PHYSICAL THEORIES AND POSSIBLE WORLDS

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1 Introduction

Formalized physical theories are not, as a rule, stated in intensional languages. Yet in talking about them we often treat them as if they were. We say for instance: 'Consider what would happen if instead of p's being true q were. In such a case r would be likely.' If we say this sort of thing p, q and r appear to stand for the meanings of sentences of the theory, but meanings in some intensional sense.

Now it is very easy to extend the syntax of the formal theory by adding all sorts of intensional operators, e.g. a modal operator; and it is possible to extend the semantics by adding a set of possible worlds and evaluating the modal formulae in the usual way [6, p. 146f]. But this procedure is open to the criticism that we are extending the theory by adding something which is not already there. In particular the criticis mwill be that the possible worlds required by the semantics seem to have no connection with the intended interpretations of the original physical theory.

The aim of this paper will be to shew how a set of possible worlds is already implicit in the intended interpretations of a formally presented physical theory and that these interpretations induce, in a comparatively direct way, an intensional semantics which corresponds to the original one.

We have in mind a first-order language $\mathcal{L}$ in which is stated a theory $\mathcal{T}$ which is intended as a total physical theory in the sense that if it should turn out that there can be distinct possible situations which models of the theory cannot distinguish between, then the theory will be held to be inadequate. For definiteness we shall assume that the primitive logical symbols of $\mathcal{L}$ are $\sim$ (negation), $\vee$ (disjunction) and the universal quantifier; and for simplicity we assume that $\mathcal{L}$ contains no individual constants or function symbols. (These can always be liminated from a first-order theory by the theory of descriptions.)

We assume that the theory makes a distinction between those sentences which are intended to stipulate the meanings of the predicates and those which are intended to make an empirical claim. (On ways of making this distinction *vide* [10], [11] and [18].) Of course philosophers who, like Quine, [12], reject the analytic/synthetic distinction will want to say that the first set should contain only the theorems of first-order logic. There is no reason why we should not allow this as a limiting case, though it is a bit like saying that 'pigs don't fly' would be false if we gave the name 'pigs' to birds.

2 Semantics of $\mathcal{T}$

Let $E(\mathcal{T})$ be a consistent set of sentences (i.e. closed formulae) of $\mathcal{L}$ and let $L(\mathcal{T})$ be a subset of $E(\mathcal{T})$. The intuitive idea is that $E(\mathcal{L})$ is the set of axioms of the physical theory and $L(\mathcal{L})$ contains those members of $E(\mathcal{T})$ which are the analytic axioms of $\mathcal{T}$ (cf. [10, pp. 88-90]). Since this paper is not concerned with axiomatizability we require no more than that $E(\mathcal{T})$ be consistent. It would be intuitively desirable, though, that both it and $L(\mathcal{T})$ be effectively specifiable. The theorems of $\mathcal{T}$ will be all the deductive consequences of $E(\mathcal{T})$ and the *logical* (or perhaps, to avoid confusion, the *analytic*) theorems of $\mathcal{T}$ will be the consequences of $L(\mathcal{T})$.

An interpretation or model $\mathbf{M}$ for $\mathcal{L}$ is an ordered pair $\langle \mathbf{D}, h \rangle$ in which $\mathbf{D}$ is a set (domain) of 'individuals' and h is a value assignment to the predicates such that:

2.1 Where π is of degree k, i.e. where π forms a wff when followed by k individual variables, then $h(\pi)$ is a set of k-tuples of $\mathbf{D}$.

Associated with $\mathbf{M}$ will be a set N of assignments to the individual variables. Where x is an individual variable and $\nu \in N$ then $\nu(x) \in \mathbf{D}$.

2.2 Where ν and μ are both in N and ν and μ coincide every except (possibly) on x, then ν and μ are called *x-alternatives.*

M induces an assignment to all wff, which can be defined as follows (where $\mathbf{M} \models_{\nu} \alpha$ can be read as 'α is true in **M** for assignment ν' and we write '$\mathbf{M} =_{\nu} \alpha$ to mean 'α is not true in **M** for assignment ν'):

2.3 Suppose α is $\pi(x_1, \ldots, x_n)$. Then

$$[\mathbf{M} \models_{\nu} \alpha \text{ iff } \langle \nu(x_1), \ldots, \nu(x_n) \rangle \in h(\pi)]$$

2.4 $\mathbf{M} \models_{\nu} \sim\alpha$ iff $\mathbf{M} =_{\nu} \alpha$

2.5 $\mathbf{M} \models_{\nu} \alpha \vee \beta$ iff $\mathbf{M} \models_{\nu} \alpha$ or $\mathbf{M} \models_{\nu} \beta$

2.6 $\mathbf{M} \models_{\nu} (x)\alpha$ iff $\mathbf{M} \models_{\mu} \alpha$ for every x-alternative μ of ν

Where α is a sentence then reference to ν is unnecessary and we may just write $\mathbf{M} \models \alpha$. In such a case **M** is a *model for* α or α is *true* in **M**. A model of $L(\mathcal{T})$ is called a *possibility model* of $\mathcal{T}$ and a model of $E(\mathcal{T})$ is called an *actuality model* sometimes just a *model*) of $\mathcal{T}$. Obviously all actuality models are possibility models but the converse does not hold in general.

3 Possible worlds

In what follows we shall be considering only those interpretations in which **D** is the intended domain of $\mathcal{T}$. This allows us to refer to a particular interpretation by citing only its value-assignment. **D** may, e.g., be the set of all space-time points but its particular nature is immaterial to what we are about to prove. We assume that $\mathcal{L}$ has only finitely many predicates, each of some specified degree. (This finiteness restriction is not necessary for the theorem we are going to prove, but it is probably a desirable constraint to place upon the language of a physical theory.) We shall assume that these predicates are indexed by an initial segment of the natural numbers. Thus we can speak of

$$\langle \pi_1, \ldots, \pi_n \rangle.$$

Where h is a value-assignment (based on the intended domain **D** of $\mathcal{L}$) and $\langle a_1, \ldots, a_n \rangle$ is an n-tuple then we say

3.1 $\langle a_1, \ldots, a_n \rangle$ is the n-tuple *determined by* h iff for every i such that $1 \leqslant i \leqslant n$, $a_i = h(\pi_i)$.
$| = | h(\pi_i)$.

Obviously not only does every h determine an n-tuple but every appropriate n-tuple determines an h. (where an 'appropriate' n-tuple is one whose i-th member is a set of k-tuples where k is the degree of π_i).

We note here that although $\langle a_1, \ldots, a_n \rangle$ represents a value-assignment to $\mathcal{L}$ it does not contain any explicit reference to $\mathcal{L}$. In fact it is made up by set-theoretical construction out of **D**. It is this fact which makes it ontologically important. For we recall that $\mathcal{T}$ is supposed to be a total theory in that the intended possibility models of $\mathcal{T}$ are supposed to represent all the ways the world could be (whether or not $\mathcal{T}$ is true). This means that where $\langle D, h \rangle$ is an intended possibility model of $\mathcal{T}$, the n-tuple determined by h in the described in 3.1 can be regarded as a 'possible world'.

3.2 Where $\langle a_1, \ldots, a_n \rangle$ is determined by h according to 3.1. and $\langle \mathbf{D}, h \rangle$ is a possibility model of $\mathcal{T}$ then $\langle a_1, \ldots, a_n \rangle$ is called the *world determined by* h and written w_h.

The idea that possible worlds can be thought of as models is not new (cf., e.g., [7, p. 2] and [3 pp. 330-335] and many others). What is interesting about the present case is that we have not used any of the 'linguistic' elements in the model but only those parts which can be made up from the domain of the intended interpretation. This means that although our notion of a possible world is related to $\mathcal{T}$ in a crucial way yet the possible worlds are not strictly, linguistic entities. A second fact of course is that our insistence that $\mathcal{T}$ be a total physical theory gives a metaphysical importance to the possible worlds constructed out of its intended possibility models.

It may be that, as Przełęcki notes on page 30 of [10], we want to restrict intended models even further by defining some predicates by ostensive means. By hypothesis such restrictions cannot be captured by sentences of $\mathcal{L}$ and so they do not affect most of what we say. (though they do have certain consequences for modal extensions of $\mathcal{T}$).

4 Intensional interpretations

Given a theory $\mathcal{T}$ and the intended domain **D** of its models, we define the *canonical intensional interpretation* **I** *based on* $\mathcal{T}$ as follows:

I is an ordered triple $\langle \mathbf{W}, \mathbf{D}, V \rangle$ in which **W** is the set of all w_h (*vide* 3.2) for which h is a possibility model of $\mathcal{T}$, (i.e. $\langle \mathbf{D}, h \rangle$ is a model of L($\mathcal{T}$)) **D** is the intended domain of $\mathcal{T}$, and V is a value assignment such that:

4.1 For each k-place predicate π of $\mathcal{L}$, V(π) (frequently written ω_π) is a function from D^k into $\mathcal{P}$ **W** such that for any $u_1, \ldots, u_k \in \mathbf{D}$ and $w_h \in \mathbf{W}$,

$$w_h \in \omega_\pi(u_1, \ldots, u_k) \quad \text{iff} \quad \langle u_1, \ldots, u_k \rangle \in h(\pi)$$

This means that a predicate will be true of a k-tuple of individuals from **D** in a world corresponding to h iff that predicate was true of that k-tuple in $\langle \mathbf{D}, h \rangle$. Since $\mathcal{L}$ and **D** are the same in the intensional model as in the original models so is the set N of assignments to the variables. We shew how to define an assignment V_ν (for every $\nu \in N$) to every wff α of $\mathcal{L}$.

4.2 For atomic formulae

$$V_\nu(\pi(x_1, \ldots, x_n)) = \omega_\pi(\nu(x_1), \ldots, \nu(x_n))$$

4.3 $V_\nu(\sim\alpha) = \mathbf{W} - V_\nu\ \alpha)$

4.4 $V_\nu(\alpha \vee \beta) = V_\nu(\alpha) \cup V_\nu(\beta)$

4.5 For any $w \in \mathbf{W}$, $w \in V_\nu((x)\alpha)$ iff for every **x**-alternative μ of ν $w \in V_\mu(\alpha)$.

As with first-order interpretations, if α is closed then $V_\nu(\alpha) = V_\mu(\alpha)$ for any ν and μ in N, and so we may speak of $V(\alpha)$. (Cf. [5, p. 87]). We can now prove a theorem of equivalence between **I** and the original models.

THEOREM 1 *For any assignment h in the intended possibility models of $\mathfrak{T}$, and any $\nu \in N$ and wff α*

$$[\langle \mathbf{D},h\rangle \models_\nu \alpha \quad \text{iff} \quad w_h \in V_\nu(\alpha)$$

The proof is by induction on the construction of α.

(1) Suppose α is $\pi(x_1,\ldots,x_n)$

Then $\langle \mathbf{D},h\rangle \models_\nu \alpha$ iff $\langle \nu(x_1),\ldots, \nu(x_n)\rangle \in h(\pi)$;
i.e. (by 4.1) iff $w_h \in \omega_\pi(\nu(x_1),\ldots,\nu(x_n))$;
i.e. iff $w_h \in V(\pi(x_1,\ldots,x_n))$
$w_h \in V_\nu((x)\alpha)$.

(2) Suppose the induction holds for α and β. Then

$$\begin{aligned} \langle \mathbf{D},h\rangle \models_\nu \sim\alpha \quad &\text{iff } \langle \mathbf{D},h\rangle \models_\nu \alpha \\ &\text{iff } \mathbf{w}_h | \notin V_\nu(\alpha) \\ &\text{iff } \mathbf{w}_h \in V_\nu(\sim\alpha) \end{aligned}$$

and
$\langle \mathbf{D},h\rangle \models_\nu \alpha \vee \beta$ iff either $\langle \mathbf{D},h\rangle \models_\nu$ or $\langle \mathbf{D},h\rangle \models_\nu \beta$
iff either $w_h \in V\ (\alpha)$ or $w_h \in V_\nu(\beta)$
iff $w_h \in V_\nu\ (\alpha \vee \beta$

(3) Suppose the induction holds for α (for all $v | \in | N$). Then $\langle \mathbf{D},h\rangle \models_v (x)\alpha$ iff $\langle \mathbf{D},h\rangle \models_\mu \alpha$, for every x-alternative μ of v

i.e. iff $w_h \in V_\mu(\alpha)$ for every x-alternative μ of v. i.e. iff

$w_h \in V_v((x)\alpha$.

This induction proves the theorem.

Corollary 1 *If α is a sentence then $\langle \mathbf{D},h\rangle \models \alpha$ iff $w_h \in V(\alpha)$.*

Corollary 2 *If h is an actuality model of $\mathcal{L}$ then for any deductive consequence α of $E(\mathcal{L})$, $w_h \in V_v(\alpha)$ for every $v \in N$.*

5 Possible worlds and atomic facts

Some views of possible worlds (e.g. those found in [19] and in [2]) would suggest that a world can be characterized by the 'atomic facts' true in it. In our present framework an atomic fact could be given by associating with each $u \in \mathbf{D}$ a sequence f (called an *atomic* sequence) of 'truth values', say 1 and 0, in such a way that the i-th member of $f(u)$ is to be 1 if u satisfies the i-th predicate and 0 if it does not. Given a complete set of these sequences, one for each $u \in \mathbf{D}$ we could reconstruct the world

$$\langle a_1, \ldots, a_n \rangle$$

which represents this set by letting (for each $u \in \mathbf{D}$) $u | \in a_i$ iff the i-th member of $f(u)$ is 1. Of couse some of these complete sets of atomic facts would represent interpretations which are not possibility models of $\mathcal{T}$, and these would not determine possible worlds.

In certain cases we can give an even simpler characterization of atomic facts; a characterization which lay behind the proposals in [4, p. 6] and [5, p. 38] to regard a possible world as a subset of a set $\mathbf{B}$ of 'basic particular situations'. By way of illustration **B** was postulated as the set of all space-time points and the illustration time points and the illustration assumed that any set of space-time points determines a possible world, in the sense that it is to be thought of as the set of those points which are occupied. It is our business in this section to take a look at what kind of physical theory would make sensible such an illustration. In particular we want to consider the conditions under which any subset of the intended domain of the theory determines a possible world.

We suppose that $\mathcal{L}$ contains a one-place predicate π, (We are putting it first in the enumeration of predicates to simplify matters) π_1 corresponds to the 'is occupied' predicate of the illustration. If there is no single predicate in $\mathcal{L}$ which does the trick but there is a complex wff α with only one free variable, say x, which does then we simply add an extra predicate π_1 to $\mathcal{L}$ and the axiom (x) $(\alpha \equiv \pi(x))$ to L($\mathcal{T}$). We then assume that it is the extended theory we have to deal with and revise everything accordingly.

All this means that any world w_h in the intensional model of $\mathcal{T}$ will be an n-tuple $\langle a_1, \ldots, a_n \rangle$ in which $a_i = h(\pi_i)$ and in which, in particular, $h(\pi_1) \subseteq$ **D**. A subset of **D** will determine a possible world if the following holds:

5.1 If $\langle a_1, \ldots, a_n \rangle$ and $\langle b_1, \ldots, b_n \rangle \in$ **W** and if $a_1 = b_1$, then $a_i = b_i$ for all $1 \leqslant i \leqslant n$

What this condition comes to is that differences in worlds are always reflected by differences in their first members. This means that a_1 determines a unique world; and since a_1 is a subset of **D** this means that every subset of **D** determines a member of **W**. In fact there is no reason why we cannot simply say that a_1 *is* a possible world. Notice that $\mathcal{T}$ may contain other predicates than π_1 and may assert all sorts of relations between them.

It may of course happen that not all possibility models of $\mathcal{L}$ satisfy 5.1 but that enough of them do, in the sense that for any w, w'$\in$**W**, where

$$w = \langle a_1,\ldots,a_n \rangle \text{ and } w' = \langle b_1,\ldots,b_n \rangle$$

5.2 If $a_1 = b_1$ then for a any wff α of $\mathcal{L}$ and any assignment $\nu \in N$, $w \in V_\nu(\alpha)$ iff $w' \in V_\nu(\alpha)$.

What this means is that although w and w' may be distinct worlds in the model yet there is no formula which will distinguish between them in the sense of being true in one but false in the other. In such a case we can contract the model, either by leaving only one 'representative' of all the worlds which are equivalent in the evaluation of formulae, or by redefining worlds in the new model as equivalence classes of worlds in the old. In the contracted model worlds will correspond with subsets of **D** as before.

6 Intensional extensions of $\mathcal{L}$

At this point one might wonder what has been gained by the elaborate construction of an intensional model for a physical theory formulated in an extensional language. If we choose to leave the language as it is the answer is that nothing has been gained. But, as our opening remarks indicated, one of the reasons for wanting an intensional semantics was so that the theory could be embedded in a richer language. One wants to be able to say, granted that $\mathcal{T}$ is the correct total theory of the physical world, how our ordinary talk, in our ordinary intensional language (of which $\mathcal{L}$ is only a small part) can be construed as talking about the same physical world which $\mathcal{T}$ reveals to us. The kinds of richer language I have in mind are those developed in detail in [5] in which it is shewn how, on the basis of a set **B** of 'basic particular situations' (**B** could be **D** if $\mathcal{T}$ were a theory of the kind satisfying 5.1) one can build up semantics for languages which are argued to be rich enough to model English.

The details of this construction, and the evidence for the claim that such a formal language can model English, are too long to be described here; we must refer the reader to [5]. We shall, however, mention at least two ways in which $\mathcal{L}$ can be extended. We could add new functors (we shall look at a moment at the necessity functor) and new predicates; and entities of higher syntactic categories could be added in the manner of [5 pp. 72 and 85]. But, more radically, we could extend the domain of the model. If we are to keep to the ontological restriction which is the main theme of this essay we should want the extended domain **D*** to be made up from set-theoretical construction out of **D**. Some hints about what sorts of things everyday objects would then be are given on pp. 94-98 of [5]. If the domain is enlarged the quantifiers of $\mathcal{L}$ would have to become restricted quantifiers ranging only over **D**. This could be done either directly or with a predicate which was true only of ultimate entities (i.e. members of **D**). We could then use an unrestricted quantifier provided **D*** was not too big.

What is important about these intensional extensions of $\mathcal{L}$ is that on the one hand they are all based on the framework required by the original scientific theory $\mathcal{T}$ i.e. they do not go beyond envisaging the world in ways which $\mathcal{T}$ allows; yet on the other hand they do not require the translation of the ordinary language into the language of $\mathcal{T}$. The expressive power of a language in which meanings can be defined in terms of things obtained out of the intensional semantics of $\mathcal{T}$ may be well beyond that of $\mathcal{T}$ itself. Historically it seems to me to have been a mistake of the logical atomists, and those who followed in their footsteps, to think that once having found the correct total physical theory, and having put it into a first-order language; then any respectable talk about the world ought to be translatable into the language of that theory. The present paper has been trying to shew how to have it both ways; i.e. how to have our ordinary language without having to give up any of the claims we might want to make for the adequacy of a first-order physical theory.

7 Modal logic

By way of illustrating an extension of $\mathcal{L}$ which makes essential use of its intensional semantics I shall consider the language $\mathcal{L}^+$ obtained from $\mathcal{L}$ by the addition of an S5 type of necessity operator. L is a one-place propositional functor. (I.e.

$\mathcal{L}^+$ is $\mathcal{L}$ with the extra formation rule that if α is a wff so is $L\alpha$.) This means that the value of L in an intensional interpretation is an operation on sets of worlds. In fact in any intensional model **I**, $V(L)$ is the function in ω such that:

7.1 For any $w \in \mathbf{W}$ and $a \subseteq \mathbf{W}$, $w \in \omega(a)$ iff $a = \mathbf{W}$. (Instead of $V(L)$ we sometimes write α_L.)

THEOREM2 *If* $\alpha \in L(\mathcal{L})$ *and* **I** *is the canonical intensional model based on* $(\mathcal{T})$ *(in the sense of section 4) then* $w \in V(L\alpha)$ *for any* $w \in \mathbf{W}$.

The proof is immediate, for we know that any w is w_h for some $\langle \mathbf{D},h \rangle$ and that $\langle \mathbf{D},h \rangle \models \alpha$ iff $w_h \in V(\alpha)$. Further we know that $\langle \mathbf{D},h \rangle$ is a possibility model for $\mathcal{T}$ and so if $\alpha \in L(\mathcal{T})$ then $\langle \mathbf{D},h \rangle \models \alpha$ for every such $\langle \mathbf{D},h \rangle$. Thus $\mathbf{w}_h \in V(\alpha)$ for every $w_h \in W$ and so by 7.1, $w \in V(L\ )$ for any $w \in W$.**QED**.

It is convenient to refer to $\{La : \alpha \in L(\mathcal{L})\}$ as $L(\mathcal{L})^+.\}$.

Theorem 2 is not surprising but it means that in $\mathcal{L}^+$ we can express the fact that the members of $L(\mathcal{T})$ are necessary truths.

We must now look at the problem of characterizing the class of wff which are true in all members of **W**. Obviously this will include all the members of $L(\mathcal{L})$ and $L(\mathcal{L})^+$). Also it will include all instances of valid S5 formulae (with the Barcan formula). The ordinary rules of quantification will be validity-preserving, as will necessitation. What we need to know is whether it will be the least set satisfying these conditions. Call this set the S5-*closure* of $\mathcal{L}$ and let it be denoted by $\mathcal{L}^+$. From standard results in modal predicate logic [6] we know that any consistent set of wff is true in some world in the model constructed by letting worlds be certain maximal consistent sets of wff. We can call a model like this a *Henkin* model. If α is not in the S5-closure of T then $L(\mathcal{L})^+ \cup \{\sim\alpha\}$ will be consistent and so α will be false in a Henkin model in which $L(\mathcal{T})^+$ is true. The nature of a Henkin model will also ensure that its worlds will correspond with the worlds of the canonical intensional model based on ($\mathcal{T}$), and this enables us to conclude that a wff α is true in all worlds of the canonical intensional model iff it is in the S5-closure of $\mathcal{T}$.

The intended interpretation of $\mathcal{L}^+$ has the same domain as those of $\mathcal{L}$. Further this domain has been constant in all worlds. This is why the Barcan formula is valid in the canonical intensional model. But of course we spoke of extending the domain in various ways. To take a rather simple case suppose that **D** is the set of all space-time points and suppose that we think of a physical object as a function from a world to the set of space-time points the object occupies in that world [5, pp. 94-96]. Then we must extend our domain to incude such functions. We suppose further that a physical object exists only in those worlds in which the set of space-time points it occupies is not empty. We can then associate with each world the set of things which exist in that world; and by means of these differing domains we can falsify the Barcan formula [pp. 170-183]. Similar remarks apply to the introduction of intensional objects and contingent identity. (*Vide* [6, p. 197f], [13, pp. 152-155] and [5, p. 69f])

8 Essentialism

It has been acknowledged that the main motive for the intensional model of $\mathcal{T}$ is that we want to be able to embed it in an intensional language without having to assume any more basic entities that $\mathcal{T}$ does. This means more that the somewhat minimal extension which the addition of a single modal operator has produced. Nevertheless there are some philosophically important issues which can be raised, even at this stage. Given the semantics we have for the modal operator then the intended model for the modal extension $\mathcal{T}^+$ of $\mathcal{T}$ is going to be unique in the sense that the value of any wff α of $\mathcal{L}^+$ in any world in the canonical intensional model based on $\mathcal{T}$ is fixed. This being so we can ask whether a theory $\mathcal{T}$ can impose any interesting *modal* features on its extension $\mathcal{T}^+$; e.g. can a theory impose or prevent essentialism.

Part of the problem here is how to characterize essentialism.

This problem is discussed by Terence Parsons in [9] (cf. also [18]) who suggests that essentialism should be regarded as the doctrine that some things have necessarily a property which other things do not have necessarily. If we restrict this to simple one-place predicates this means that for some π the following holds;

8.1 $(\exists x)L\pi x \;.\cdot\; (\exists x)\sim L\pi x$

(Parsons has a more complicated formulation because he wants to consider predicates of higher degree.)

The fact that we need an atomic predicate is not important for if we have a theory in which a complex wff α (with only x free) is an essentialist formula we can simply extend the the language by adding an extra predicate and add to $L(\mathcal{G})$ the formula

8.2 $(x)\,(\alpha \equiv \pi(x))$

When restricted to one-place predicates Parsons' formulation is only trivially different from ours. What Parsons shews is that an essentialist sentence is false in what he calls a maximal model. He has also shewn that there is a maximal model for any consistent set of closed non-modal formulae (e.g. the models of $L(\mathcal{T})$. This means that no physical theory entails essentialism with respect to the predicates of the theory. (Provided of course that **W** consists of all models of $L(\mathcal{T})$. If we restrict the intended models by means other than by adding axioms it is not clear that we can still prevent essentialism. Suppose, e.g., that there is some u such that, in every intended model, $h(\pi) = \{u\}$. No sentence of the original first-order theory could capture this but it would entail that if we made **W** out of only intended possibility assignments we would have $(\exists x)L\pi x \;.\cdot\; (\exists x)\sim L\pi x$ as true.)

But of course the point of an intensional model is to allow us to extend the language, and there is nothing to prevent our adding a new predicate ψ with the following semantics:

8.3 Given some $u \in \mathbf{D}$ and some $w^* \in \mathbf{W}$ we suppose, for every $w \in \mathbf{W}$

(1) $w^* \in \omega_\psi(u)$

(2) For any u' other than u

(a) $w^* \in \omega_\psi(u')$

(b) If $w \neq w^*$, $w \notin \omega_\psi(u')$

This gives us the truth of an even stronger formula that any of Parsons' for in w^* the following will be true:

8.4 $(\exists x)L\psi x \,.\, (\exists x)(\psi x \,.\, \sim\psi x)$

I.e. there is a predicate which one thing has necessarily and which another thing has contingently.

What this seems to mean is that even if essentialism is not 'out there' in the world there is nothing to prevent us from introducing it if we want to. As noted in [5, p. 84n] the really interesting question is whether and how the predicates of ordinary language introduce it. The believer in 'scientific realism' need not fear essentialism, for it does not affect any of his claims for the adequacy of his theory as a complete framework for a physical description of the world; indeed our discussion has proceeded on the assumption that it *is* a complete framework, otherwise models of $L(\mathcal{T})$ would not exhaust the possible worlds. Essentialism comes from the way we talk about the world; it comes from the language we erect upon the language of the physical theory. All that $\mathcal{L}$ does is provide the entities we need for the semantics of that language.

9 Other extensions

We have defined the semantics of *L* in terms of truth in all possible worlds. One can also define an empirical necessity operator *E* in terms of truth in all worlds which correspond to the actuality models of $\mathcal{T}$, (i.e. the models of $E(\mathcal{T})$). This theory will contain $L(\mathcal{T})$ and $E(\mathcal{T})$ together with all the principles of S5 (for both operators) except $Ep \supset p$ (that would fail in all worlds which are not actuality models of $\mathcal{T}$). In this theory *L*-necessitation will sometimes fail (for the axioms of $\mathcal{T}$ are not all in L($\mathcal{L}$).) though *E*-nesessitation will always hold. This will make explicit the distinction between those sentences which, according to $\mathcal{L}$, are empirically necessary and those which are logically necessary.

What is perhaps a more interesting extension of $\mathcal{L}$ would involve the addition of a counterfactual condional operator $>$ in the manner of [14] and [15]. $\alpha > \beta$ is true in a world *w* iff in the nearest world to *w* in which α is true β is also. (More sophisticated versions deny the assumption that there is a unique nearest world, e.g. [8] and [1].) In the development of these theories the 'choice function' which picks out the nearest world in which the antecedent is true, is specified by the model. It must satisfy certain minimal conditions but mostly it is left quite undetermined. This procedure is perfectly acceptable when studying conterfactual *logics,* for it amounts to deciding to investigate those principles which are true no matter what choice function we use. However the canonical intensional model based on $\mathcal{T}$ is supposed to represent how the physical world works and has a structure imposed on it by the nature of $\mathcal{T}$. If we are to adopt the Stalnaker/Thomason theory of the conditional as a viable theory of counterfactuals for the physical world then what we need to look out for are features of the canonical intensional model based on $\mathcal{T}$ which can be used to define the choice function required in a natural kind of way. I have no suggestions to offer as to how this might be done but am concerned merely to stress that the existence of canonical intensional models for physical theories, models whose nature is closely related to the structure of those theories, could provide a framework in which counterfactuals involving the physical world can be given a non-arbitrary semantics.

10 Higher-order languages

It might be held that we have unduly restricted ourselves in considering only theories stated in first-order languages. Certainly many theories have been envisaged as stated most naturally in lanuages of higher order. Indeed the position of the logical atomists was not cited correctly above since they tended to have in mind the theory of types of *Principia Mathematica* as their preferred language. One way of dealing with such a comment would be to indicate how what we have said could be adapted to deal with models for higher-order languages. There is nothing in principle impossible about such extensions but their details would be rather complex. The alternative, and, I think, the preferable approach, is to assume that these theories are reformalized in the language of a first-order set theory.

If $\mathcal{T}$ includes the axioms of set theory (presumably as logical axioms) then its intended domain will have to be rather bigger than the domains we have been considering. In fact **D** will have to be so big that the possibility of extending it by set-theoretical construction will be ruled out; for the extra members are in **D** already. This means that the only extensions which we need to consider are those which consist in adding new symbols to $\mathcal{L}$ and giving semantic rules for them. The domain of such a theory will be the universe class which is based on a set **B** of 'individuals'. (For even if nothing but sets and classes are needed for mathematica we almost certainly need individuals for the intended domains of physical theories.) Some predicates might be assigned always subsets of **B**. If the π_1 of section 5 is such a predicate and if 5.1 holds of $\mathcal{T}$ then we can define a possible world as a subset of **B**.

It may be of course that the idea that a total physical theory is possible is a Utopian one. On this fundemental issue I would not like to comment. All I want to insist is that the intensional extensions of such a theory are in no worse position than the original theory and that the pursuit of precision in philosophy, so far from requiring that these extensions are somehow not respectable, demands on the contrary that they be followed up as fully as possible.

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