"OUGHT" AND CONDITIONALS

Soshichi Uchii *

1. *Introduction.*

Although many papers have been written on deontic logic since von Wright's 1951 paper "Deontic Logic" ([18]), it seems to me that there are some important aspects of "ought" which have been strangely neglected by logicians. For example, no one seems to have tried to incorporate into his system of deontic logic the presence of two components ou "ought," i.e. its *normative* and *descriptive* components, which have been pointed out by many moral philosophers. (1) Another aspect of "ought" which has been neglected is the *inference by modus ponens* from a conditional "ought" and the statement that the condition obtains to the conclusion of that "ought." For instance, the following inference seems valid: If you borrowed money from him, you ought to return it; but you in fact borrowed money; hence you ought to return it. In order to give a satisfactory formalization of "ought" with respect to this modus ponens inference, we need a careful examination of conditional "ought" as well as unconditional "ought." This will require references to the analysis of conditionals in descriptive contexts (counterfactuals, subjunctives, etc.).

In this paper, I should like to present a formal analysis of "ought" and conditional "ought" which takes at least the following two into account: (i) the descriptive as well as the normative component of "ought" and (ii) the modus ponens inference containing "ought." My analysis owes its impetus to Arthur Burks's analysis of causal conditional statements. So to begin with, let us briefly summarize his analysis of "if ... then" in causal contexts.

2. *Burks's Analysis of Causal Conditionals.*

Burks's analysis tries to catch the *causal connection* between the antecedent and the consequent of a conditional like

(1) If this gold ring should be placed in aqua regia, it would dissolve.

For this purpose, he constructed a logic of causal modalities which takes *logical necessity* $\Box$ and *causal necessity* $\Box_c$ as primitive. Logical possibility and strict implication can be defined in terms of $\Box$; i.e.

$$\Diamond A = \sim\Box\sim A$$
$$A \rightarrow B = \Box(A \supset B).$$

Causal possibility $\Diamond_c$ and causal implication $\underset{c}{\rightarrow}$ can be defined analogously in terms of $\Box_c$. And the relation between $\Box$ and $\Box_c$ is determined by the axiom

$$\Box A \supset \Box_c A;$$

i.e. $\Box$ and $\Box_c$ may overlap. (2)

Then the causal connection can be formalized by what Burks calls the *nonparadoxical causal implication* **npc** ([2], p. 176).

(2) $A \mathbf{\ npc\ } B = \sim(A \rightarrow B) \ \& \ (A \underset{c}{\rightarrow} B) \ \& \ \Diamond_c A \ \& \sim\Box_c B.$

That is to say, the causal connection between A and B, being an *empirical* connection, is not logically necessary $(\sim(A \rightarrow B))$ but only causally necessary $(A \underset{c}{\rightarrow} B)$; moreover, it is a *genuine* connection in the sense that A or B alone is not sufficient for the connection $(\Diamond_c A \ \& \sim\Box_c B)$; for if $\sim\Diamond_c A$ or $\Box_c B$ is true, $A \underset{c}{\rightarrow} B$ becomes true by virtue of the antecedent or the consequent alone. Notice that the following two

(3) $\sim\Diamond_c A \supset (A \underset{c}{\rightarrow} B)$

(4) $\Box_c B \supset (A \underset{c}{\rightarrow} B),$

which can be proved in Burks's system (3) and are exactly analogous to the so-called *paradoxes of strict implication*, are the reason for adding the last two conjuncts in (2).

The **npc** is adequate for formalizing those conditionals in which the antecedent states a causally sufficient condition for the consequent. Thus if we may assume that, in (1), being made of gold (*G*) and in aqua regia (*A*) is causally sufficient for this ring to dissolve (*D*), (1) can be translated as follows:

(5) *G* & (*GA* **npc** *D*).

(We will often omit "&" in the following.) The conjunct *G* is necessary because (1) implies that this ring is in fact made of gold.

However, not all conditionals specify causally sufficient conditions in the antecedent. As a matter of fact, it is safe to say that *most* conditionals specify only a *part* of causally sufficient conditions in the antecedent and in this sense are *elliptical*. For example, if we do not explicitly state that this ring is made of gold in (1), it becomes:

(6) If this ring should be placed in aqua regia (*A*), it would dissolve (*D*).

Then **npc** cannot be used to translate (6); for

(7) *A* **npc** *D*

becomes false since *A* is not causally sufficient for D, whereas (6) is true if (1) is true. Hence we need some extra device in order to translate such elliptical conditionals.

In order to look for a better translation of (6), let us compare (6) and its explicit form (1). Assuming the same context, (6) takes it for granted that this ring is made of gold, and the truth of (6) essentially depends on this tacit assumption. However, since this assumption is *tacit*, we cannot explicitly name condition G in our translation. Thus our translation has to state that *there is* a tacit condition which *in fact* holds and, *together with* the condition explicitly stated in the antecedent, is sufficient for bringing about the event described in the consequent. This can be done by using a second order quantifier over statements. Then (6) may become as follows:

(8) $(EX)(X \,\&\, (XA\ \mathbf{npc}\ D))$.

Notice that (5) entails (8) just as (1) entails (6) in informal discourse.

With the same idea as (8), Burks's *elliptical causal implication* (**ec**) can be defined ([2], p. 180).

(9) $A\ \mathbf{ec}\ B = (EX)(X \,\&\, (XA\ \mathbf{npc}\ B))$.

As is clear from what we have seen above, the **ec** is adequate for translating elliptical conditionals. Using **ec**, (8) then becomes:

(10) $A\ \mathbf{ec}\ D$.

For the convenience of later references, we will list a few of the important properties of **npc** and **ec**. (4)

(11) $(AB\ \mathbf{npc}\ C) \supset (A \supset (B\ \mathbf{ec}\ C))$
(12) $(AB\ \mathbf{ec}\ C) \supset (A \supset (B\ \mathbf{ec}\ C))$
(13) $(A\ \mathbf{ec}\ B) \supset (A \supset B)$.

3. *Formal System D.*

With these preliminaries, we now turn to the analysis of "ought." Since we need a framework of formal analysis, let us start from a rather simple system of deontic logic. We will take *moral necessity* $\Box_m$ as the primitive deontic notion; this roughly corresponds to the notion of moral obligation or duty. *Moral possibility* (permissibility) $\Diamond_m$ and *moral implication* $\underset{m}{\rightarrow}$ are defined as follows:

(1) $\Diamond_m A = \sim \Box_m \sim A$
(2) $A \underset{m}{\rightarrow} B = \Box_m (A \supset B)$.

Then formal system D can be given as follows:

A0. All tautologies of propositional calculus (abbreviated as PC)

A1. $\Box_m(A \supset B) \supset (\Box_m A \supset \Box_m B)$

A2. $\Box_m A \supset \Diamond_m A$

R1. From A and $(A \supset B)$ to infer B

R2. From A to infer $\Box_m A$.

The semantics for D can be given by using the technique of Kripke's model theory for modal logic, and the completeness can be proved (5); but we need not go into detail (see Appendix).

Although, as we will see shortly, D is not quite sufficient to handle "ought" and conditional "ought," it still can provide the basic framework of our analysis. As a matter of fact, we will make use of the theorems of D in the following.

4. *Conditions for a Formal Definition of "Ought."*

Now let us make our objectives more precise. "Ought"-statements can be divided into two classes on the syntactic basis: unconditional "ought"-statements like "It ought to be the case that B," and conditional "ought"-statements like "If A then it ought to be the case that B." So let us express the former by "$\Box_o B$" and the latter by "$A \underset{o}{\rightarrow} B$." Then our objectives are to give formal definitions of these two within the framework of D or its extension. And we will impose the following conditions (desiderata), which seem desirable on intuitive grounds, on our definitions:

(i) "Ought"-statements, conditional or unconditional, must have two components, *normative* and *descriptive*. First, "ought" must contain in some way $\Box_m$, because "ought" depends on, or expresses, moral norms or obligations. In this sense we will say that "ought" has a *normative* component. Secondly, we can point out that certain fact sare relevant to an "ought"-statement, so that if one asserts an "ought"-statement, he must be ready to admit that certain descriptive (factual) statements are true. This aspect of "ought" will be called *descriptive*. The point will become clearer with examples. If one says "You might to return the money to me," he usually has *reasons* for this "ought" (e.g. "you" in fact borrowed money from "me" and "you" now have enough money to return, etc.); i.e. there are some descriptive statements which are in fact true and relevant to this "ought." The same holds for conditional "ought" like "If you borowed money, you ought to return it." For conditional "ought" likewise has reasons, and moreover, conditionals are usually elliptical in that they require, in addition to the ontecedent, some extra conditions which are assumed to hold (cf. sec. 2, (6)).

(ii) Our second requirement is the modus ponens inference: one must be able to deduce $\Box_o B$ from $A \underset{o}{\rightarrow} B$ and A. I.e.

(1) $(A \underset{o}{\rightarrow} B) \supset (A \supset \Box_o B)$

must be a theorem of our system. Then it is clear that we cannot interpret $\Box_m$ and $\underset{m}{\rightarrow}$ as $\Box_o$ and $\underset{o}{\rightarrow}$, respectively. For in that case (1) becomes

$$(A \underset{m}{\rightarrow} B) \supset (A \supset \Box_m B),$$

and substituting A for B in this formula, we can obtain

$$(A \underset{m}{\rightarrow} A) \supset (A \supset \Box_m A).$$

But, since $A \underset{m}{\rightarrow} A$ is a theorem of D, we must then admit that $A \supset \Box_m A$ is also a theorem, which is quite disastrous.

(iii) Next, moral necessity must entail "ought." That is,

(2) $\Box_m A \supset \Box_o A$

must hold. (But this may need a modification; see sec. 10, (7).)

(iv) Finally, "ought" must imply moral possibility (permissibility); for it is unreasonable if what we ought to do is not morally permissible. Hence we must have

(3) $\Box_o A \supset \Diamond_m A$.

5. The Paradoxes of Moral Implication.

With the preceding four conditions in mind, we will now look for adequate definitions of $\Box_o$ and $\underset{o}{\rightarrow}$. First, from conditions (i) and (ii), we can notice an interesting parallelism between **ec** and "ought." The descriptive component of "ought" corresponds to the existence of tacit conditions in **ec**, and the modus ponens requirement (1) is quite analogous to (11) or (12) of sec. 2. Then one may offer the following as the simplest candidates for $\Box_o$ and $\underset{o}{\rightarrow}$.

(1) $(EX)(X \,\&\, (X \underset{m}{\rightarrow} A))$ for $\Box_o A$.

(2) $(EX)(X \,\&\, (XA \underset{m}{\rightarrow} B))$ for $A \underset{o}{\rightarrow} B$.

However, it is easy to see that the condition (iv) fails on these definitions, because we have the *paradoxes of moral implication* [6] in D (analogous to (3) and (4) of sec. 2):

(3) $\sim\Diamond_m A \supset (A \underset{m}{\rightarrow} B)$

(4) $\Box_m B \supset (A \underset{m}{\rightarrow} B)$.

Of these two, (3) is disastrous. And indeed it is (3) that makes (1) inadequate as a definition of "ought." Suppose something which is morally impossible has actually happened ($(EX)(\sim\Diamond_m X \,\&\, X)$); then, according to (1), another thing A which is also morally impossible becomes an "ought" because of (3), thus:

(5) $\sim\Diamond_m A \,\&\, (EX)(X \,\&\, \sim\Diamond_m X) \supset (EX)(X \,\&\, (X \underset{m}{\rightarrow} A))$.

Hence (3) of sec. 4 does not hold. Also, undesirable features are produced by (2) and (3) with respect to $\underset{o}{\rightarrow}$.

On the other hand, (4) seems innocuous. If B is morally necessary, certainly it should be an "ought" under any circumstances. Indeed, (4) turns out to be conductive to condition (iii), as we will see shortly.

Then we can draw the following suggestion from the preceding considerations: The descriptive component of "ought" (conditional or unconditional), which corresponds to reasons for "ought," must *normally* express *morally permissible* states of affairs. Notice that this requirement is analogous to the element $\Diamond_c A$ in A **npc** B (sec. 2, (2)). In the following section, we will incorporate this requirement into our definitions of "ought."

However, we have to point out that this maneuver does not completely resolve the puzzle generated by (3). For we can still ask "What ought we to do if a morally impossible thing happens ?" This is a genuine moral question. Although we cannot pursue this question in this paper, a suggestion how to handle it may be in order here. My suggestion is to introduce the notion of *relative* moral necessity $\Box_m(B/A)$ (read: B is morally necessary given that A happened), which is analogous to von Wright's dyadic O-operator $O(q/p)$ (see [19], pp. 4, 22 ff.). That is, if $\sim\Diamond_m A$ and A actually happens, we have to consider what we ought to do *relative to this situation*, since we cannot change what has happened. So our moral consideration must shift to a *different level*, i.e. from absolute norm ($\Box_m \sim A$) to relative norm ($\Box_m(B/A)$). Then "ought" in a new level can be analysed (in terms of relative moral necessity) in a manner analogous to what we will show in the following. Incidentally, I am inclined to restrict the use of relative moral necessity to those cases where morally impossible things have happened.

6. *Definitions of "Ought."*

First we will define *normal moral implication* (**nm**) in order to escape the influence of (3) of sec. 5.

(1) $A \ \mathbf{nm} \ B = \Diamond_m A \ \& \ (A \underset{m}{\rightarrow} B)$.

Given this definition, we can derive in D

(2) $A \mathbf{nm} B \supset \Diamond_m(AB)$.

Then we can define "ought" and conditional "ought":

(3) $\Box_o A = (EX)(X \& (X \mathbf{nm} A))$
(4) $A \underset{o}{\rightarrow} B = (EX)(X \& (XA \mathbf{nm} B))$.

It should be noticed that $A \underset{o}{\rightarrow} B$ is not equivalent to $\Box_o(A \supset B)$. We will show that these definitions in fact satisfy the four conditions of sec. 4.

Condition (i) is satisfied. The only remark we need to make with respect to this condition is that the descriptive component of "ought" may be vacuous, i.e. logically true, in a special case.

That condition (ii), the modus ponens requirement, is satisfied can be proved (informally) as follows (7):

(a)	$A \underset{o}{\rightarrow} B$	Premise
(b)	A	Premise
(c)	$(EX)(X \& (XA \mathbf{nm} B))$	(a), (4)
(d)	$X \& (XA \mathbf{nm} B)$	(c), existential instantiation
(e)	$XA (XA \mathbf{nm} B)$	(b), (d)
(f)	$(EY)(Y \& (Y \mathbf{nm} B))$	(e), existential generalization
(g)	$\Box_o B$	(f), (3) Q.E.D.

In addition to

(5) $(A \underset{o}{\rightarrow} B) \supset (A \supset \Box_o B)$,

we can also prove that

(6) $(AB \underset{o}{\rightarrow} C) \supset (A \supset (B \underset{o}{\rightarrow} C))$

holds; compare this to (11) and (12) of sec. 2. Since the proof is quite analogous to the above one, we need not repeat.

We will next show that (iii) is satisfied. Suppose $\Box_m A$ is true. Then, since

$$(B \vee \sim B) \,\&\, ((B \vee \sim B) \underset{m}{\rightarrow} A)$$

is true (by (4) of sec. 5) and $\Diamond_m (B \vee \sim B)$ (which is a theorem of D) is also true, we obtain

$$(B \vee \sim B) \,\&\, ((B \vee \sim B) \mathbf{\ nm\ } A).$$

By existential generalization, the last yields

$$(EX)(X \,\&\, (X \mathbf{\ nm\ } A)).$$

This shows that

(7) $\Box_m A \supset \Box_o A$

holds. Notice that (4) of sec. 5 plays an essential role in this proof.

Finally, it is obvious that condition (iv) is satisfied, since (3) and (2) entail

(8) $\Box_o A \supset \Diamond_m A.$

Thus our definitions (3) and (4) meet all the four requirements. So next we have to examine the logical properties of $\Box_o$ and $\underset{o}{\rightarrow}$ more closely.

7. *"Ought" and Negation.*

An interesting corollary of our definitions is that $\Box_o A$ and $\Box_o \sim A$, or $A \underset{o}{\rightarrow} B$ and $A \underset{o}{\rightarrow} \sim B$, are compatible. [8] And we can prove the following two:

(1) $\Box_o A \Box_o \sim A \supset (EX)(EY)(XY \,\&\, \Diamond_m X \Diamond_m Y \,\&\, \Box_m \sim (XY))$

(2) $(A \underset{o}{\rightarrow} B)(A \underset{o}{\rightarrow} \sim B) \supset (EX)(EY)(XY \, \& \Diamond_m(XA)\Diamond_m(YA) \, \&$
$(XY \underset{m}{\rightarrow} \sim A))$.

This fact has an interesting implication to the problem of the *conflict of obligation*. As is clear from (1), $\Box_0 A$ and $\Box_0 \sim A$ both hold only if there are two facts such that each is permissible by itself but the two are morally incompatible. This shows that the validity of $\Box_0 A$ depends on *partial* considerations of reality (facts). And in this respect, our $\Box_0$ resembles informal "ought." For, in informal level also, two conflicting acts sometimes are equally justified as an "ought" when each has a supporting reason.

But how should we resolve such conflict of obligation? In informal moral considerations, we might let one "ought" *override* another, or decide to make our moral principles *more specific* so that we may know on what criteria we should make exceptions. In any case, we need a decision to shift from old norms (principles) to a new set of norms. If this is the case, a similar move can be made in our formal model also, along the line suggested in the last paragraph of sec. 5. Namely, we can shift from moral necessity $\Box_m$ to relative moral necessity $\Box_m(\ /\ )$. Notice that $\Box_0 A \Box_0 \sim A$ implies by (1) that a morally impossible thing has occurred.

The case of conditional "ought" (2) is a little bit different. We have to distinguish two cases with respect to XY in (2): (i) $\Diamond_m(XY)$ and (ii) $\sim\Diamond_m(XY)$. If (i) is the case, then $(A \underset{o}{\rightarrow} B)$ $(A \underset{o}{\rightarrow} \sim B)$ implies $\Box_0 \sim A$, so that we can prevent the conflict between $\Box_0 B$ and $\Box_0 \sim B$ by bringing about the state $\sim A$. If (ii) is the case, on the other hand, this demands the shift to relative moral necessity as in the case of (1).

Now from (1) and (2) above, we can specify a sufficient condition under which any conflict of "ought" cannot arise:

(3) $\sim (EX)(X \,\&\, \Box_m \sim X) \supset (\Box_0 A \supset \sim \Box_0 \sim A)$
(4) $\sim (EX)(X \,\&\, (X \underset{m}{\rightarrow} \sim A)) \supset ((A \underset{o}{\rightarrow} B) \supset \sim (A \underset{o}{\rightarrow} \sim B))$.

Also, it should be clear that $\Box_0 A \,\&\, \Box_0 \sim A$ is not equivalent with $\Box_o(A{\sim}A)$. The latter is a contradiction; i.e.

(5) $\Box_o(A{\sim}A) \equiv A{\sim}A$.

Likewise,

(6) $(A \underset{o}{\rightarrow} B{\sim}B) \equiv B{\sim}B$.

8. *Transitivity and Contraposition of Conditional "Ought."*

Another consequence of our definitions is that $\underset{o}{\rightarrow}$ is not transitive ; i. e.

$$(A \underset{o}{\rightarrow} B)(B \underset{o}{\rightarrow} C) \supset (A \underset{o}{\rightarrow} C)$$

does not hold. In this respect, $\underset{o}{\rightarrow}$ is in accord with conditional "ought" in informal discourse. For consider the following example: Let

O = Your parents are very old
C = You take care of them
T = Your parents thank you.

And suppose we assert both "If O then it ought to be the case that C" and "If C then it ought to be the case that T." But this by no means commits us to assert "If O then it ought to be the case that T," especially if $O{\sim}C{\sim}T$ is actually true.

But notice that both $\underset{m}{\rightarrow}$ and **nm** are transitive, which clearly shows that our (informal) conditional "ought" has a different structure from them.

Next, we can point out that the law of contraposition ("If A then B" implies "If ${\sim}$B then ${\sim}$A") does not hold for $\underset{o}{\rightarrow}$. I.e.

$$(A \underset{o}{\rightarrow} B) \supset ({\sim}B \underset{o}{\rightarrow} {\sim}A)$$

does not hold. Thus the antecedent and the consequent of conditional "ought" are not symmetrical. This is also a merit of our analysis. Using the same example as above, it should be clear that

$$(C \underset{0}{\rightarrow} T) \supset (\sim T \underset{0}{\rightarrow} \sim C)$$

does not, and should not, hold. However, notice that

(1) $(A \underset{0}{\rightarrow} B) \supset (\sim \Box_0 B \supset \sim A)$

does hold; this is a consequence of the modus ponens requirement (sec. 6, (5)).

9. *Other Properties of Conditional "Ought."*

Finally, we will mention the important properties of $\underset{0}{\rightarrow}$ with respect to a conjunction in the consequent and a disjunction in the antecedent. That

(1) $(A \underset{0}{\rightarrow} BC) \supset (A \underset{0}{\rightarrow} B)(A \underset{0}{\rightarrow} C)$

holds is obvious. However, the *converse* of (1) does not hold, because $(A \underset{0}{\rightarrow} B)(A \underset{0}{\rightarrow} \sim B)$ is not, whereas $(A \underset{0}{\rightarrow} B \sim B)$ is, a contradiction (sec. 7).

Next,

$$((A \vee B) \underset{0}{\rightarrow} C) \supset (A \underset{0}{\rightarrow} C)(B \underset{0}{\rightarrow} C)$$

may seem to hold but does not; only a much weaker

(2) $((A \vee B) \underset{0}{\rightarrow} C) \supset (A \underset{0}{\rightarrow} C) \vee (B \underset{0}{\rightarrow} C)$

holds. For $\Diamond_m(A \vee B)$ implies only $(\Diamond_m A \vee \Diamond_m B)$, not $\Diamond_m A \Diamond_m B$. Further,

$$(A \underset{0}{\rightarrow} C)(B \underset{0}{\rightarrow} C) \supset ((A \vee B) \underset{0}{\rightarrow} C)$$

seems desirable but this does not hold either. One may think that this is a defect of our definition, but notice that

(3) $(A \underset{o}{\rightarrow} C)(B \underset{o}{\rightarrow} C) \supset ((A \vee B) \supset \Box_o C)$

holds, so that we can infer $\Box_o C$ from three premises $(A \underset{o}{\rightarrow} C)$, $(B \underset{o}{\rightarrow} C)$ and $A \vee B$.

10. *"Ought" and Alethic Modalities.*

So far we have used system D with (informal) second order quantification as the framework of our analysis. And because of this, our definitions turn out to be still defective in some important respects. For in D,

(1) $A \underset{m}{\rightarrow} A$

is a theorem so that we have, according to our definitions,

(2) $\Diamond_m A \supset (A \underset{o}{\rightarrow} A)$.

Hence we can also derive

(3) $\Diamond_m A \supset (A \supset \Box_o A)$,

which clearly shows that $\Box_o$ is a bit too weak. This defect, however, can be remedied by introducing alethic modalities, e.g. logical modalities.

We can then define *genuine moral implication* **gm**:

(4) $A \mathbf{\,gm\,} B = \Diamond(A \sim B)\Diamond_m A(A \underset{m}{\rightarrow} B)$.

(If causal modalities are also introduced, $\Diamond$ in (4) should be replaced by $\Diamond^c$.) Compare (4) with **npc**. Tten we can modify our difinitions of $\Box_o$ and $\rightarrow$ as follows:

(5) $(EX)(X \,\&\, (X \mathbf{\,gm\,} A))$ for $\Box_o A$

(6) $(EX)(X \,\&\, (XA \mathbf{\,gm\,} B))$ for $A \underset{o}{\rightarrow} B$.

With this modification, (2) and (3) are excluded while all nice properties of the old definitions are preserved, except that

(7) $\sim \Box A \supset (\Box_m A \supset \Box_o A)$

replaces $\Box_m A \supset \Box_o A$. [9]

Then in this extended framework, how should we fix the relation between $\Box_m$ and $\Box$? I would naturally recommend the following:

(8) $\Box A \supset \Box_m A$

(or $\Box_c A \supset \Box_m A$, in case we introduce $\Box_c$ also). For (8) enables "ought" to imply "can"; i.e.

(9) $\Box_o A \supset \Diamond A$

holds by virtue of $\Diamond_m A \supset \Diamond A$, which is nothing but another way to express (8).

11. *Concluding Remarks.*

The formal analysis of "ought" sketched in this paper seems to catch the essential properties of "ought" with respect to its two components and the modus ponens inference. And it also seems to be intuitively sound with respect to several other properties of "ought," e.g. that "ought" implies permission, the nontransitivity, and the asymmetry between the antecedent and the consequent, of conditional "ought," etc. However, there are still other problems of "ought" which must be considered by logicians and moral philosophers alike. I should like to mention only two such problems here: (i) the relation between "ought" and imperatives, and (ii) the universalizability of "ought." I am still unclear about the nature of (i), so I will refrain from making any comments on it. [10] On the other hand, I have some suggestions how to handle (ii).

The universalizability of "ought" may be roughly formulated thus: If (person) a ought to do F, anyone, who is in similar circumstances as those of a (in the relevant respects), ought to do F.[11] Now it seems to me that the universalizability of moral "ought" is essentially that of moral necessity; that is, letting A be any act or state of affairs realizable by a,

(1) $\Box_m Aa$ implies $(x)\Box_m Ax$.

In other words, the individuals are symmetrical with respect to moral necessity.

I believe that the symmetry of individuals is (epistemologically) an indispensable requirement of any kind of necessity; and I have elsewhere shown that the analogues of (1) in terms of $\Box$ and $\Box_c$ can be constructed within a logic of causal modalities ([15], sec. 4 and [17], sec. 11). For example,

(2) $\Box_c Ac \supset (x)\Box_c Ax$

(where c is an individual *thing*) and, if at most x is free in Ax and Ax is without any individual constant,

(3) $(Ex)\Box_c Ax \supset (x)\Box_c Ax$

hold in that logic.

Then the universalizability of "ought" may be roughly formalized in (an extension of) our framework as follows:

(4) $\Box_o Fa = (EX)(Xa \;\&\; (Xa \;\mathbf{nm}\; Fa))$, where X is a predicate variable; and $\Box_o Fa$ is universalizable in the sense that $(EX)\,(Xa \;\&\; (Xa \;\mathbf{nm}\; Fa) \supset (EX)\,(Xa \;\&\; (x)\,(Xx \;\mathbf{nm}\; Fx))$ holds.

Notice that (4) pretty well corresponds to the preceding informal formulation o the universalizability.[12] In order to establish the universalizability in the sense of (4), we need

(5) $\Diamond_m Aa \supset (x)\Diamond_m Ax$

as well as (1). But (5) can be derived from the analogue of (3), i.e.

(6) $(Ex)\Box_m Ax \supset (x)\Box_m Ax$,

which yields (by contraposition and substitution)

(7) $(Ex)\Diamond_m Ax \supset (x)\Diamond_m Ax$.

Although (4) above is only a rough suggestion, it shows that the universalizability of "ought" can be handled in principle within an extension of our framework.

Kyoto University

APPENDIX
ON SECOND ORDER MODAL PROPOSITIONAL LOGIC

Since we have used in our analysis the technique of second order quantification, our framework is essentially that of second order modal propositional logic. In the following, we will show how it can be rigorously formalized; for the sake of simplicity, we will understand that $\Box$ can be interpreted either as logical, causal, or moral necessity depending on our purposes.

(I) Formation Rules

Suppose the list of denumerably infinite propositional variables p_1, p_2, ... is given for language *L*. We will take $\sim$, $\supset$, $\Box$, and (X) (universal quantifier, where X is any propositional variable) as the primitive connectives. Then a *wff* (well-formed formula) can be defined as follows:

(1) (a) Any propositional variable is a wff.
(b) If A is a wff, so are $\sim A$ and $\Box A$.
(c) If A and B are wffs, so is $A \supset B$.
(d) If A is a wff and X is a propositional variable, (X)A is a wff.

We will say that the wffs obtained by (a)-(c) alone are *first order* wffs.

We can make the distinction of *bound* and *free* (occurrences of) variables as usual. And we will introduce the notation "A^X_B" of the substitution for free X in A.

(2) Let A and B be wffs such that (i) B is a first order wff and (ii) for each free variable Y in B, X does not occur as a free variable within the scope of (Y) in A; then A^X_B stands for the result of replacing all free occurrences of X in A with B. If (i) and (ii) are unsatisfied, $A_B = A$.

(II) Axiomatic Systems

Various systems can be formed by taking diferent combinations of the following:

A0. All tautologies (PC)
A1. $\Box(A \supset B) \supset (\Box A \supset \Box B)$
A2. $\Box A \supset \Diamond A$
A3. $\Box A \supset A$
A4. $\Box A \supset \Box\Box A$
A5. $A \supset \Box\Diamond A$
A6. $\Diamond A \supset \Box\Diamond A$
A7. $(X)A \supset A^X_B$
A8. $(X)(A \supset B) \supset (A \supset (X)B)$, where X is not free in A
A9. $(X)\Box A \supset \Box(X)A$
R1. From A and $A \supset B$ to infer B
R2. From A to infer $\Box A$
R3. From A to infer $(X)A$.

As is well known, various first order systems can be defined as follows:

D = {A0, A1, A2, R1, R2}
T = {A0, A1, A3, R1, R2}
S4 = {A0, A1, A3, A4, R1, R2}
BS = {A0, A1, A3, A5, R1, R2}
S5 = {A0, A1, A3, A6, R1, R2}.

If we add A7 - A9 and R3 to these, the second order systems II D, II T, II S4, II BS, and II S5 are obtained, respectively.

Remarks: (i) A4 and A5 are theorems of S5; more precisely, A4 plus A5 are deductively equivalent to A6 relative to system T. However, these three are independent relative to D. (ii) The converse of A9 (A9 is an analogue of the Barcan formula)

$$\Box(X)A \supset (X)\Box A$$

is provable even in II D. (iii) The second order system without modalities, i.e.

$$\text{II PC} = \{A0, A7, A8, R1, R3\},$$

is virtually equivalent to PC itself; because for any A (without modalities), letting T = Y⊃Y and F = ∼T (where Y does not occur in A),

$$(X)A \equiv A^{X}_{T} \,\&\, A^{X}_{F}$$

can be proved in II PC (see Church [4], pp. 151-152, 298-299). (iv) Systems with more than two kinds of modalities can easily be constructed; but we do not go into detail.

(III) Model Theory

The semantics for the preceding systems can be given by Kripke's model theory (Hughes & Cresswell [8] is a very good introduction to the Kripke model theory). In the triple (w_a, W, R), let $W \neq \varnothing$, $w_a \in W$, and R be a dyadic relation over W. The intuitive idea is that W is the set of possible worlds, w_a the actual world, and R(w_i, w_j) means that w_j is *possible relative to* w_i (i.e. if A is true in w_j, A is posible in w_i). A *model* is any such triple.

Each wff can be evaluated in a model (w_a, W, R); the essential part of the evaluation rule is this:

(3) (a) If A = □B, then A is true in $w_i \in W$ iff in each w_j such that R(w_i, w_j), B is true.
(b) If A = (X)B then A is true in w_i iff for each *first order wff* C, B^{X}_{C} is true in w_i.

By imposing various conditions on R, we can obtain different types of model. Namely, (w_a, W, R) is a (a) II D model, (b) II T model, (c) II S4 model, (d) II BS model, or (e) II S5 model according as

(a) for each $w_i \in W$, there is a $w_j \in W$ such that $R(w_i, w_j)$
(b) R is reflexive
(c) R is reflexive and transitive
(d) R is reflexive and symmetrical
(e) R is an equivalence relation (i.e. reflexive, transitive, and symmetrical).

We will say that a wff A is *true in a model* (w_a, W, R) iff A is true in w_a; and A is *valid in* a system (II D - II S5) iff A is true in *every* model of the corresponding type (II D model - II S5 model). The *completeness* of each of these systems can be proved (I am planning to publish a proof).

REFERENCES

[1] BURKS, A. W. "The Logic of Causal Propositions." *Mind* 60 (1951): 363-382.

[2] ——. "Dispositional Statements." *Philosophy of Science* 22 (1955): 175-193.

[3] ——. *Cause, Chance, and Reason.* Unpublished book originally copyrighted in 1963 and 1964. References are made to the latest revised version, 1971.

[4] CHURCH, A. *Introduction to Mathematical Logic, Vol. I.* Princeton University Press, 1956.

[5] HARE, R. M. *The Language of Morals.* Oxford University Press, 1952.

[6] ——. "Universalizability." *Proceedings of the Aristotelian Society* LV (1954-5): 295-312.

[7] ——. *Freedom and Reason.* Oxford University Press, 1963.

[8] HUGHES, G. E. and CRESSWELL, M. J. *An Introduction to Modal Logic.* London: Methuen, 1968.

[9] KRIPKE, S. A. "Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculi." *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 9 (1963): 67-96.

[10] ——. "Semantical Considerations on Modal Logic." *Acta Philosophica Fennica* 16 (1963): 83-94.

[11] LEMMON, E. J. "Algebraic Semantics for Modal Logics I." *Journal of Symbolic Logic* 31 (1966): 46-65.

[12] Prior, A. N. "The Paradoxes of Derived Obligation." Mind 63 (1954): 64-65.
[13] SELLARS, W. *Science and Metaphysics: Variations on Kantian Themes.* London: Routledge & Kegan Paul, 1968.
[14] STALNAKER, R. C. "A Theory of Conditionals." In *Studies in Logical Theory*, ed. by N. Rescher, pp. 98-112. *American Philosophical Quarterly*, Monograph Series No. 2, 1968.
[15] UCHII, S. "Inductive Logic with Causal Modalities: A Probabilistic Approach." *Philosophy of Science* 39 (1972), June.
[16] ——. "Yōsō Ronri to Jōkenhō" [Modal Logics and Conditional Statements]. *Zinbun Gakuhō* [*Journal of Humanistic Studies*] 35 (1972): 21-66. Research Institute for Humanistic Studies, Kyoto University.
[17] ——. "Inductive Logic with Causal Modalities: A Deterministic Approach." *Synthese*, fortcoming.
[18] VON WRIGHT, G. H. "Deontic Logic." *Mind* 60 (1951): 1-15.
[19] ——. *An Essay in Deontic Logic and the General Theory of Action. Acta Philosophica Fennica* 21 (1968). Amsterdam: North-Holland Publishing Co.

NOTES

* The substance of this paper is drawn from ch. v of my [16] written in Japanese. However, many changes have been made.

(1) See, for example, Hare [5], pp. 159-160, [7], pp. 26-29; according to his theory, the normative component is *prescriptive*, like imperatives.

(2) For a full presentation of Burks's system, see [1].

(3) See Burks [1], pp. 376-378. Incidentally, "(P_{21}) $p \supset . q\, c\, p$" on p. 377 contains a typographical error: its antecedent should be $\Box_c p$.

(4) For proofs of (11)-(13), see Burks [3], 7.3.3. Also cf. the proof of (5) of sec. 6 below. Incidentally, Stalnaker's theory of conditionals [14] is based on a similar idea as that of **ec** in that it incorporates the elliptical character of conditionals. And in some respects his conditional connective "$>$" (in his C2) is intuitively superior to **ec**. However, the analogue of (12), i.e.

$$(AB > C) \supset (A \supset (B > C))$$

does not hold; hence **ec** is better than $>$ in this respect. If this formula is added as an extra axiom to Stalnaker's system, $A \supset \Box A$ becomes derivable. For a detailed examination and some modifications of Stalnaker's system, see my [16], ch. iv.

(5) For Kripke's model theory, see [9] and [10], and Hughes & Cresswell [8]. For the completeness of D, see Lemmon [11]; our D corresponds to his system T(D).

(6) These are called «the paradoxes of derived obligation» in most literature of deontic logic. See Prior [12] and von Wright [19], p. 4, etc.

(7) This proof can be rigorously formalized in II D of Appendix.

(8) Likewise, A **ec** B and A **ec** ~B are compatible. See Burks [3], 7.3.3., theorem [24a].

(9) One may object that, since, according to (6), $A \underset{0}{\rightarrow} A$ *never* holds, even when $\Box_m A$ is true, (b in top restrictive, for in $\Box_m A$ is true, we will indeed assert "If A is the case, it is as it ought to be." However, this seeming difficulty of (6) can be removed by distinguishing $A \underset{o}{\rightarrow} A$ and $A \supset \Box_0 A$. Although (6) makes

$$\Box_m A \supset (A \underset{o}{\rightarrow} A)$$

always false (unless $\Box_m A$ is false),

$$\Box_m A \supset (A \supset \Box_0 A)$$

is always true (unless, of course, $\Box A$ is true).

(10) For a discussion of the relation between "ought" and imperatives, see Hare [5], secs. 11 and 12; and also Sellars [13], ch. vii.

(11) For the universalizability of "ought," see Hare [6] and [7], pp. 10-14, 30-40.

(12) If the reader has a difficulty in seeing the correspondence, it can be explained as follows: Suppose $\Box_0$Fa is true by virtue of the circumstance C; i.e.

Ca & (Ca **nm** Fa).

Then for *any* individual b such that Cb is true, $\Box_0$Fb is true. For Ca **nm** Fa implies (x)(Cx BF Fx) (by (1) and (5)) and hence Cb BF Fb, so that

Cb & (c. bc. B.F.F.x).

is true. That is, if a ought to do F, then anyone, who is in a similar circumstance as that of a, ought to do F.