bronx Zoo in 1964. Was the

(1) I derive 3 in this fashion, rather than more directly by EG on '$þx(x = x) = þx(x = x)$', to avoid making use of the assumption that generic descriptions may be substituted without restriction for bound variables.

lion brought to the Bronx Zoo in 1964 ? 'Yes' probably isn't the right answer; but 'no' seems more odd than correct. That just isn't the sort of thing people ask about the lion. What sort of thing does one ask about the lion ? A not implausible answer is 'things that are true of all lions or no lions'. Just this restriction on the things we can say about genera emerges if we treat genera as the products of abstraction in the sense of Lorenzen [DI §§2-3]. Notice, incidentally, that with this restriction on 'G' the anomaly of Twardowski's principle G1 evaporates. «Converse subalternation»,

$$\exists x(Fx \,\&\, Gx) \supset \forall x(Fx \supset Gx),$$

is in fact equivalent to the very restriction suggested.

21. Abstraction. The method of abstraction articulated by Lorenzen is closely related to other notions of abstraction developed in modern logic. (For the history, cf. Scholz and Schweitzer [DA], 1. Hauptstück.) In order to abstract from a domain of items, we must first find an *equivalence relation* with that domain as its field. With respect to the equivalence relation *R*, we may distinguish those statements about (or, *mutatis mutandis*, properties of) the original items which are *invariant*. An invariant statement A is one which, if true of an item, is also true of all *equivalent* items:

$$A(x) \,\&\, xRy \supset A(y)$$

In other words, an invariant statement is oone which holds of *all or none* of the members of each equivalence class generated by the equivalence relation *R*. (It will be remembered that equivalence classes are mutually exclusive and jointly exhaustive of the field of the corresponding equivalence relation.) (2) In these circumstances we may «abstract» from the (for this purpose) inessential properties distinguishing items within each equivalence class, thereby obtaining one abstract item per equivalence class. At that point we are in a position to introduce names for the newly won abstractions as well as new variables ranging over them. These names I propose as a reconstruction of the geneeric descriptions of ordinary language. The new names and variables obey familiar logical laws privided that we restrict our use of them to such statements as were distinguished as *invariant* with respect to *R*. To put it another way, we must confine our statements about genera to those which preserve the *indiscernibility of equivalents.* (3)

22. An example. Let the domain be made up of individual animals. An equivalence relation over this domain is that of belonging to the same species (species-identity). The property of being a mammal is invariant with respect to this equivalence relation, while the property of having been brought to the Bronx Zoo in 1964 is not. Species-identity divides animals up into several equivalence classes. We now proceed to posit one genus for each equivalence class. If we can find some predicate the application of which coincides with a given equivalence class, we can form a name of the corresponding genus by prefixing the generic 'the'. Since the application of 'lion' coincides with one of the equivalence classes, 'the lion' names the corresponding genus. Te genera generated by the equivalence relation in question are none other than the species of zooloogy. We may only make *invariant* statements about the new species (genera), however. Thus

The lion was brought to the Bronx Zoo in 1964 is ruled out, while

The lion is a mammal

is meaningful and true..

23. Classes inessential. In expounding abstraction, I have referred freely to classes. Though that is the way it is usually done (e.g. by Whitehead and Russell [PM 442], Carnap [AL 48]),

(2) With Scholz and Schweitzer [DA 54 n. 2, 59f] I exclude the null class from the set of equivalence classes.

(3) The abstractive approach seems to be at the bottom of Langford's informal account of the semantics of generic descriptions. Cf. 'An object is always some kind of invariance; and a book [type] is a literary invariance' [IUT 114].

it is not essential. Once we have some items about which we can make some meaningful statements, among them the attribution of an equivalence relation, we have the wherewithal to abstract. Indeed, Lorenzen gets classes by abstraction [DI 22ff] rather than *vice versa:* classes are *one kind* of genus. Once classes are available, it is erhaps natural to identify genera with equivalence classes.

24. The third man. It would seem natural to maintain that when a predicate such as 'mammal' is extended in application from its original items to genera of those items, it is being used in a new and different way. The lion is not precisely the same kind of mammal that Leo is; i.e., 'mammal' applies to the lion in a different way than it applies to Leo. Otherwise, how could we ever keep track of numbers ? If there were n mammals to begin with, and we are precluded from identifying the lion with any of them (since '= Dobbin', '= Towser',..., '= Leo',... will never be invariant contexts except when the corresponding equivalence classes are unit classes), should we not come out with $n + 1$ mammals once the lion was on hand ? But surely abstraction does not create new mammals («geheimnisvoll am lichten Tag», as Scholz and Schweitzer put it [DA 33] ! The $(n + 1)$st mammal is as dubious as the third man.

25. Possible solutions. Three ways seem to be open to us. (1) Admit that abstraction creates additional exemplifications of the properties that were invariant with respect to the equivalence relation used. (2) Distinguish two senses of the predicates expressing those properties, e.g. '*mammal*' as true of embodiments and 'mammal,' as true of genera (in this case species). (3) Specify two disjoint sorts of variable ranging over genera and embodiments respectively. (1) seems like kidding ourselves. (2) is preferable from a formal point of view but takes the excitement out of self-predication. (3) is not so simple as it might seem. Take a predicate that is invariant with respect to species-identity, say 'mammal'. Then the following statement form is also invariant:

$$\exists x(\text{mammal } x \,\&\, x = \ldots)$$

Thus we should still get

$$\exists x(\text{mammal } x \,\&\, x = \text{the lion}).$$

But this implies

$$\exists x(x = \text{the lion}).$$

This defeats the purpose of carefully sundering the sort of variable for which such expressions as 'the lion' are substitutable from the sort ('x', 'y',...) for which 'Leo' is substitutable. For the last formula suffices to warrant the inference

$$\forall x A(x)$$

$$\therefore A\ (\text{the lion})$$

26. The third man at bay. The only way I can see around this is to restrict the invariance principle by extending only *elementary* invariant statements to genera upon abstraction. (4) We could thus affirm that

mammal (Leo)

and

$$\exists x(\text{mammal } x \,\&\, x = \text{Leo}),$$

both of them invariant statements, but only the first could be extended to the lion upon abstraction:

mammal (the lion).

As a new sort of variable ranging over animal species, we

(4) As long as we're willing to entertain such restrictions here, a much simpler way of retrieving Twardowski's G1 (§ 10) suggests itself. Why not restrict '*Gx*' there to elementary statements. For that evidently keeps the argument of § 11 from getting off the ground. However, the restriction would lead to such anomalies as the following. Neither of the primitive predicates 'white' or 'black' applies to all swans. Hence by G1 the swan is neither white nor black. Ordinarily, on the other hand, we should say that the swan *is* either white or black, on the grounds that all swans are one of the two.

introduce 's',... Now we can validly infer

$\exists s$(mammal s & s = the lion).

Finally, if before abstraction we had

$\exists_n$x(mammal x),

after abstraction we could consistently add that

$\exists_m s$(mammal s)

where $m \neq n$.

So why not introduce another sort of super-universal variable 'u',... whose range would be the union of those of 'x' and 's'? We could then conclude that

$\exists_{m+n} u$(mammal u).

After all that work, (2) seems like a more attractive alternative after all. (3) has the further drawback that it would require us to find a complete analysis of a statement before extending it to genera. Otherwise we should not know whether the statement was elementary or not. While that is no problem with a formal system, it is inconvenient when we wish to apply the theory to statements given in natural language.

27. Importance of the equivalence relation. If genera are products of abstraction, a given genus might be generated by two different equivalence relations. Thus an adequate notation would have to make explicit reference to the equivalence relation used. Suppose, e.g., that there is just one library on Antarctica. Then either

is in the same library as ("L")

or

is on the same continent as ("C")

would express equivalence relations over the field of books, of which Antarctican books formed one equivalence class. Accordingly, two genera corresponding to the Antarctican book would be generated; we might call them «the$_L$ Antarctican book» and «the Antarctican book» respectively. But the predicate 'is numbered according to the Dewey Decimal System' is invariant with respect to the one equivalence relation and not the other, and hence meaningfully applicable to the$_L$ Antarctican book but not to the$_C$ Antarctican book. Thus whetheer or not

> The Antarctican book is numbered according
> to the Dewey Decimal System

is to be admitted as a meaningful sentence depends on which equivalence relation furnished the basis for abstraction.

28. Failure of the abstractive interpretation. I believe that *some* generic descriptions *are* used to refer to abstractions in Lorenzen's sense. Examples are the use of quotation expressions to refer to syntactic types and the use of abstract singular terms to refer to properties and relations. (For the details, see [GDD §§3.6 ; GCU §55].) However, I hold that such examples constitute regimented oro recoonstructed usage. The regimentation consists in the imposition of the invariance requirement.

Despite its promise, however, the abstractive approach to genera is too stringent. The invariance requirement would rule out as meaningless such paradigm generic assertions as

> The lion is tawny.

For though all or no lions are tawny — in fact all (normal ones) are — it is not so that all or no rabbits are tawny. Tawniness is thus *not* invariant with respect to species-identity. (5) In order to justify the above assertion by abstraction, we should have to find some other equivalence relation of which lions were an equivalence class and with respect to which tawniness was invariant. That hardly seems like a hopeful line of attack.

29. Two hopeful attempts to render genera tenable by formalization thus founder. Although the second approach yields some useful spin-offs, it regiments ordinary genera like the lion beyond recognition. *Sic transeunt genera.*

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(5) I owe this insight to Marcelo Dascal.

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