# QUANTIFICATION INTO EPISTEMIC CONTEXTS

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In *Knowledge and Belief* ([1]) Hintikka sets out a system of quantified epistemic logic. One of the basic features of his sistem is a set of recommendations for formalizing ordinary language into the formulae of the system. Thesee recommeendations have important consequences for the system. The most noticeable of these consequences is the complexity of the logic. Although Hintikka begins by setting out a relatively simply logic he proceeds to modify, revise, and make the system more complex as he discusses questions of quantificatioon into epistemic contexts.

In this paper I will argue that Hintikka's recommendations for formalizing do not provide the best basis for quantified epistemic logic. There are two main considerations to be taken into account. The first is the question of the «fit» of language to the formulae of the system, the second is the question of the consequences for the logic of particular recommendations for formalizing.

As I proceed to deal with these questions I will set out what seem to be better recommandations for formalizing, and show how these do not have the same consequences for a quantified epistemic logic, but keep the system far simpler, yet preserve the features which Hintikka claims to be desirable.

We begin by turning our attention to Hintikka's recommendations for formalizing. These can be set out in the form of a dictionary made up from various sections of KB, and reasonable conjectures based on what is said in KB. The following is the dictionary. Items marked with (H) are to be found in the text.

| | | | |
|---|---|---|---|
| (1) | $K_a (Ux) Fx$ | (i) | *a* knows that everything is *F* |
| (2) | $K_a (Ex) Fx$ | (ii) | *a* knows that at least one thing is *F* |

(3) $K_a (Ex)(x = b)$ (iii) *a knows that b exists*
((H) *a* knows that there is a *b*)

(4) $(Ux) K_a Fx$ (iv) Everything known to *a* is known by *a* to be *F*
((H) $(Ux) K_a p$ Of each x known to *a* he knows that *p*)

(5) $(Ex) K_a Fx$ (v) Something known to *a* is known by *a* to be *F*
((H) $(Ex) K_a$ p *a* knows who is such that *p*)

(6) $(Ex) K_a (x = b)$ (vi) *a* knows who *b* is (H)
(In what follows we abbreviate $(Ux)$ to $(x)$ ).

The first thing to be noted is that although Hintikka's logic is a free logic, this is not made clear in the dictionary, except for the third entry. There is a failure to make clear *at every point* that bound variables range only over *existing* individuals whilst constants can designate non-existent individuals.

Although it is claimed by many logicians that the existential quantifier automatically carries an existential loading, so that $K_a (Ex) Fx$ has to be read as «*a* knows that there exists at least one F», it is just this claim which free logic tries to make quite explicit. It then beomes possible, although Hintikka does not do it, to quantify over non-existent individuals and to formalise sentences such as «some things do not exist». (2) Furthermore, the claim about the existential quantifier has consequences for the universal quantifier when it is maintained, as does Hintikka, that

$$(Ex)\alpha \equiv \sim (x) \sim \alpha\,.$$

These consequences should be made quite explicit for a free logic.

We need:

(1) $K_a (x)Fx$ (i') *a* knows that every existing x is *F*

(2) $K_a (Ex)Fx$ (ii′) *a* knows that there exists at least one *F*

where the epistemic operator is not in the scope of the quantification.

But where the epistemic operator is in the scope of the quantification, and there is also a variable in the scope of the epistemic operator bound by the quantifier outside the scope of the epistemic operator, the revision proposed is not so straightforward. In order to simplify matters as much as possible, I shall proceed in two moves. In the first move I simply revise Hintikka's recommendations by making the free logic interpretation of the quantifiers explicit. Then I shall reconsider the recommendations in the light of other matters. So, in the first move we have:

(4) $(x)\, K_a Fx$ (iv′) Every existing thing known to *a* is known by *a* to be *F*.

(5) $(Ex)\, K_a Fx$ (v′) There exists at least one thing known to *a* which is known by *a* to be *F*.

(6) $(Ex)\, K_a (x = b)$ (vi′) *b* exists and *a* knows who *b* is.

In fact, the reading of (6) as (vi′) is just what Hintikka suggests in '«Knowing oneself» and other problems in epistemic logic' (3) in reply to a point made by Castañeda.

The insistence on an explicit reading of the existential import of the quantifiers is very important to the question of whether the logic «fits» the language it is supposed to formalize. The word «fits» is the best I can find and by it mean that the structure and logic of the formulae should be reasonably clear from our ordinarly language reading of the formulae. So, the existential import of the quantifiers should be cleear, especially as the logic is a free logic. Also, by «fits» I mean that the construction of the formulae from its well-formed parts should be open to matching in language, step by step in the construction of the formulae.

This latter point is best demonstrated with examples. To construct (1) we begin with *Fa*. *Fa* matches with: *a* is *F*. Then we can move to (x) *Fa* (x/*a*), which matches with: every existing thing is *F*. We can match the construction of (1) with that of (i′).

We now turn to (5) and (v′). We can begin with:

(7) $K_a Fb$

which matches with:
(vii) *a* knows that *b* is *F*.
We then add the quantifier to get:

(5) $(Ex)\ K_a Fx$

which would match with:
(viii) There exists at least one thing such that *a* knows that it is *F*.
Now it must be clear that (viii) does not say that *a* knows *b* (i.e. where *b* is the one existing thing which is *F*), nor does (viii) say that *a* knows that *b* exists. What (viii) does say is that what *a* knows to be *F* actually exists. Another way of putting this would be
(ix) At least one, of what *a* knows to be *F*, exists.
For example, *a* may know that Romeo, Juliet and Caesar are characters in Shakespeare's plays so we can say:

*(Ex)* $K_a$ *x is a character in Shakespeare's plays.*

The difference between (viii) and (ix) on the one hand and (v′) on the other is that in the latter the domain of quantification is greatly restricted.

Similarly with (6) and (vi′), we begin with:

(8) $K_a\ (c = b)$

which matches with:

(x) *a* knows that *c* is *b*.

We then add the quantifier to get:

(6) $(Ex)\, K_a\, (x = b)$

which would match with:

(xi) There exists at least one thing such that *a* knows that it is *b*.

Again, (xi) does not say that *a* knows *b*, or that *a* knows *b* exists. But clearly *a* has to know that something either existent or non-existent is *b*, and whatever it is that *a* knows as *b*, actually exists. This can be ut as:

(xii) What *a* knows as *b* exists.

There is a world of difference between (xii) and (vi'), or between (xii) and *a* knows who *b* is.

Before discussing the readings of (5) and (6) as (ix) and (xii) respectively we turn to

(4) $(x) K_a Fx$

This can be matched with:

(xiii) Each and every existing x is such that *a* knows x is *F*.

For all practical purposes we could not truthfully substitute the name of some human being for *a* in (xiii). The name of some omniscient being (in some sense) would be required. We can note here that while we have read (1) $(K_a\, (x)\, (Fx)$ as indicating that *a* knows a universal generalization, we should refrain in (4) $((x)\, K_a\, Fx)$ from attributing to *a* any knowledge of the universal generalization $(x)Fx$. It just happens that when we consider the list of things of which *a* knows that they are *F* we discover that they exhaust the domain of existing things. The sense of (xiii) could be, therefore, made even clearer by

(xiii') The things which *a* knows to be *F* constitute the totality of existing things.

In terms of a dictionary of formalizations we can summarise our recommendations as:

| | | |
|---|---|---|
| (1) $K_a\, (x)\, Fx$ | *a* knows that every existing x is *F* | (i') |
| (2) $K_a (Ex)\, Fx$ | *a* knows that there exists at least one *F* | (ii') |
| (3) $K_a\, (Ex)\, (x = b)$ | *a* knows that *b* exists | (iii) |

| | | |
|---|---|---|
| (4) $(x)\, K_a\, Fx$ | The things which *a* knows to be *F* constitute the totality of existing things | (xiii′) |
| (5) $(Ex)\, K_a\, Fx$ | At least one, of what *a* knows to be *F* exists | (ix) |
| (6) $(Ex)\, K_a\, (x = b)$ | What *a* knows as *b* exists | (xii) |

Although we might accept these formalizations for the sake of our linguistic intuitions, this alone will not be a sufficient reason for acceptance, even though it is a powerful reason.

There are two main consequences of following Hintikka's recommendations. One is, as we have seen, that the domain of quantification is restricted when epistemic operators are in the scope of the quantifier; the other is that «'knowing who' behaves like 'knowing that'.»

We turn first to the question of the restriction of the domain of quantification. For the purposes of this discussion we shall call any variable, in the scope of an epistemic operator, 0, bound by a quantifier outside the scope of 0, an e-variable.

It would not be quite so problematic if the restriction of the domain of quantification suggested by Hintikka's (iv) and (v) was simply that the ranges of e-variables were restricted to a domain denoted by an abstraction predicate indicated by an epistemic operator in the appropriate context. But this is not so.

Consider (4), (5) and (6). If we accept that the existential quantifier has as its domain only persons known to *a*, then although we should read (4) and (5) almost as Hintikka suggests, we should read (6) as:

(xiv) Someone known to *a* is known by *a* to be *b*

which is obviously synonymous with:

(xv) *a* knows someone and knows him (or her) to be *b*.

This is not to say that *a* knows who *b* is, for *a* can know his next door neighbour, know him to be Mr. Jones, but not know who Mr. Jones is, because Mr. Jones is really the Lord Mayor of Brisbane and *a* does not know that. There is a strong sense in which one can know (be acquainted with) people and yet not know who they are. Knowing who someone is is not a straightforward matter.

Since writing KB Hintikka has indicated that he wants to change the kind of restriction on the domain of quantification in formulae such as (6). In KB

> «It was in effect said that in expressions like '(*E*x) [*a* knows that (*b* = x)]' (i.e., in expressions in which one quantifies into a knowledge context) the bound variable in a sense ranges over individuals known to *a*. What was intended was not the set of *a*'s acquaintances, but something that can be expressed more appropriately by speaking of individuals of whom *a* knows who they are.» (4)

If we accept that the quantifier quantified over those individuals of whom we are to say that *a* knows who they are, instead of quantifying over existing individuals, then we *should* read (3) ($K_a$ (*E*x) (x = *b*)) as:

(xiv) *a* knows that there is at least one person out of those of whom *a* knows who they are who is *b*.

But it might be said that only when a quantifier binds e-variables does it quantify in this way. Then we still have to read

(6) ((*E*x)$K_a$ (x = *b*)) as:

(xv) At least one person from those of whom *a* knows who they are *a* knows to be *b*.

Now, the important thing about (xv) is that it shows clearly that we cannot read (6) as: *a* knows who *b* is. It may be true, because of the domain of (*E*x), that *a* knows who x (someone) is, but it does not follow that *a* knows who *b* is.

For example, let us draw up a list of individuals such that *a* knows of each who he is. We declare this list to be the domain of the quantifier. In that list there is the name 'c' which is read as 'Dr. Jekyll'. Of course, *a* knows who Dr. Jekyll is (it does not follow that he knows Dr. Jekyll). Now let it be the case that *a* has heard of Mr. Hyde, but does not know who he is.

Surely *we* can truly say of *a*

(9) $(Ex)K_a(b = x)$, where '*b*'is read as 'Mr. Hyde'.

(9) is true because, although *a* does not know it, *b* does designate one of the individuals in the domain of the quantifier. But we cannot read (9) as «*a* knows who *b* is.»

The general point needs to be made that when the quantifier, no matter what its domain, is outside the scope of the epistemic operator it is best to read the bound variable in such a way that we make clear that, for example in (9), *a* does not necessarily know that the abstraction predicate for the domain of the quantifier is predicated of some individual designated by x. In the case of (9), read as (xv), the abstraction predicate is '*a* knows who ... is.»

Now, if Hintikka's epistemic logic is to be a free logic, whatever abstraction predicates are to be added to the quantifiers in certain contexts, these predicates should be over and above the predicate of existence. If not, then the quantifiers begin to look like utterly different symbols in different contexts, or as Åqvist calls them «variable quantifiers». [5]

This can be made even more clear when we consider formulae like $(Ex)K_aFx$ and $(Ex)K_bFx$. Here we could have quite different domains of quantification. We can then see that the interpretation of the quantifiers in

(10) $(Ex)\ (Ey)K_aK_b\ (x = y)\ \&\ (y = e))$ could be extremely difficult. These problems *could* be overcome. But the logic would be very complex. Rules would proliferate, as indeed they are already tended to. [6] The question then arises, why have more rules so that the domain of the quantifier can be changed every time there is a change in the epistemic operator ?

If we accept the association of (4) with (xiii'), (5) with (ix) and (6) with (xii) there will be no need to invent elaborate modifications for the original simple rules for quantifiers as set out in KB. [7]

Also, the only restriction of the domain of quantification would be the free logic restriction to a domain of existing objects, and this would be uniform for all formulae.

Uniformity in this matter is something about which Hintikka himself is not altogether clear. In some places such as KB and OKB he opts for restrictions of various kinds. In KOPEL it is not quite clear what is proposed. But in KOPEL he certainly acknowledges the failure in KB to make explicit the existing object domain of quantification in formulae such as (6). In *Models for Modalities* Hintikka denies that he wants restricted quantification for quantification into modal contexts. [8] But then, this might not be relevant to epistemic contexts.

However, whatever the latest opinion is, and whatever the latest complex rule is, our suggestions avoid all of that complexity and uncertainty. We propose a simple logic with a free-logic interpretation of both quantifiers in all contexts.

In that simpler logic neither the Barcan formula nor its converse are valid (self-sustaining). [9] This accords with our reading of the formulae and out intuitive evaluation of the implications. Consider first.

(11) $(x)\ K_a\ Fx \supset K_a\ (x)\ Fx$ (Barcan Formula).

This could be the formalization of a conditional, the antecedent being «The things which *a* knows to be coloured constitute the totality of existing things», and the consequent being «*a* knows that all existing things are coloured».

Clearly, *a* might claim that some non-coloured things exist and so it would be false that he knows that all existing things are coloured. Yet his claim that non-coloured things exist does not falsify the antecedent of the conditional.

Similarly for

(12) $K_a\ (x)\ Fx \supset (x)\ K_a\ Fx$ .

There is nothing wrong (or contradictory) in saying both «*a* knows that every existing thing is coloured», and also «It is false that the things which *a* knows to be coloured constitute the totality of existing things». This is clear in the case where *a* knows that every existing thing is coloured, but does not know that *b* exists and believes that *b* is not coloured.

When we turn to the formulae which affirm the commutation of existential quantifier and epistemic operator we see also that neither are self-sustaining. (9) This also accords with our intuitive evaluation of the implications.

First there is

(13) $K_a(Ex)\,Fx \supset (Ex)K_aFx$ .

This could be the formalization of a conditional, the antecedent being «*a* knows that there actually existed at least one person who was a character in Shakespeare's plays», the consequent being «there actually existed at least one person of whom *a* knows that he was a character in Shakespeare's plays».

Clearly, someone could know that Shakespeare wrote plays in which historical characters appeared without knowing who any of the characters were in any play. Of such a person we could not say that, of the characters he knows to be Shakespearean, at least one existed, because he knows none of the Shakespearean characters. Of such a person we can affirm the antecedent of (13) and deny the consequent.

Similarly for

(14) $(Ex)K_aFx \supset K_a\,(Ex)\,Fx$ .

There is nothing counter-intuitive in saying, «The person known by *a* as a character in a play actually existed but *a* does not know that some characters in plays actually exist». Once again we can affirm the antecedent and deny the consequent.

We now turn to the behaviour of «knowing who». Hintikka heads section 6.13 of KB with *«Knowing who» behaves like «knowing that»*. He writes «that 'knows who' behaves, as far as the technique of model systems is concerned, exactly in the same way as 'knows that'.» (10) A set of rules is proposed which formalizes this feature of «knowing who».

A consequence of these rules is that (14) is valid but (13) is not. Yet one could ask the question, If «knowing who» behaves *exactly* like «knowing that», why are $K_a$ $(Ex)$ $Fx$ and $(Ex)$ $K_a$ $Fx$ not equivalent ? The answer is, of course, that the behaviour of «knowing who» is problematic when formalized as either (5) ($(Ex)K_aFx$) or (6) ($(Ex)$ $K_a$ $(x = b)$), and Hintikka has never really solved the problem.

*Prima facie,* it does seem strange that each of (2) ($K_a$ $(Ex)$ $Fx$), (3) ($K_a$ $(Ex)$ $(x = b)$), (5) and (6) should be read as:

*a* knows ...

This raises the question of the «main» operator.

The «main» operator question is an important one. From a purely syntactic point of view, the main operator in both (5) and (6) is the quantification. But from the point of view of interpretation it looks as though the epistemic operator is the main operator in (6) at least. In KB, Section 6.13, Hintikka introduces two rules, (C.EK = EK = EK =*) and (C.EK =), which have the effect of making the epistemic operator in (6) the main operator. These rules have the same effect as would be the case if we assumed the epistemic operator to be the main operator, and then applied the rules (C.KK*) and/or (C.K).

This gives one the feeling that in Hintikka's logic, not only is the quantifier different from place to place, but also, the epistemic operator is different from place to place. In order to see just what has gone wrong we go back in KB to the point where the idea of reading (5) as a «knowing who» sentence was first mooted.

Hintikka writes:

> «Under what circumstances is it true to say of you, with respect to a certain property, «He knows who has this property» ? For example, when is it true to say of you, «He knows who is the murderer of Toto de Brunel» ? Clearly you know this only if you know a right answer to the question: Who killed Toto ? And this you can do only if there is someone of whom you know that he (or she) killed Toto. The translation of «*a* knows who killed Toto» into our symbolism is

therefore «$(Ex)\ K_a$ (x killed Toto).» In general, *a* knows who has the attribute defined by the expression *p* (which contains the variable x) if and only if the sentence

$$«(Ex)K_a p»$$

is true. This sentence therefore constitutes a translation of the sentence «*a* knows who is such that *p*». ([11])

We have already discussed one defiociency of this suggestion — a deficiency which Hintikka acknowledges indirectly in KOPEL. Although this may not appear to have great effect upon the suggested formalization, nevertheless there is a clear difference between
(xvi) *a* knows who is such that *p*
and
(xvii) *a* knows $who_1$ is such that *p*, and $he_1$ exists.
(The subscripts are to clarify the pronominal reference).

When Hintikka turns to (6) he writes:

«The difference between «*a* knows who the dictator of Portugal is» and «*a* knows that there is a dictator of Portugal» (more idiomatically, «*a* knows that Portugal is a dictatorship»), in short, the difference between
(100) «$(Ex)K_a$ (the dictator of Portugal = x)»
and
(101) «$K_a(Ex)$ (the dictator of Portugal = x)»
illustrates the distinction involved.» ([12])

Here also there is a failure to interpret explicity the quantifier. There is a clear difference between (vi) and
(xviii) *a* knows $who_1$ is such that $he_1$ is *b*, and *b* exists.

Also, we must decide whether in general knowing who is such that *p* or knowing who is such that he is *b* is a sufficient condition for saying that it is known who is *p* or it is known who *b* is. If *p* is taken as x *F*'s, then it seems reasonable to read (5) as implying: *a* knows at least one person who *F*'s. If *F* is read as «murdered Toto», then (5) reads, a little more precisely, as:

(xix) *a* knows at least one person who murdered Toto.
This is not quite
(xx) *a* knows who is *the* murderer of Toto.
Hintikka has simply assumed that contexts like *p*, or *F*x, are functional contexts; that is, that *F* is a functional variable (or constant) and not a predicate variable (or constant).

This assumption is explicit, but not discussed, in (100) and (101) where a definite description is used. [13] If we take examples where we use simple names, the effect is different. Hintikka said that we know who killed Toto only if there is someone of whom we know that he (or she) killed Toto. By analogy, we know who Brown is only if there is someone of whom we know that he (or she) is Brown. That is, we know who Brown is only if $(Ex)K_w$ (x is Brown).

But this will not do. We might have heard Brown mentioned in a news broadcast, or a conversation, and so there is someone of whom we know that he is Brown. In other words, there is someone whom we know is *called* «Brown». But this is not a sufficient condition for knowing who Brown is. Indeed it is not even a necessary condition when we interpret the quantifier as saying that Brown exists. For we might know who Brown is in the same way as we know who Pickwick is, or what Pegasus is, and $\sim(Ex)$ (x = Brown), and indeed $\sim(Ex)K_w$ (x is Brown).

According to our recommendation this last formula would read as:

(xxi) The person we know as Brown does not exist.
And of course, this does not say that we know who Brown is, or that we do not know who Brown is.

Accepting a reading of (6) as (xii), and not as (vi′) (*a* knows who *b* is), leaves us with the problem of formalizing (vi′). A suitable formalization might be based on Hintikka's approach to this statement in KB in terms of an indirect question, «Who is *b* ?» [14] Normally the answer would be of the form «*b* is the *F*» for some predicate *F*. So if *a* knows who *b* is, for some predicate *F*, *a* knows that *b* is the *F*.

Such an approach is likely to lead into higher-order logic.

The phrase «for some predicate *F*» would indicate such. So we leave this problem for the time being.

We have looked at the way in which Hintika's recommendations for reading formulae do not do justice either to free quantification nor to the way in which formulae are built up from their well-formed parts. We have seen how this results in complexities, and in changes in meaning for various operators.

Our recommendations do justice to free-quantification and to the way in which formulae are built up from their well-formed parts. Also there is no need to make the logic complex in order to try to deal with odd consequences. By means of our dictionary we can retain the simplest logic consonant with providing an appropriate explanatory model for ordinary discourse.

If the logic is to provide a «genuine theory of the meaning of the words and expressions» [15] of ordinary language there will have to be some reasonably firm way of relating the theory to the data it is to explain. In discussing epistemic logic too little attention has been paid to this question and the result has been he awkardness of the relationship between model and language.

By adopting the kind of relationship between model and language set out in the revised dictionary above the simpler logic becomes appropriate. Also we preserve a more uniform interpretation of quantifiers and modal operators.

## APPENDIX

We show that neither the Barcan formula (11) nor its converse (12) is self-sustaining, nor are (13) and (14). We proceed by showing that we can have the negation of the formula in question as a member of a model set whose membership is subject to the rules mentioned in footnote 7 and (CP*), (C.KK*), (C.K), (C.~P), and (C.~K). (If we cannot have the negation as a member of a model set then the formula is self-sustaining).

The model system for the negation of (11) is

$\Omega_1$ where $\Omega_1 = \{\mu_1, \mu_2\}$
and $\mu_1 = \{(x)K_aFx, \ \sim K_a(x)Fx, \ P_a(Ex) \sim Fx\}$
and $\mu_2 = \{(Ex) \sim Fx, \ (Ex)\ (x = b), \ \sim Fb\}$

Comment: since there is no formula of the form $(Ex)\ (x = b)$ in $\mu_1$, we cannot «instantiate» $(x)K_aFx$ to $K_aFb$ and, in this way, get an inconsistent $\mu_2$.

The model system for the negation of (12) is

$\Omega_2$ where $\Omega_2 = \{\mu_3, \mu_4\}$
and $\mu_3 = \{K_a(x)Fx, \ \sim(x)K_aFx, \ (Ex)P_a \sim Fx, \ (Ex)\ (x = b), \ P_a \sim Fb, \ (x)Fx, \ Fb\}$
and $\mu_4 = \{\sim Fb, \ K_a(x)Fx, \ (x)Fx\}$

Comment: since there is no formula of the form $(Ex)\ (x = b)$ in $\mu_4$, we cannot «instantiate» $(x)Fx$ to $Fb$ and, in this way, get an inconsistent $\mu_4$. In $\mu_3$, it is possible, for all $a$ knows that $b$ is not $F$. This accords with our example.

$\Omega_3$ where $\Omega_3 = \{\mu_5, \mu_6\}$
and $\mu_5 = \{K_a(Ex)Fx, \ \sim(Ex)K_aEx, \ (x)P_a \sim Fx, \ (Ex)Fx, \ (Ex)\ (x = b), \ Fb, \ P_a \sim Fb\}$
and $\mu_6 = \{\sim Fb, \ K_a(Ex)Fx, \ (Ex)Fx, \ (Ex)\ (x = c), \ Fc\}$

Comment: we cannot «instantiate» $(Ex)Fx$ to $Fb$ and, in this way, get an inconsistent $\mu_6$. This model is of interest because there is a reading of (13) which could incline us to think (13) is self-sustaining. The reading is: If $a$ knows that at least one existing thing is $F$ then at least one existing thing is known by $a$ to be $F$.

A careful look at this reading shows that for the consequent to be true there has to be something, say $b$, such that $K_aFb$ and $(Ex)\ (x = b)$. But if we look at the antecedent, and $\mu_5$, we see that $K_a(Ex)Fx$ does not imply $K_aFb$ even though it implies $(Ex)Fx$ and $Fb$. The antecedent implies only that what $a$ knows is that some existing thing is $F$, but the consequent means that what $a$ knows is that a particular thing, say $b$, is $F$ and $b$ exists.

The model system for the negation of (14) is

$\Omega_4$ where $\Omega_4 = \{\mu_7, \mu_8\}$
and $\mu_7 = \{(Ex)K_aFx, \sim K_a(Ex)Fx, P_a(x) \sim Fx, (Ex)(x = b), K_aFb, Fb\}$
and $\mu_8 = \{(x) \sim Fb, K_aFb, Fb\}$

Comment: we cannot «instantiate» $(x) \sim Fx$ to $\sim Fb$ in $\mu_7$, and in this way make $\mu_8$ inconsistent.

FOOTNOTES

(1) Jaakko HINTIKKA: *Knowledge and Belief, An introduction to the Logic of the two notions* (Cornell University Press, Ithaca) 1962. Chapter 6. [abbreviated hereafter as KB].

(2) cf. R. ROUTLEY: «Some things do not exist» *Notre Dame Journal of Formal Logic* 7 (1966). In this article quantifiers are proposed with no existential import.

(3) *Theoria* 32 (1966) pp. 4-5. [abbreviated hereafter as KOPEL].

(4) «Objects of Knowledge and Belief: Acquaintances and Public Figures.» *The Journal of Philosophy* 67 (1970) p. 880. [hereafter abbreviated as OKB].

(5) Lennart ÅQVIST: «Modal Logic with Subjunctive Conditionals and Dispositional Predicates» *Journal of Philosophical Logic* 2 (1973) p. 53.

(6) See Jaakko HINTIKKA: «Existential Presuppositions and Uniqueness Presuppositions» *Philosophical Problems in Logic* ed. Karel LAMBERT (Reidel, Dordrecht) 1970.

(7) The rules are (C.~E), C.~U), (C.U.), (C.E.), (C.=), (C.Self≠), (C.=K) and (C.=P).

(8) Jaakko HINTIKKA: *Models for Modalities* (Riedel, Dordrecht) 1969, p. 125.

(9) Model systems can be constructed for the negation of each formula. See the Appendix for these systems and some comments on them.

(10) p. 160.

(11) p. 131.

(12) p. 141.

(13) Cf. Jaakko HINTIKKA: «Existence and identity in epistemic contexts: A comment on Føllesdal's paper» *Theoria* 33 (1967), p. 142.

(14) p. 141.

(15) Jaakko HINTIKKA: «Epistemic logic and the methods of philosophical analysis» *Australasian Journal of Philosophy* 46, (1968), p .50.