AUTHORITY: A MATHEMATICAL LOGICAL ANALYSIS

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One science that has received scant attention from symbolic logicians is sociology. Yet whatever the reason may be for this lack of attention it cannot be due to the fact that formal logicians have nothing to contribute to that science. For sociologists deal with formal, as well as material, features of various types of social relationship. And these formal features, which include such relations as transitivity and symmetry, are expressible in mathematical logical notation. In particular, the relationships holding between members of an authority hierarchy are easily expressed in this manner. And the purpose of this paper is to investigate the dyadic relation «in authority over» with the aid of the notation and techniques of mathematical logic.

My reason for introducing mathematical tools into this field is not that such a manœuvre may give the work a scientific air, but, rather, that there are grounds for believing that these tools can be used with profit in the investigation of types of social relationship. Merely expressing such relationships with the aid of mathematical symbols will not, of course, by itself ensure a greater understanding of their nature. But, on the other hand, there are sciences (notably mathematics and physics, and, in our own time, linguistics) that have been greatly aided by the introduction into them of new notation, if only because such innovations have stimulated a changed approach to the subject matter of those sciences; and in this connection one very relevant example that may be cited is the application of the mathematical theory of games to political science. There is thus reason to believe that the application of symbolic logic to sociology may lead to an advance in that science also.

I shall begin by giving an informal account of the logic of «authority» or *Authority Logic* (henceforth abbreviated to AL), after which AL will be presented as an axiomatic system. System AL may be regarded as a basic systematic account of the concept of «authority». To satisfy the methodological need for such an account at this stage of the investigation, the authority hierarchy I shall be describing will be taken to have a rule limiting each of its members to only one office or role.

The elements of AL

E1. The elements of the classical propositional calculus (Polish notation).

E2. a, b, c, ... ad inf. (individual constants).

E3. Y (dyadic relational symbol).

E4. *A*, *K* (dyadic relational symbols).

The terms of AL

T1. a, b, c, ... are terms.

T2. If t_1, t_2 are terms then At_1t_2, Kt_1t_2 are terms.

T3. Something not a term by T1 or T2 is *ipso facto* not a term.

The wffs of AL

W1. All the *wffs* in the propositional calculus (printed in Polish notation) are *wffs* in AL.

W2. If t_1, t_2 are terms then Yt_1t_2 is a wff.

W3. If α, β are *wffs* then $N\alpha$, $K\alpha\beta$,
$A\alpha\beta$, $C\alpha\beta$, $E\alpha\beta$ are *wffs*.

W4. Something not a *wff* by W1, W2, or W3 is *ipso facto* not a *wff*.

Use will be made of conjunctions and disjunctions both of propositional variables and propositional compounds and of constants. Conjunctions and disjunctions of propositions can be investigated for truth value in the normal ways; but conjunctions and disjunctions of individual constants are truth valueless. Hence it is necessary to distinguish between the two types of conjunction and disjunction. The distinction will be made typographically by using plain A and K as usual to symbolise the propositional operators 'or' (inclusive) and 'and' respectively, and *A* and *K* to symbolise the individual operators 'or' and 'and' respectively.

'a', 'b', 'c', etc., symbolise names of members of the authority hierarchy. 'Y' symbolises the dyadic relation «is coordinate with or in authority over», i.e. «is not subordinate to». We can use Y as follows to define three dyadic relational symbols, namely, U, G and S: Uab =df NYba, Gab =df KYabYba, Sab =df NYab. Thus Uab, Gab and Sab symbolise respectively «A is in authority over B», «A is coordinate with B» and «A is subordinate to B».

Among the theses we would wish to include in AL are the following:

1) AAKUabNAGabSabKGabNAUabSabKSabNAUabGab, i.e. for any two members, A and B, of the authority hierarchy, A is either in authority over, coordinate with, or subordinate to B. These alternatives are jointly exhaustive and mutually exclusive.

2) CUabNUba, i.e. U is asymmetrical. From 2) it can easily be proved that U is irreflexive, i.e. 3) NUaa. U is transitive, i.e.

4) CUabCUbcUac. G is both symmetrical and transitive, i.e.

5) CGabGba; 6) CGabCGbcGac. Thus, while U and G are both transitive, U is asymmetrical and G is symmetrical.

7) EUa*K*bcKUabUac.

8) EUa*A*bcAUabUac.

9) CU*A*abcAUacUbc.

10) CU*A*ab*A*cdAAAUacUadUbcUbd. It must be noted that

10) cannot be expressed as a *bi*conditional, for otherwise the following line of argument could be advanced (see p. 46 for the rules of inference):

A) NUaa (= 3))
A. a/*A*ab = B.

B) NU*A*ab*A*ab
10. right to left = C.

C) CAAAUacUadUbcUbdU*A*ab*A*cd
C. c/a, d/b = D.

D) CAAAUaaUabUbaUbbU*A*ab*A*ab

E) CCAAApqrstCNtNAAApqrs (prop. calc.)

E. p/Uaa, q/Uab, r/Uba, s/Ubb, t/U*A*ab*A*ab = Ea; Ea, D, (*RD*) = F.

F) CNU*A*ab*A*abNAAAUaaUabUbaUbb
F, B, (*RD*) = G.

G) NAAAUaaUabUbaUbb

H) CNAAApqrsKKKNpNqNrNs (prop. calc.)
H. p/Uaa, q/Uab, r/Uba, s/Ubb = Ha; Ha, G, (*RD*) = I.

I) KKKNUaaNUabNUbaNUbb

J) CKKKpqrsKqr (prop. calc.)
J. p/NUaa, q/NUab, r/NUba, s/NUbb = Ja; Ja, I, (*RD*) = K.

K) KNUabNUba.

K) states that nobody is in authority over anybody. In other words if 10) were made a biconditional the authority hierarchy would collapse. It can be proved that this situation would also arise if 9) were made a biconditional, i.e. if A) CAUacUbcU*A*abc could also be asserted. The proof is as follows:

A) CAUacUbcU*A*abc (= 10) right to left)
A. c/b = B.

B) CAUabUbbU*A*abb
3. a/b = C.

C) NUbb

D) CNqCCApqrCpr (prop. calc.)

D. p/Uab, q/Ubb, r/U*A*abb = Da; Da, C, (*RD*) = Db; Db, B, (*RD*) = E.

E) CUabU*A*abb

F) CEpqCqp (prop. calc.)
F. p/Ua*A*bc, q/AUabUac = Fa; Fa, 8, (*RD*) = G.

G) CAUabUacUa*A*bc
G. b/a, c/b = H.

H) CAUaaUabUa*A*ab

I) CCApqrCNpCqr (prop. calc.)

I. p/Uaa, q/Uab, r/Ua*A*ab = Ia; Ia, H, (*RD*) = Ib; Ib, 3, (*RD*) = J.

J) CUabUa*A*ab
J. a/*A*ab = K.

K) CU*A*abbU*A*ab*AA*abb
K. (*RTS*) = L.

L) CU*A*abbU*A*ab*A*ab

M) CCpqCCqrCpr (prop. calc.)

M. p/Uab, q/U*A*abb, r/U*A*ab*A*ab = Ma; Ma, E, (*RD*) = Mb; Mb, L, (*RD*) = N.

N) CUabU*A*ab*A*ab

O) CCpqCNqNp (prop. calc.)
0. p/Uab, q/U*A*ab*A*ab = Oa; Oa, N, (*RD*) = P.

P) CNU*A*ab*A*abNUab
3. a/*A*ab = 3a; P, 3a, (*RD*) = Q.

Q) NUab.

An interesting problem is raised by the formula CU*K*abcKUacUbc. Now: «A and B are in authority over C» would normally be taken to imply that neither A nor B is not in authority over C. But if the matter is considered from the point of view of the scope of the authority possessed, a different line can be taken. Within the hierarchy, if A is in authority over C then A and B together have more authority than C, and this is so however B is related to C in the hierarchy. It follows from this that it cannot be concluded from the fact that two people together are in authority over a third that each is in authority over the third. All that can be concluded is that one or other of the two is in authority over the third. I shall, therefore, not adopt CU*K*abcKUacUbc as a valid implication, while noting that a strong «ordinary language» argument can be made out in its favour. The formula I shall accept is the following: 11) CU*K*abcAUacUbc. Though 11) is true of many types of authority hierarchy it is not true of those providing a mechanism whereby two offices may exercise joint authority over a third though neither by itself is in authority over the third.

From what has been said in justification of 11) it follows that if one or other of two people is in authority over a third, the scope of the authority of the two combined must be greater thant that of the third. Therefore, the two together are in authority over the third. That is, 12) CAUacUbcU*K*abc. As 11) and 12) are both theses of AL the following equivalence must also be a thesis: 13) EU*K*abcAUacUbc. It can be proved by hypothetical syllogism from 9) and 12) that 14) CU*A*abcU*K*abc. Since 9) is not a biconditional the reverse of 14) cannot be proved. Hence U*A*abc and U*K*abc are not equivalent.

The calculus I have been outlining is self-consistent, since all the theses 1) to 14) are valid when transformed so that the name constants becomes propositional variables, Uab becomes KNpq (i.e. one-way implication from left to right), Gab becomes Epq, Sab becomes KpNq (i.e. one-way implication from right to left), *A* becomes A, and *K* becomes K. (This is not, of course, to say that all *wffs* of AL which, when transformed according to the above rules, are theorems of the propositional calculus, are also valid in AL).

Now that certain of the more important theses I wish to include in AL have been constructed and discussed, I shall attempt an axiomatisation of the system.

The axioms of AL

A0. Any complete axiomatic basis for the classical propositional calculus.

A1. CYabCYbcYac[1]

A2. AYabYba[1]

A3. EYa*K*bcKYabYac

A4. EYa*A*bcAYabYac

A5. EY*K*abcAYacYbc

A6. EY*A*abcKYacYbc

Rules of inference

Rule of Variable Substitution (RVS): If α is a thesis of AL, and a propositional variable in α is replaced by any *wff*, the *wff* derived thus from α is also a thesis of AL, provided that the same substitution is made wherever that variable appears in α.

Rule of Constant Substitution (RCS): If α is a thesis of AL, and an individual constant a' in α is replaced by any term, the *wff* derived thus from α is also a thesis of AL, provided that the same substitution is made wherever a' appears in α.

Rule of Detachment (RD): If in AL $\vdash \alpha$, $\vdash C\alpha\beta$, then $\vdash \beta$ in AL.

Rule of Replacement (RR): If α is a thesis of AL containing a *wff* A, and A is equivalent to B, then A can be replaced by B in α. The *wff* derived thus from α is also a thesis of AL.

Rule of Term Simplification (RTS): If α is a thesis of AL containing the term (1) Kt_1t_1, (2) At_1t_1, (3) Kt_1t_2, (4) At_1t_2, (5) $KKt_1t_2t_1$, (6) $AAt_1t_2t_1$, (7) $KAt_1t_2t_1$ or (8) $AKt_1t_2t_1$, and if β is derived from α by replacing (1), (2) ..., or (8) by t_1, t_1, Kt_2t_1,

At_2t_1, Kt_1t_2, At_1t_2 t_1 or t_1 respectively, then β is a thesis of AL. [2]

Definitions

Df. U: Uab = NYba

Df. G: Gab = KYabYba

Df. S: Sab = NYab

Theorems of AL

To be proved: T1. NUaa

1) AYabYba (= A2)
1. a/b, b/a = 2.

2) AYbaYab

3) CApqCNpq (prop. calc.)
3. p/Yba, q/Yab = 3a; 3a, 2, (*RD*) = 4.

4) CNYbaYab
4. Df. U = 5.

5) CUabYab

6) CCpqCpNNq (prop. calc.)
6. p/Uab, q/Yab = 6a; 6a, 5, (*RD*) = 6b; 6b. Df. U = 7

7) CUabNUba
7. b/a = 8.

8) CUaaNUaa

9) CCpNpNp (prop. calc.)
9. p/Uaa = 9a; 9a, 8, (*RD*) = 10.

10) NUaa = T1. Q.E.D.

To be proved: T2. EUabSba

1) EUabNYba (by df. U)

2) ESabNYab (by df. S)
2. a/b, b/a = 3.

3) ESbaNYba

4) CEprCEqrEpq (prop. calc.)

4. p/Uab, q/Sba, r/NYba = 4a; 4a, 1, (*RD*) = 4b; 4b, 3, (*RD*) = 5.

5) EUabSba = T2 Q.E.D.

To be proved: T3

AAKUabKNGabNSabKGabKNUabNSabKSabKNUabNGab

1) CKpqNNp (prop. calc.)

1. p/Yab, q/Yba = 1a; 1a. df. G, df. S = 2.

2) CGabNSab

3) CKpqNNq (prop. calc.)

p/Yab, q/Yba = 3a; 3a. df. G, df. S = 4.

4) CGabNSba

4. T2, (*RR*) = 5.

5) CGabNUab

6) CCpqCCprCpKqr (prop. calc.)

6. p/Gab, q/NUab, r/NSab = 6a; 6a, 5, (*RD*) = 6b; 6b, 2, (*RD*) = 7.

7) CGabKNUabNSab

8) CNpNKqp (prop. calc.)

8. p/Yba, q/Yab = 8a; 8a. df. U, df G = 9.

9) CUabNGab

10) CApqCNpq (prop. calc.)

10. p/Yab, q/Yba = 10a; 10a, A2, (*RD*) = 11.

11) CNYabYba

12) CCNpqCNqNNp (prop. calc.)

12. p/Yab, q/Yba = 12a; 12a, 11, (*RD*) = 13.

13) CNYbaNNYab

13. df. U, df. S = 14.

14) CUabNSab

6. p/Uab, q/NGab, r/NSab = 14a; 14a, 9, (*RD*) = 14b; 14b, 14, (*RD*) = 15.

15) CUabKNGabNSab

16) CNpNKpq (prop. calc.)

16. p/Yab, q/Yba = 16a; 16a. df. S, df. G = 17.

17) CSabNGab

13. a/b, b/a = 18.

18) CNYabNNYba

18. df. S, df. U = 19.

19) CSabNUab

6. p/Sab, q/NUab, r/NGab = 19a;

19a, 19, (*RD*) = 19b; 19b, 17, (*RD*) = 20.

20) CSabKNUabNGab

21) AANpKpqNq (prop. calc.)

21. p/Yba, q/Yab = 22.

22) AANYbaKYabYbaNYab

22. df. U, df. G, df. S = 23.

23) AAUabGabSab

24) CpCqCrKKpqr (prop. calc.)

24. p/CUabKNGabNSab, q/CGabKNUabNSab, r/CSabKNUabNGab = 24a;

24a, 15, (*RD*) = 24b; 24b, 7. (*RD*) = 24c; 24c, 20, (*RD*) = 25.

25) KKCUabKNGabNSabCGabKNUabNSabCSabKNUabNGab

26) CAApqrCKKCpKNqNrCqKNpNrCrKNpNqAAKNqNrKN pNrKNpNq (prop. calc.)

26. p/Uab, q/Gab, r/Sab = 26a; 26a, 23, (*RD*) = 26b;

26b, 26, (*RD*) = 27.

27) AAKNGabNSabKNUabNSabKNUabNGab

28) CAApqrCAAKNqNrKNpNrKNpNqAAKpKNqNrKqKN pNrKrKNpNq (prop. calc.)

28. p/Uab, q/Gab, r/Sab = 28a; 28a, 23, (*RD)* = *28b;*

28b, 27, (*RD*) = 29.

29) AAKUabKNGabNSabKGabKNUabNSabKSabKNUabNGab = T3. Q.E.D.

To be proved: T4. ENSabYab

1) ESabNYab (by df. S)

2) CEpNqENpq (prop. calc.)

2. p/Sab, q/Yab = 2a; 2a, 1, (*RD*) = 3.

3) ENSabYab = T4. Q.E.D.

To be proved: T5. EUa*K*bcKUabUac

1. a/b, b/c, c/a = 2.

2) EY*K*bcaAYbaYca

T4. a/b, b/a = 2a; 2a, 2, (*RR*) = 2b;

T4. a/c, b/a = 2c; 2c, 2b, (*RR*) = 2d;

T4. a/*K*bc, b/a = 2e; 2e, 2d, (*RR*) = 3.

3) ENS*K*bcaANSbaNSca

4) CENpANqNrENpNKqr (prop. calc.)

4. p/S*K*bca, q/Sba, r/Sca = 4a; 4a, 3, (*RD*) = 5.

5) ENS*K*bcaNKSbaSca

6) CENpNqEpq (prop. calc.)

6. p/S*K*bca, q/KSbaSca = 6a; 6a, 5, (*RD*) = 7.

7) ES*K*bcaKSbaSca

T2. b/*K*bc = 8.

8) EUa*K*bcS*K*bca

8, 7, (*RR*) = 9.

9) EUa*K*bcKSbaSca

T2, 9, (*RR*) = 10.

10) EUa*K*bcKUabSca

T2. b/c = 10a; 10a, 10, (*RR*) = 11.

11) EUa*K*bcKUabUac = T5. Q.E.D.

To be proved: T6. EU*K*abcAUacUbc

1) EYa*K*bcKYabYac (= A3)

1. a/c, b/a, c/b = 2.

2) EYc*K*abKYcaYcb

T4. a/c, b/*K*ab = 2a; 2a, 2, (*RR*) = 3.

3) ENSc*K*abKYcaYcb

T4. a/c, b/a = 3a; 3a, 3, (*RR*) = 4.

4) ENSc*K*abKNScaYcb

T4. a/c = 4a; 4a, 4, (*RR*) = 5.

5) ENSc*K*abKNScaNScb

6) CENpKNqNrENpNAqr (prop. calc.)

6. p/Sc*K*ab, q/Sca, r/Scb = 6a; 6a, 5, (*RD*) = 7.

7) ENSc*K*abNAScaScb

8) CENpNqEpq (prop. calc.)

8. p/Sc*K*ab, q/AScaScb = 8a; 8a, 7, (*RD*) = 9.

9) EScKabAScaScb

T2. a/Kab, b/c = 9a; 9a, 9, (RR) = 9b;

T2. b/c = 9c; 9c, 9b, (RR) = 9d;

T2. a/b, b/c = 9e; 9e, 9d, (RR) = 10.

10) EUKabcAUacUbc = T6. Q.E.D.

To be proved: T7. EUaAbcAUabUac

1) EYAabcKYacYbc (= A6)

1. a/b, b/c, c/a = 2.

2) EYAbcaKYbaYca

T4. a/Abc, b/a = 2a; 2a, 2, (RR) = 2b;

T4. a/b, b/a = 2c; 2c, 2b, (RR) = 2d;

T4. a/c, b/a = 2e; 2e, 2d, (RR) = 3.

3) ENSAbcaKNSbaNSca

4) CENpKNqNrENpNAqr (prop. calc.)

4. p/SAbca, q/Sba, r/Sca = 4a; 4a, 3, (RD) = 5.

5) ENSAbcaNASbaSca

6) CENpNqEpq (prop. calc.)

6. p/SAbca, q/ASbaSca = 6a; 6a, 5, (RD) = 7.

7) ESAbcaASbaSca

T2. b/Abc = 7a; 7a, 7, (RR) = 7b; T2. 7b, (RR) = 7c;

T2. b/c = 7d; 7d, 7c, (RR) = 8.

8) EUaAbcAUabUac = T7. Q.E.D.

To be proved: T8. EGabGba

1) EKpqKqp (prop. calc.)

1. p/Yab, q/Yba = 2.

2) EKYabYbaKYbaYab

3) EGabKYabYba (by df. G)

3, 2, (*RR*) = 4.

4) EGabKYbaYab

3. a/b, b/a = 4a; 4a, 4, (*RR*) = 5.

5) EGabGba = T8. Q.E.D.

To be proved: T9. CGabCUbcUac

1) CYabCYbcYac (= A1)

1. a/c, b/a, c/b = 2.

2) CYcaCYabYcb

3) CCpCqrCpCKqsr (prop. calc.)

3. p/Yca, q/Yab, r/Ycb, s/Yba = 3a; 3a, 2, (*RD*) = 4.

4) CYcaCKYabYbaYcb

5) CCpCqrCpCNrNq (prop. calc.)

5. p/Yca, q/KYabYba, r/Ycb = 5a; 5a, 4, (*RD*) = 6.

6) CYcaCNYcbNKYabYba

7) CCpCqrCqCpr (prop. calc.)

7. p/Yca, q/NYcb, r/NKYabYba = 7a; 7a, 6, (*RD*) = 8.

8) CNYcbCYcaNKYabYba

9) CCNpCqNrCNpCrNq (prop. calc.)

9. p/Ycb, q/Yca, r/KYabYba = 9a; 9a, 8, (*RD*) = 10.

10) CNYcbCKYabYbaNYca

10. df. U, df. G. = 11.

11) CUbcCGabUac

7. p/Ubc, q/Gab, r/Uac = 11a; 11a, 11, (*RD*) = 12.

12) CGabCUbcUac = T9. Q.E.D.

To be proved: T10. CUabCUbcUac

1) AYabYba (= A2)

2) CApqCNqp (prop. calc.)

2. p/Yab, q/Yba = 2a; 2a, 1, (*RD*) = 3.

3) CNYbaYab

4) CCCpqCqCrsCpCrs (prop. calc.)

4. p/NYba, q/Yab, r/Ybc, s/Yac = 4a;

4a, 3, (*RD*) = 4b; 4b, A1, (*RD*) = 5.

5) CNYbaCYbcYac

6) CCpCqrCpCNrNq (prop. calc.)

6. p/NYba, q/Ybc, r/Yac = 6a; 6a, 5, (*RD*) = 7.

7) CNYbaCNYacNYbc

7. df. U = 8.

8) CUabCUcaUcb

9) CCpCqrCqCpr (prop. calc.)

9. p/Uab, q/Uca, r/Ucb = 9a; 9a, 8, (*RD*) = 10.

10) CUcaCUabUcb

10. c/a, a/b, b/c = 11.

11) CUabCUbcUac = T10. Q.E.D.

To be proved: T11. CGabCGbcGac

1) CYabCYbcYac (= A1)

1. a/c, c/a = 2.

2) CYcbCYbaYca

3. p/Ycb, q/Yba, r/Yca = 3a; 3a, 2, (*RD*) = 4.

3) CCpCqrCqCpr (prop. calc.)

4) CYbaCYcbYca

5) CCpCqrCCsCtuCKpsCKqtKru (prop. calc.)

5. p/Yab, q/Ybc, r/Yac, s/Yba, t/Ycb, u/Yca = 5a;

5a, 1, (*RD*) = 5b; 5b, 4, (*RD*) = 6.

6) CKYabYbaCKYbcYcbKYacYca

6. df. G = 7.

7) CGabCGbcGac = T11. Q.E.D.

The proof of T11 completes our exposition of AL. Although AL does describe certain features of authority, it is unable to cope with all the relational complexities of authority hierarchies. One complication in particular must be mentioned, though I shall do no more than adumbrate the kind of system needed to deal with it.

AL describes a hierarchy in which there is an amalgamation of relations of rank and relations of authority, in the sense that if one person has a higher rank than another *ipso facto* he is in authority over the other, and *vice versa*. An authority hierarchy containing several fields of jurisdiction may, however, lack the amalgamation of relations of rank and relations of authority which is characteristic of AL. In the more complex type of case here envisaged, though in the overall hierarchy A may have a higher rank than B, he may lack authority over B if B's office lies outside A's field of jurisdiction, i.e. if A and B do not occupy offices in a common chain of command. With regard to this type of hierarchy, (henceforth referred to as «H'»), the theorem T3 of AL is invalid, since Uab, Gab and Sab may all fail to capture the relation between A and B in H'. A fourth type of relation is required, which is similar to Hallden's «— is not comparable with —», and which I shall symbolise «H».[3] Since T3 is not a thesis of the logic of H', (abbreviated to «HAL»), AL is not a fragment of HAL.

Since HAL contains a theorem not present in AL, namely, TH1) J_1UabGabSabHab,[4] HAL is not a fragment of AL. How-

[3] *On the Logic of Better*, esp. pp. 45-7.

[4] J_1pqrs is true iff one and only one of p, q, r, s is true; i.e. for the truth table laid out in binary numbers whose 16 decimal equivalents are 15, 14, ..., 1, 0, J_1pqrs can be defined as (0000000100010110)pqrs.

ever, AL and HAL do have some theorems in common, e.g. EUabSba. Hence AL and HAL each contain only a proper subset of the other.

The formal definition of H in terms of Y presents difficulties. Given that one theorem of HAL is TH1) J_4UabGabSabHab, it follows that Hab is equivalent to a) NAAUabGabSab which is equivalent to b) KKYabYbaNGab. But b) is equivalent to KGabNGab, which is self-contradictory. Hence it is not possible for HAL to contain the definitions of U, G and S that were given in AL, together with a definition of H. Thus it would seem that at least one of the definitions of U, G and S must be altered if HAL is to be self-consistent. Uab cannot be defined as NYba in HAL, since Uab excludes Hab as well as Yba; and Sab cannot be defined as NYab in HAL, since Sab excludes Hab as well as Yab.

I shall end this paper with some brief remarks about the logical properties of H. H shares with G the property of symmetry; CHabHba seems to be valid. H shares with U the property of irreflexivity; NHaa seems to be valid. There is however one property that H shares with neither U, G nor S, namely, non-transitivity. With regard to the hierarchy schematised in Fig. 1 CHabCHbcHac is not valid since Uac.

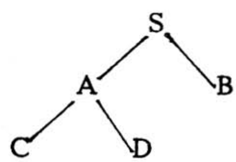

Fig I

On the other hand H is not intransitive (as opposed to 'non-transitive') since, where A, B and C are in different fields of jurisdiction, KKHabHbcHac might be true.

Let us suppose that A is coordinate with B, and hence that A and B are in the same field of jurisdiction. It seems that whoever is not in the same field of jurisdiction as A cannot be in the same field of jurisdiction as B. Thus one logical thesis that relates H to G is (a) CGabCHacHbc. If C is in authority over (or subordinate to) B he must also be in authority over (orsubordinate to) anyone coordinate with B. Likewise if C is coordinate with B he must be coordinate with anyone coordinate with B. Hence, assuming Gab, if Hbc is false so also is Hac. Consequently (a) is valid. By a similar line of reasoning it may also be concluded that (b) CUabCHacHbc is valid. (b) is, however valid only on a certain assumption. There are degrees of «H-ness» since there are what might be termed «degrees of coordination». A may be neither in authority over nor coordinate with C, yet not totally non-coordinate with C either, for although each has authority over someone over whom the other does not have authority, there is someone over whom both are in authority with regard to the same thing. In this situation Hac holds since Uac, Gac, Sac are all false; and, furthermore, both Uab and Ucb may hold though Hac does not, (see Fig. II).

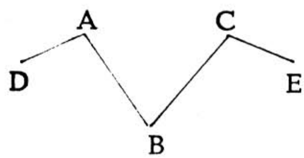

Fig. II

Hence, where H symbolises the relation of partial coordination as well as of full non-coordination, (b) CUabCHacHbc is invalid. (b) appears to hold only where H symbolises the relationship such that Hab is true iff (1) NUab, (2) NSab and (3) A and B are not even partially coordinate. On the other hand (a) CGab CHacHbc appears to be valid even where H is taken to cover the case of not-more-than-partial-coordination.

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