SOME REMARKS ON TRUTH AND BIVALENCE

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In an earlier article, I discussed the problem of future contingents and proposed a logic that is neither bivalent nor truth functional [1]. Arguing that propositions predicting the outcome of future events are not *yet* true or false, I proposed a system based on the following definitions, where «A» and «B» range over formulas and «V(A)» designates the value of A:

(1) $V(\sim A)$ = t iff $V(A)$ = f or A is a classical contradiction.
= f iff $V(A)$ = t or A is a classical tautology.
Undefined otherwise.

(2) $V(A \supset B)$ = t iff $V(A)$ = f or $V(B)$ = t or $A \supset B$ is a classical tautology.
= f iff $V(A)$ = t and $V(B)$ = f, or $A \supset B$ is a classical contradiction.
Undefined otherwise.

One advantage of this system is that it validates all those and only those tautologies of classical, two-valued logic. But a diffiult problem arises as soon as one rejects bivalence for any reason. Let T be an operator and let TA be interpreted as saying «It is true that A.» Now consider the following argument: [2]

(3) $A \equiv TA$ assumption [3]

(4) $\sim A \equiv T\sim A$ (3)

(5) $A \vee \sim A$ excluded middle

(6) $TA \vee T\sim A$ (3), (4), (5), and substitution.

The problem with this argument is that even though all of the steps seem plausible, the conclusion, $TA \vee T\sim A$, asserts the principle of bivalence. Since I have argued for a non-bivalent system which accepts excluded middle as well as all other classical tautologies, it is clear that I must reject (3). But if the validity of (3) is rejected, what, if anything, could the relation between A and TA be ? This paper will consider the problem of defining the relation between A and TA in a logic that is not bivalent.

I will begin, as I did in my earlier article, by looking at the three-valued system of Lukasiewicz. For him we can infer the necessity of a proposition from its truth and the impossibility of a proposition from its falsity. [4] Let SP say «It is necessary that P» and IP say «It is impossible that P.» Łukasiewicz then has the following matrix:

P	SP	IP
1	1	0
$^1/_2$	0	0
0	0	1

Clearly there is an inherent connection between truth and necessity. Łukasiewicz has $SP \supset P$ as a thesis, but we saw that $P \supset SP$ fails when P is $^1/_2$. Prior suggests that the relation between P and SP be formalized as

(7) $P \supset (P \supset SP)$. [5]

Yet even (7) is problematic since its validity rests on a highly questionable aspect of Lukasiewicz' system: that *all* conditionals whose terms both take $^1/_2$ are to be counted as true. Another shortcoming of Łukasiewicz' system is that it is possible for the outcome of the future to change the value of a proposition not just from $^1/_2$ to 1 or 0, but from 1 to 0 or from 0 to 1. [6] This result is definitely at odds with our intuitions. It makes good sense to say that a proposition takes the third value now but will become either true or false tomorrow. However it defeats the purpose of introducing the third value to allow the outcome of the future to change the value of a proposition that is assigned one of the traditional values today. [7] It would seem that once a proposition becomes true or false, its value should remain unchanged: the only propositions whose value should be affected by the outcome of the future are those which take $^1/_2$.

Leaving Łukasiewicz, let us consider the relation between A and TA in the system which I have proposed. Certainly the material equivalence of A and TA will hold as long as TA⋁T∼A holds, i.e. as long as A is either true or false. But what should the value of TA be if A lacks a value ? One suggestion is that if A lacks a value, then TA ought to lack one as well. In this case, A will always have the same value as TA and will lack a value iff TA lacks one; however A and TA will still not be materially equivalent. If the vaule of A is undefined, then TA will be undefined, making both A⊃TA and TA⊃A undefined as well. On the whole, this suggestion will probably appeal to those who, for philosophic reasons, wish to claim that TA does not really assert anything more than A asserts. (8) The weak point of this suggestion is that it does not allow one to distinguish between a proposition's being false and its not being true, i.e. between T∼A and ∼TA. If TA lacks a value when A does, then T∼A and ∼TA will always take the same value as shown below:

A	∼A	TA	T∼A	∼TA
t	f	t	f	f
f	t	f	t	t
—	—	—	—	—

But surely a system that is not bivalent ought to have a way of distinguishing between the case when A is false and the case when A lacks a value and thus is not true. Without such a distinction, much of the force of rejecting bivalence is lost.

Suppose, therefore, that we allow TA to be false when A has no value. We will then get the following matrix:

A	∼A	TA	T∼A	∼TA	∼T∼A
t	f	t	f	f	t
f	t	f	t	t	f
—	—	f	f	t	t

This alternative forces us to say that propositions concerning the truth or falsity of A are bivalent even though A itself can be neither true nor false. Put another way,

(8) $TA \vee \sim TA$

is valid while $TA \vee T \sim A$, as we have seen, is false when A lacks a value. No doubt the bivalence of propositions concerning the truth or falsity of A is something that sits well with our intuitions. As Prior has argued, questions about the truth or falsity of A are questions about present and thus determinate fact. [9] It is, after all, a determinate fact whether A is true or false *now*. If A has no value, for example, then it is a fact that at the present time A is not true and also that A is not false. The importance of distinguishing between $T \sim A$ and $\sim TA$ is that we get a square of opposition for the operator T.

TA	$T \sim A$
$\sim T \sim A$	$\sim TA$

Here the usual relations between contraries and contradictories hold, i.e. $TA \supset \sim T \sim A$, $T \sim A \supset \sim TA$, TA is the contradictory of $\sim TA$, and $T \sim A$ is the contradictory of $\sim T \sim A$. What is more, A will not have a value if and only if

(9) $\sim T \sim A \wedge \sim TA$

is true. Thus T behaves very much like the square in modal logic. Indeed, all of the following are valid:

(10) $T(A \supset B) \supset (TA \supset TB)$

(11) $TA \supset A$

(12) $TA \supset TTA$

(13) $TA \supset T \sim T \sim A$,

making this system similar to Lewis' S5.

One interesting consequence of denying the material equivalence of A and TA is that

(14) There will be a sea-fight tomorrow

and (15) It is true that there will be a sea-fight tomorrow cannot have the same sense. [10] If (14) lacks a value, then (15) will not lack a value but will be false. Since (15) can be false when (14) is not, ,it is clear that (15) must assert something different from what is asserted by (14). Thus the present analysis is inconsistent with a redundancy theory of truth. Prefacing a sentence with «It is true that ...» creates an entirely new sentence, not a reiteration of the orginal. Indeed, the phrase «It is true that...» creates an opaque context where one would least expect to find it. Following Frege, (15) has a reference (truth value) even though (14) does not. The reference of «there will be a sea-fight tomorrow» in (15) would have to be its customary sense in order for (15) to be false when (14) is neither true nor false. Of course a follower of Frege could argue that (14) does refer when the occurrence of the sea-fight is still contingent by postulating the existence of an object, the Undefined, to join the True and the False. [11] This alternative might have appealed to Łukasiewicz, but it is not consistent with the present analysis since future contingent propositions do not take a peculiar third value but lack a value entirely.

It remains to be seen, however, what the relation between A and TA could be. A particularly illuminating suggestion has been put forward by Bas van Fraassen. [12] According to this view, we must distinguish between both

(16) $A \Vdash TA$

and (17) $TA \Vdash A$

on the one hand and

(18) $\Vdash A \equiv TA$

on the other. [13] Taken together, (16) and (17) say *only* that if A is assigned the value true, then TA is assigned the value true; and if TA is assigned the value true, then A is assigned the value true. They do *not* say that A and TA are always assigned the same value. Thus we can accept (16) and (17) and still maintain that TA is false when A lacks a value. On the other hand, (18) will not hold if we allow the value of TA to be other than that of A. According to the semantic analogue of the deduction theorem, (18) would follow (16) and (17). But in a logic that is not bivalent, (18) amounts to a stronger claim than the conjunction of (16) and (17). When A lacks a value, then $A \supset TA$ will lack one as well.. Hence A and TA will not be materially equivalent even though each is assigned the value true whenever the other one is. The advantages of van Fraassen's suggestion are considerable since it allows us to reject bivalence but still give a plausible account of the relation between A and TA.

Unfortunately, an apparent difficulty is created by rejecting (18) but accepting (16) and (17). As we saw a moment ago, $A \supset TA$ will not have a value if A does not have one. Yet a peculiarity develops in that the value that A will take cannot possibly affect the value that $A \supset TA$ will take. If A becomes true, then $A \supset TA$ will become true. But $A \supset TA$ will also become true if A becomes false. Thus $A \supset TA$ will become true regardless of what value A takes in the future. [14] Now since the outcome of the future can never make the value of $A \supset TA$ anything but true, there is reason to doubt whether $A \supset TA$ should lack a value just because A does. If $A \supset TA$ could become either true or false depending on the value that A takes, then there would be good grounds for saying that $A \supset TA$ should lack a value if A does. But there is only one way that $A \supset TA$ can turn out, namely true. Why not simply claim validity for $A \supset TA$ on the grounds that its truth is inevitable ? The reason why the validity of $A \supset TA$ must be rejected is that by accepting the validity of this formula, we would be accepting the validity of (18) which, when combined with excluded middle, entails bivalence. [15] The problem is that rejecting the validity of $A \supset TA$ requires us to say that a proposition can lack a truth value in the present even if we can be certain what value it will take in the future. Put another way, $A \supset TA$ cannot be a tautology, but neither can it be contingent: it must have a unique status.

Though someone might object that it is paradoxical to say that a proposition can lack a truth value even though we can becertain what value it will take in the future, I wish to argue that this feature is not a weakness of the present system but a strength. While «There will be a sea-fight tomorrow» lacks a truth value at the present, we can be certain that it will obey those laws which are characteristic of bivalent systems when it acquires a value in the future. Since $A \supset TA$ would hold in a bivalent logic, there can be no question that it will become true regardless of what value A is assigned. Hence the inevitability of assigning $A \supset TA$ the value true reflects the fact that propositions dealing with past and present events are either true or false, i.e. that $TA \vee T{\sim}A$ holds for that class of propositions. Far from a weakness, this fact is an essential part of the position I am defending: temooral assymetry. If $A \supset TA$ could become false or remain undefined after A takes a value, then either T would have to have very strange properties or the past and present would have to be «open» in the way that the future is.

One further consequence of accepting (16) and (17) but rejecting (18) is that we have some grounds for saying that the meaning of a sentence cannot be the conditions under which we assign it the value true. ([16]) As we saw above, «There will be a sea-fight tomorrow» and «It is true that there will be a sea-fight tomorrow» cannot have the same sense because the latter will be false if the former lacks a truth value. Yet any situation in which we assign one the value true will be a situation in which we assign the other the value true as well. Thus it is possible for two propositions to have the same truth conditions but to assert different things. ([17]) What is more, it is possible for two propositions to have identical truth conditions but to have different conditions under which they are assigned the value false. TA and A have the same truth conditions, but clearly it is possible for TA to be false though A is not. There ason why these somewhat strange consequences follow is that I have rejected one of the original assumptions of Łuaksiewicz: to be meaningful a proosition must have a truth value. It is possible for A to lack a truth value but to be meaningful nonetheless. Since TA cannot lack a truth value whereas A might lack one, TA must assert something different from what A asserts. Yet, as van Fraassen has shown, we can still hold that A|⊢TA and that TA|⊢A. Thus van Fraassen has provided us with a way of rejecting the argumeent given in (3) - (6) but of retaining a close connection between A and TA.

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NOTES

(1) «Many-valued logic and future contingencies,» *Logique et Analyse*, LVI (1971), pp. 759-773.

(2) Variations of this same argument can be found in B. van Fraassen, «Singular Terms, Truth-Value Gaps, and Free Logic,» *Journal of Philosophy*, LXIII (1966), pp. 493-495; «Presupposition, Implication, and Self-Reference,» *Journal of Philosophy* LXV (1968), pp. 143-146; *Formal Semantics and Logic* (New York: Macmillan, 1971), pp. 163-166; as well as Richmond Thomason, «Philosophical Applications of Logical Theory,» 1970, unpublished lecture notes, pp. 24-25. Notice that in *Formal Semantics and Logic*, van Fraassen interprets T(*A*) as «[*A*] is true» and *not* as «It is true that A.» Thus his version of the argument given in (3)-(6) is not quite the same as mine. I would maintain that (3) appears plausible because it is an analogue of Tarski's principle, not because it is a statement of that principle. Cf. Thomason, *op. cit.*

(3) $V(A \equiv TA) = t$ iff $V(A \supset TA) = t$ and $V(TA \supset A) = t$.

(4) Once again it is worth noting that Łukasiewicz is not talking about logical necessity, but rather the necessity we associate with past and present events.

(5) A. N. Prior, *Formal Logic*, 2nd ed. (1955; rpt. Oxford: Clarendon Press, 1962), p. 249.

(6) Examples of propositions whose values change from 0 to 1 are: (a) $\sim(((\sim P \supset P) \supset P) \supset \sim((\sim P \supset P) \supset P))$, (b) $\sim(\sim(P \wedge \sim P) \supset (P \wedge \sim P))$, (c) $\sim((P \vee \sim P) \supset \sim(P \vee \sim P))$. If P is $1/2$, then (a), (b), and (c) are 0. But if the value of P changes to 0 or 1, then (a), (b), and (c) will take the value 1. Examples of propositions whose values change from 1 to 0 are the negations of (a), (b), and (c).

(7) Obviously this criticism does not apply to formulas containing modal operators.

(8) Examples of those expressing sympathy for this view are: Frege, «On Sense and Reference,» *Translations from the Philosophical Writings of Gottlob Frege,* ed. Geach and Black (1952; rpt. Oxford: Basil Blackwell, 1970), p. 64; F. P. Ramsey, «Facts and Propositions,» *Proceedings of the Aristotelian Society,* Suppl. VII (1927), pp. 153-170; A. J. Ayer, *The Concept of a Person* (New York: St. Martin's Press, 1963), pp. 162-187; P. F. Strawson, «Truth,» *Analysis,* IX (1949), pp. 83-97. Strawson claims that to say that a statement is true is not to make a further statement but denies that to say that a statement is true is not to do something different from just making the statement.

(9) A. N. Prior, «Three-Valued Logic and Future Contingents,» *Philosophical Quarterly* III (1953), pp. 323-324.

(10) For an interesting discussion of this issue, see M. Dummett, «Truth,» in *Philosophical Logic,* ed. P. F. Strawson (Oxford: University Press, 1967), pp. 49-68. Also see Peter Woodruff, «Logic and Truth Value Gaps,» in *Philosophical Problems in Logic,* ed. Karel Lambert (Dordrecht: D. Reidel, 1970), pp. 134-137.

(11) Postulating the existence of this object would allow us to say that «It is true that...» does *not* create an opaque context.

(12) See van Fraassen, *op. cit.*

(13) For an account of the symbol «$\mid\vdash$» see Richmond Thomason, *Symbolic Logic* (New York: Macmillan, 1970), pp. 34 ff.

(14) If A is a sentence like «Pegasus has wings», then $A \supset TA$ will never become true since, presumably, «Pegasus» will never refer. In other words, this problem arises in the context of future contingents but not in that of presupposition failures.

(15) The validity of $TA \supset A$ is not in question since TA is false when A does not have a value.

(16) Cf. *Tractatus* 4.024 and 4.063. Also cf. Dummett, *op. cit.,* pp. 55-56.

(17) By «truth conditions» I mean *only* those conditions under which we assign a proposition the value true.