EPISTEMIC LOGIC AND MERE BELIEF

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In a previous article [1] I argued that the distinction which epistemologists look for between knowing and believing is actually the distinction between knowing and *merely* believing, where, unlike belief, mere belief is incompatible with knowledge. Using Fred Sommers' notion of predicate negation, [2] where the negation of a predicate is equivalent to the disjunction of all those predicates incompatible with it, I formulated several epistemic statements and drew out ten conditionals which should at least be theorems of any epistemic calculus. In what follows I want to set up the axioms for an epistemic calculus. A few of these will come from my previous list of conditionals. I then want to show that in such a system the necessary conditions for *mere belief* can be adequately formulated.

The list of conditionals which I gave (where "$a\overline{Kp}$" is the predicate negation of "aKp" and reads "a fails to know that p") is:

(a) $aKp \supset \sim aK - p$

(b) $aK - p \supset \sim aKp$

(c) $aKp \supset \sim a\overline{Kp}$

(d) $a\overline{Kp} \supset \sim aKp$

(e) $aK - p \supset \sim a\overline{K - p}$

(f) $\overline{aK - p} \supset \sim aK - p$

(g) $aK - p \supset a\overline{Kp}$

(h) $\sim a\overline{Kp} \supset \sim aK - p$

(i) $aKp \supset a\overline{K - p}$

(j) $\sim a\overline{K - p} \supset \sim aK - p$

Accepting that "p" can range over any proposition, thus allowing a "$-p/p$" substitution, and a little elementary logic will show that all of the conditionals easily reduce to just three: (a), (c) and (i). In effect (a) says that a knower cannot know a contradiction, (c) says that a knower does not fail to know what he knows and (i) says that a knower fails to know what is contradictory to what he knows. (3)

I also listed the following three biconditionals.

(k) $\sim\sim aKp \equiv aKp$
(l) $aKp \equiv aKp$
(m) aK — — $p \equiv aKp$

However, I do not believe that all of these can or should be added to our list (a), (c), and (i). (k) is not needed since it is already available to us in the fund of logical tools offered by propositional logic. A careful look at (m) reveals that it is actually an instance of the more general formula

(m') $p \equiv q \,.\supset.\, aKp \equiv aKp$

since it assumes "— — p" is true. Now (m') seems open to counter examples. Consider: in mathematics "$4^{1/2} = 2 \equiv 2 = 2$" is a valid biconditional; thus, by (m'), anyone who knows that $2 = 2$ must also know that $4^{1/2} = 2$, which is clearly not the case when we consider all those ignorant of the role of exponents in algebra but who are likely to know that $2 = 2$. (4) So it would seem prudent to reject (m). This leaves (1), which is simply the double negation analogue for a calculus allowing predicate negation (the more general form is "$Px \equiv \bar{\bar{P}}x$"), to be added to (a), (c) and (i).

I believe the following three should also be added to our list.

(n) $aKp \supset p$
(o) $p \supset \Diamond p$
(p) $aK(p \,.\, q) \equiv aKp \,.\, aKq$

(n) is fairly obvious and is found in nearly all epistemic logics. It simply says that a knower cannot know what is not the case. In fact, since (a) can easily be deduced from (n), we can safely remove (a) from our list now that we have (n).

Formula (o) is usually taken as an axiom of modal logic. However, while we may not wish to accept any available modal system, I believe we would nevertheless accept (o). Furthermore, (o) is included in our epistemic calculus because we will want to distinguish between the case where p is known to hold and the case where p is only known to be possible. We will also see that (n) and (o) will facilitate the formulation of the necessary conditions for mere belief.

Formula (p) simply says that a knower knows individually just what is included in his "fund of knowledge" (i.e. the set of all facts which he knows). [5] It might be argued that along with (p) we should include

$$\text{(q)} \quad aK(p \vee q) \equiv aKp \vee aKq$$

But counter examples to (q) are extremely easy. For example, I do not know that Armstrong placed the first U.S. flag on the moon and I do not know that Aldrin placed the first U.S. flag on the moon.

We have, then

$$\text{(c)} \quad a\overline{Kp} \supset \sim aKp$$
$$\text{(i)} \quad aKp \supset a\overline{K - p}$$
$$\text{(l)} \quad a\overline{\overline{Kp}} \equiv aKp$$
$$\text{(n)} \quad aKp \supset p$$
$$\text{(o)} \quad p \supset \Diamond p$$
$$\text{(p)} \quad aK(p \,.\, q) \equiv aKp \,.\, aKp$$

My claim has been that these six, along with the propositional calculus, would be sufficient as a basis for an epistemic calculus. The set is independent and consistent and will generate all those theorems which I believe should hold for epistemology (note that "$aKp \supset aKaKp$" would not be a theorem).

Finally, and very briefly, I want to formulate the necessary conditions for mere belief. I have said that *mere* belief is incompatible with knowledge. If a predicate $\varnothing$ is incompatible with a predicate ψ then "$(x) \sim (\varnothing x \,.\, \psi x)$" is valid, from which it follows that "(x) $(\varnothing x \supset \sim \psi x)$" is valid. Thus our formulation of mere belief must be such that the attribution of mere belief that p to a knower entails that the attribution of knowledge that p to that knower does not hold. Let "aMp" read "a merely believes that p". I will say that a knower merely believes that p only if he fails to know that p but does know that p is possible. Formally:

$$aMp \supset a\overline{Kp} \,.\, aK\Diamond p$$

From "$a\overline{Kp} \,.\, aK\Diamond p$" it follows, by (c), that "$\sim aKp$", so that it is not possible for a knower to know what he merely believes (i.e. mere belief is incompatible with knowledge). Since "aMp" entails "$aK\Diamond p$", which further entails "$\Diamond p$", we can say that a knower cannot merely believe what is impossible; but nothing prevents him from merely believing what is not in fact the case.

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FOOTNOTES

(1) "Knowledge, Negation and Incompatibility", *Journal of Philosophy*, Vol. 66 (1969).

(2) "Predicability," in Max BLACK, ed., *Philosophy in America* (Ithaca, N.Y., 1965), and "On a Fregean Dogma," in *Problems in the Philosophy of Mathematics* (Amsterdam, 1967).

(3) Those who do not see the distinction between (a) and (i) should consult the articles referred to in the first two footnotes.

(4) For a similar argument see E. J. LEMMON, "If I Know, Do I Know That I Know ?", in Avrum STROLL, ed., *Epistemology* (N.Y., 1967), pp. 77-78.

(5) For a fairly mild rejection of this view see Lemmon *op. cit.*, p. 77.