SYLLOGISTIC AND CALCULUS OF CLASSES

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TIMOTHY SMILEY (2) has translated Aristotle's syllogistic into the many-sorted logic. In his translation all the traditional *A, E, I, O* forms have *existential import*. That is, if we again translate the many-sorted logic into the ordinary single-sorted logic, *A* and *I* are represented by $(x)\ (A(x) \supset B(x))$ and $(Ex)\ (A(x) \& B(x))$, respectively, (*E* and *O* are negations of *I* and *A*, respectively), and each formula of the form $(Ex)A(x)$ is logically true. ((2)esp.pp66-68). STORRS McCALL (4) has presented a deductive system which provides a model for Aristotle's syllogistic. In it also all the traditional forms have existential import. On the other hand, since the mediaeval age it is known that the traditional forms are represented by the following schema, (which is commonly called «square of opposition»), and the traditional relations of contradiction, contrariety, subcontrariety, and subalternation all hold. ((3)esp.p52).

A	*E*
$(Ex)A(x) \& (x)(A(x) \supset B(x))$	$\sim (Ex)(A(x) \& B(x))$
I	*O*
$(Ex)(A(x) \& B(x))$	$\sim (Ex)A(x) \vee (Ex)(A(x) \& \sim B(x))$

The purpose of this paper is to show the following fact. With the above representations of the traditional forms, (and without adding any peculiar axioms), Aristotle's syllogistic is completely translated into the monadic predicate calculus or the calculus of classes in the sense that all the true Aristotlian syllogisms are provable and all the rejected forms are not provable in it. The decision procedure for it is extremely simple and like the traditional one.

§ 1 The true syllogisms

The symbolism used in this paper is that of HILBERT and ACKERMANN (1). The symbolic representations of *A, E, I,* and *O* forms are:

Aab	*Eab*
$\neg(\alpha \subset \overline{\alpha}) \land (\alpha \subset \beta)$	$\alpha \subset \overline{\beta}$
Iab	*Oab*
$\neg(\alpha \subset \overline{\beta})$	$(\alpha \subset \overline{\alpha}) \lor \neg(\alpha \subset \beta)$

A, O and *E, I* are two pairs of *contradictories.* As the following formula is provable, *I* and *O* are *subalterns* of *A* and *E,* respectively, i.e. «If *Aab*, then *Iab*.», and, «If *Eab*, then *Oab*.»

$$\neg(\alpha \subset \overline{\alpha}) \land (\alpha \subset \beta) \rightarrow \neg(\alpha \subset \overline{\beta}).$$

A and *E* are *contraries. I* and *O* are *subcontraries,* i.e. contradictories of contraries.

As *E* and *I* are symmetric in α and β, *E* and *I* are convertible by *simple conversion.*

The four true syllogisms in the first figure Barbara, Celarent, Darii, and Ferio are represented by the following formulas, respectively, and obviously provable.

$$\neg(\beta \subset \overline{\beta}) \land (\beta \subset \gamma) \land \neg(\alpha \subset \overline{\alpha}) \land (\alpha \subset \beta) \rightarrow \neg(\alpha \subset \overline{\alpha}) \land (\alpha \subset \gamma)$$
$$(\beta \subset \overline{\gamma}) \land \neg(\alpha \subset \overline{\alpha}) \land (\alpha \subset \beta) \rightarrow (\alpha \subset \overline{\gamma})$$
$$\neg(\beta \subset \overline{\beta}) \land (\beta \subset \gamma) \land \neg(\alpha \subset \overline{\beta}) \rightarrow \neg(\alpha \subset \overline{\gamma})$$
$$(\beta \subset \overline{\gamma}) \land \neg(\alpha \subset \beta) \rightarrow (\alpha \subset \overline{\alpha}) \lor \neg(\alpha \subset \gamma)$$

According to Aristotle only these four syllogisms are perfect. The other true syllogisms are reducible to one of them. The traditional reduction procedure is admissible in our system. For example, we show that other true syllogisms in the universal mood, (of which both the premisses and the conclusion are universal) are reducible to Celarent.

From Celarent, by conversion of *E,* we obtain Cesare. Also from Celarent, by interchanging of α and γ, (and owing to the commutative law of conjunction), we obtain Camestres:

$$\neg(\gamma \subset \overline{\gamma}) \wedge (\gamma \subset \beta) \wedge (\alpha \subset \overline{\beta}) \rightarrow (\alpha \subset \overline{\gamma}).$$

From Camestres, by conversion of *E*, we obtain Calemos.

Remark: The traditional reduction procedure is not applicable to Baroco and Bocardo. We can easily show that these two syllogisms sre provable. But, as is seen in the next paragraph, we have only to show that all the true syllogisms in the universal mood are provable. The five subaltern moods Barbari, &c. are obviously provable.

In conclusion, each of the four figures has 6 true syllogisms, and there are in all 24 true syllogisms.

§ 2 The rejected forms

The decision problem has been solved for the calculus of classes. In our system a more elementary decision procedure is available.

We classify all the possible combinations into the following six cases.

[Case 1] Both the premisses are universal

[1.1] The conclusion is universal

[1.2] The conclusion is particular.

[Case 2] One of the premisses is universal and the other is particular.

[2.1] The conclusion is universal.

[2.2] The conclusion is particular.

[Case 3] Both the premisses are particular.

[3.1] The conclusion is universal.

[3.2] The conclusion is particular.

Owing to the equivalence of $M \wedge N \rightarrow C$ and $\neg C \wedge N \rightarrow \neg M$, and the substitution of variables, (and occasionaly owing to the commutative law of conjunction), we can see that to any one of the moods in the case [1.1] there correspond the two moods in the case [2.2]. The correspondence is seen from the following table: P, Q, and R are the particular forms which contradict the universal forms U, V, and W, respectively.

$U \wedge V \rightarrow W$		
(in the 1st figure)	$R \wedge V \rightarrow P$ (in the 3rd figure)	$U \wedge R \rightarrow Q$ (in the 2nd figure)
(in the 2nd figure)	$V \wedge R \rightarrow P$ (in the 3rd figure)	$U \wedge R \rightarrow Q$ (in the 1st figure)
(in the 3rd figure)	$R \wedge V \rightarrow P$ (in the 1st figure)	$R \wedge U \rightarrow Q$ (in the 2nd figure)
(in the 4th figure)	$V \wedge R \rightarrow P$ (in the 4th figure)	$R \wedge U \rightarrow Q$ (in the 4th figure)

Now we show that the following combinations of the premisses are rejected.

EE in all the figures — (1)

Proof: A syllogism is of the form $M \wedge N \rightarrow C$, where M, N, and C contain β and γ, α and β, and, α and γ, respectively. None of M, N, and C is universally valid, and therefore, 1-gültig. ((1)pp47-57). The consequent C can be false in the domain of one individual. Take $\beta = O$, and $M \wedge N \rightarrow C$ comes to be false.

EO, OE, and *OO* in all the figures — (2)

Proof: by (1) and the subalternation of *O* to *E*.

Hence we have a traditional *canon of valid inference:* «From two negative premisses nothing can be inferred.»

AA in the 2nd figure — (3)

Proof: C can be false in the domain of two individuals, without giving α and γ the value O. Take $\beta = \overline{O}$, and the antecedent $\neg(\gamma \subset \overline{\gamma}) \wedge (\gamma \subset \beta) \wedge \neg(\alpha \subset \overline{\alpha}) \wedge (\alpha \subset \beta)$ comes to be true.

AI in the 2nd and 4th figures — (4)

Proof: by (3), and by conversion of *I*.

IA in the 2nd and 1st figures — (5)
The following moods are rejected.
EAA in all the figures — (6)
Proof: 1/2 (of *EO*)
(Read «This mood in the 1st figure is transformed into the *EO* combination of the premisses in the 2nd figure.») 2/1 (of *EO*), 3/2 (of *OE*), 4/4 (of *OE*)
AEA in all the figures — (7)
Proof: 1/3 (of *OE*), 2/3 (of *EO*), 3/1 (of *OE*), 4/4 (of *EO*)
AEE in the 1st and 3rd figures — (8)
Proof: 1/2 (of *AI*), 3/2 (of *IA*)
EAE in the 3rd and 4th figures — (9)
Proof: 3/1 (of *IA*), 4/4 (of *AI*)
AAE in all the figures — (10)
Proof: 1/2 (of *AI*), 3/1 (of *IA*), 4/4 (of *AI*)
rejected in the 2nd figure by (3)
AAA in the 2nd, 3rd, and 4th figures — (11)
Proof: This mood in the 3rd figure is:

$$\neg(\beta\subset\bar{\beta})\wedge(\beta\subset\gamma)\wedge\neg(\beta\subset\bar{\beta})\wedge(\beta\subset\alpha)\rightarrow\neg(\alpha\subset\bar{\alpha})\wedge(\alpha\subset\gamma)$$

and in the 4th figure is:

$$\neg(\gamma\subset\bar{\gamma})\wedge(\gamma\subset\beta)\wedge\neg(\beta\subset\bar{\beta})\wedge(\beta\subset\alpha)\rightarrow\neg(\alpha\subset\bar{\alpha})\wedge(\alpha\subset\gamma).$$

Both of these formulas come to be false in the domain of two individuals, when we take $\beta = \gamma = \{1\}$ and $\alpha = \bar{O}$.
As to the [Case 3], all the moods in this case are rejected, as follows.

II is rejected, by (3), and by conversion of *I*s.
IOO (or *IOI* in the 1st and 3rd figure is transformed into *AOE* (or *EOE*) in the 3rd and 1st figure, respectively. Therefore, (and by conversion of *I*), *IO* is rejected in all the figures. Similarly for *OI*.

We have a canon of valid inference: «From two particular premisses no conclusion can be drawn.»

To any one of the moods in the case [2.1], there corresponds at least one mood in the case [3.2], and conversely, so that all the moods in the case [2.1] are rejected. We have a canon of valid inference: «If one premiss is particular, the conclusion must be particular.»

All the true syllogisms in the universal mood (case [1.1]) are transformed into the true syllogisms in the case [2.2], as follows:

Barbara / *OAO* (in the 3rd figure), *AOO* (in the 2nd figure)
Celarent / *IAI* (in the 3rd figure), *EIO* (in the 2nd figure)
Cesare / *AII* (in the 3rd figure), *EIO* (in the 1st figure)
Camestres / *EIO* (in the 3rd figure), *AII* (in the 1st figure)
Calemes / *EIO* (in the 4th figure), *IAI* (in the 4th figure)

As to the case [1.2], this is a *weakened* case of either [1.1] or [2.2]. Counting all the possible weakened conbinations, we obtain the following table of the valid moods.

	1st	2nd	3rd	4th
Case [1.1]	Barbara Celarent	Cesare Camestres		Calemes
Case [2.2]	Darii Ferio	Festino Baroco	Datisi Feriso Disamis Bocardo	Fresison Dimatis
Case [1.2]	Barbari Celaront	Cesaro Camestros	Felapton Darapti	Bamalip Fesapo Calemos

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