

# EPISTEMIC LOGIC, LANGUAGE AND CONCEPTS

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## I

There are two main problems associated with the propositional epistemic logic which Hintikka sets out in *Knowledge and Belief*. <sup>(1)</sup> First there is the KK-thesis <sup>(2)</sup> which makes it a feature of the logic that  $K_a P \equiv K_a K_a P$ . Secondly, there is that feature of the logic which makes it indefensible to say of any person  $a$  that he does not know  $T$  where  $T$  is any tautology of propositional calculus or any self-sustaining formula of HS4.  $\sim K_a T$  is indefensible.

Although these two problems show HS4 to be an unsatisfactory logic, a logic can be produced which overcomes these problems and is far more appropriate. This system is set out in this paper, and we show that from at least two points of view the logic is more appropriate to 'knows' than HS4.

First, we take note of the two ways in which we can approach the question of discovering, or inventing an epistemic logic. In the course of a voluminous discussion of the KK-thesis <sup>(3)</sup> Hintikka has set out various methodological points in order to defend HS4. These points seem to indicate two ways of setting up and justifying epistemic logics.

In "Epistemic Logic and the Methods of Philosophical Analysis" <sup>(4)</sup> he sets out what could be called a linguistic approach. But in "Semantics for propositional attitudes" <sup>(5)</sup> and in "Knowing that one knows" reviewed" <sup>(6)</sup> he sets out what could be called a conceptual approach.

I shall argue here that from either viewpoint the KK-thesis is not justifiable, and so HS4 is inappropriate, and that from either viewpoint there tends to be a convergence as to what would be a suitable logic for the notion of knowledge.

## II

Whatever may have been Hintikka's intentions in ELMPA, he gives there the impression that the formal logic used in the analysis of epistemic (and doxastic) notions is to be applied particularly to epistemic (and doxastic) language. He remarks that,

"The philosophically interesting concepts which we want to study are largely embedded in our ordinary language. The question of the relevance of formal methods is thus closely related to the question of the applicability of these methods to the study of ordinary discourse." (7)

As to how formal logic can be applied to the study of ordinary discourse Hintikka gives the answer, after first considering and rejecting the suggestions that logic is a regimentation of ordinary discourse, or that logic helps us to revise ordinary language. His answer is that,

"A branch of logic, say epistemic logic, is best viewed as an *explanatory model* in terms of which certain aspects of the workings of our *ordinary language* \* can be understood." (8).

This point is emphasised in several ways. He refers to what he calls the "depth logic" of ordinary language. He leans on Wittgenstein's notion of a language game, and quotes with approval from the Blue Book

"When we look at such simple forms of language, the mental mist which seems to enshroud our ordinary use of language disappears." (9)

Hintikka's methodological justification for doing what he did in *Knowledge and Belief* is strongly in terms of the analysis of *language*. Also, he maintains that such an analysis will provide "a genuine theory of the meanings of the words and expressions involved." (10)

Furthermore, in ELMPA, Hintikka sets out in some detail what he conceives to be the relation between the explanatory model and ordinary language. They are related in much the same way as any theory is related to the data which is explained. The theory, above all, is not expected to account for every slight variation in the data, but simply to give an adequate explanation.

The theory of meaning set out in an explanatory model is to be seen as an explanation of the *basic meaning* of the words and expressions. But in actual discourse this basic or theoretic meaning will be modified by various pressures such as the desire to emphasise the limitations on knowledge, and other pressures of context. Modification will create a difference between *basic* and *resulting* meaning, the difference as it were between theory and practice. The actual extent of this difference is called residual meaning. Since all ordinary discourse is "in context," "the basic meaning of an expression is not always, and perhaps not even usually, its normal (most frequent) meaning. It may even happen that an expression never has its basic meaning in ordinary discourse, at least not outside philosopher's discourse." (1)

In ELMPA, it is in the light of these distinctions between basic, resulting and residual meaning that Hintikka defends the KK-thesis. His defense amounts to saying that basically, in spite of various contextual modifications of meaning, if one knows that  $p$  then one knows that one knows that  $p$ .

If we are to attack the model, HS4, or some part of it, such as the KK-thesis, then one thing that we must be able to do is to see whether the model does give an explanation that best fits the locutions about which it is a theory, albeit a theory of their meaning. Now it seems to me that the KK-thesis just does not give the best explanation for the meaning of locutions in which there is an iteration of the epistemic operator. There is a better theory.

In a nutshell, the KK-thesis says that, theoretically, the meaning of "John knows that  $p$ " is equivalent to the meaning of "John knows that he knows that  $p$ ". In order to attack this

thesis let us first consider HS4 as compared with a logic which does not have the KK-thesis.

It has been pointed out that HS4 is isomorphic with axiomatic S4. "The distinctive axiom of S4 is ' $\Box p \supset \Box \Box p$ ', and the epistemic counterpart of this formula is ... (KK-thesis)  $K_a p \supset K_a K_a p$ " (<sup>12</sup>). The distinctive axiom is the axiom, which, when added to the axioms of the Feys-von Wright modal system M, or Lemmon's system T, will give us S4.

If we set out the following axiom schemata and rules:

- A1  $A \supset (B \supset A)$
- A2  $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$
- A3  $(\sim A \supset \sim B) \supset (B \supset A)$
- A4  $\Box (A \supset B) \supset (\Box A \supset \Box B)$
- A5  $\Box A \supset A$
- A6  $\Box A \supset \Box \Box A$
- A7  $A \supset \Box \Diamond A$

$$R1 \quad \frac{A, A \supset B}{B}$$

$$R2 \quad \frac{A \supset B}{\Box A \supset \Box B}$$

$$R3 \quad \frac{A}{\Box A},$$

then we can define T and S4 by:

$$T = \{A1 - A5; R1, R3\}$$

and

$$S4 = \{A1 - A6; R1, R3\}. (<sup>13</sup>)$$

We can also define a semantics for T, which can be called 'HT', by taking the C rules as set out in *Knowledge and Belief* and modifying (C.KK\*). Whereas (C.KK\*) is:

If " $K_a q \in \mu$  and if  $\mu^*$  is an alternative to  $\mu$  (with respect to  $a$ ) in some model system, then " $K_a q \in \mu^*$ ;  
we need a rule (C.K<sup>+</sup>):

If " $K_a q$ "  $\in \mu$  and if  $\mu^*$  is an alternative to  $\mu$  (with respect to  $a$ ) in some model system, then " $q$ "  $\in \mu^*$ .

The rules common to both HS4 and HT would be:

(C. $\sim$ ), (C.&), (C.v), (C. $\sim \sim$ ), (C. $\sim$ &), (C. $\sim$ v), (C.P\*), (C. $\sim$ K), (C. $\sim$ P) and (C.K).<sup>(14)</sup> HS4 would have the additional rule (C.KK\*), and HT the rule (C.K<sup>+</sup>).

Just as A6 is not a theorem of T, the KK-thesis is not self-sustaining in HT. (" $K_a K_a p \supset K_a p$ " is self-sustaining in HT.) So " $K_a p$ " is not virtually equivalent to " $K_a K_a p$ ". If we were to accept HT as the appropriate theory for the explanation of the basic meaning of epistemic locutions then we would say that John's knowing that  $p$  is not theoretically equivalent to John's knowing that he knows that  $p$ . So, we have an alternative explanatory model available for consideration. In order to evaluate HT let us consider Hintikka's defence of HS4, which is HT plus the KK-thesis.

One of the main criticisms which Hintikka directed at those who denied the equivalence of  $K_a p$  and  $K_a K_a p$  was that they must consider that there are two senses of 'knows'.<sup>(15)</sup> But, this is an ill-considered side track. The fact is that HT gives one and only one basic meaning to knows, but has the additional flexibility that it also gives different senses to  $K_a p$  and  $K_a K_a p$ . Hintikka seems to assume in ELMPA that the only way to have two senses for  $K_a p$  and  $K_a K_a p$  is to have two senses of knows, or, if there is only one sense for knows then there must be some residual meaning present in the ordinary use of iterated epistemic locutions. But, by using HT we can get, with only one basic sense of knows, which seems desirable, different senses for  $K_a p$  and  $K_a K_a p$ , and it is clear that they do have different senses.<sup>(16)</sup>

The various examples which Hintikka uses in *Knowledge and Belief* and ELMPA are examples of a difference in meaning between  $K_a p$  and  $K_a K_a p$ . He maintains that the difference can be explained in terms of residual meaning and HS4. HT can explain the differences in these examples without resort to residual meaning, and this must count in favour of HT as a better theory than HS4.

Furthermore, in HS4 we are faced with the idea that the iteration of epistemic locutions is no more than a signalling that residual meaning may be modifying the basic sense. So we have two kinds of epistemic locutions suggested: ones with basic meaning (perhaps modified), and ones that have only residual meaning and modify other locutions. It would seem far more desirable and in accordance with the aims of formal analysis as set out in ELMPA if we could get a logic which did not divide the epistemic locutions in such a way, but could explain the meaning of both simple and iterated locutions in a univocal way. HT does just this.

### III

In " 'Knowing that one knows' reviewed" (<sup>17</sup>) Hintikka disallows appeals to ordinary language for the settling of the question as to which formal analysis is best. He writes:

"The ultimate court of appeal in deciding whether a logical principle governing some given concept is acceptable is not ordinary usage, however regimented, but rather whether the principle helps the concept in question to serve the purpose or purposes it in fact is calculated to serve in our conceptual repertoire, and whether these purposes are themselves worth our effort. By spelling out these purposes and by using them to evaluate various logical principles an analyst can perfectly well disagree with ordinary usage *and even attempt to reform it*". (<sup>18</sup>) (my emphasis)

There is here quite a change of emphasis from ELMPA. Hintikka is here interested in reforming language, something which was rejected in ELMPA. But above all, he sees his task not as one of giving a theory of the meaning of epistemic locutions in ordinary language, but as one of propounding a theory for a strong sense of knows, a sense which he feels is in ac-

cord with the most worthwhile purposes which this epistemic concept could serve.

Nevertheless, this change is not quite as radical as first it seems. All along Hintikka has maintained that his logic is not for any lay sense of knows but for a stricter sense, a more philosophical sense. <sup>(19)</sup> In KKR Hintikka sets out some of the considerations which should be taken into account when providing a logic for this stricter sense of knows.

But since we are forbidden any appeal to ordinary language, and presumably to philosophical usage, we may well ask what are we to appeal to. From what was quoted above it would appear that we have to base any discussion upon the purposes a concept serves in our conceptual repertoire. Just what this involves can be seen from SPA.

"What I take to be the distinctive feature of all use of propositional attitudes is the fact that in using them we are considering more than one possibility concerning the world ... It would be more natural to speak of different possibilities concerning our "actual" world than to speak of several possible worlds ... In our sense, whoever has made preparations for more than one course of events has dealt with several 'possible courses of events' or 'possible worlds'." <sup>(20)</sup>

The concept of knowledge can be seen as serving certain purposes which relate the various possible worlds in a particular fashion. According to Hintikka the purposes which the concept serves result in relations such that, whatever is known to be the case in a given possible world will be the case and be known to be the case in any alternative, and whatever is possible for all that one knows must be the case in at least one other possible world. HS4 formally abides by all three conditions, HT by the first and last.

In drawing up a logic which will reflect this notion of the purpose of the concept of knowledge "the semantical rules that govern epistemic concepts, ..., can all be formulated in terms of the relations that obtain between a given fixed pos-

sible world  $M$ , in which certain persons are stated to know certain things, and a number of 'epistemic alternatives' to this world which we have to consider to make sense of these statements." <sup>(21)</sup>

In order to see why the possible worlds should, or should not be related as in HS4 we must consider the purposes which the concept of knowledge serves. One consideration can be seen in terms of a contrast between what Hintikka calls "a weak sense of knowledge in which knowing merely amounts to being correctly informed (knowledge as true belief)" <sup>(22)</sup> and the strong sense where "knowing presupposes that some objective criteria are satisfied by the knower's grounds (evidence, judgement, and what not)". <sup>(23)</sup>

Hintikka suggests, tentatively, it must be admitted, that in the case of the weak sense of knowing, if  $Kap \in \mu$  where  $\mu$  partly describes  $M$  then only  $p \in v$  where  $v$  partly describes some epistemic alternative,  $N$ , to  $M$ . This is so because the whole cash value of  $Kap$  is  $p$ , and there are no objective grounds on the basis of which  $a$  truly believes  $p$ . On the other hand, it is suggested, that in the case of the strong sense of knowing, because there are objective criteria, if  $Kap \in \mu$  then  $Kap \in v$ , and for this we need the KK-thesis.

I fail to see how this contrast has any bearing on the relationship between possible worlds. It seems to me that in the case of a weak sense of knowledge what we need are rules which will make  $Kap$  virtually equivalent to  $p \& Bap$ . In the case of the strong sense of knowledge we simply need rules which will not only make  $Kap$  imply  $p$  but also imply  $Eap$  (where ' $Ea$ ' is to be read as something like ' $a$  has good evidence that') or some other such condition which can count as objective criteria.

It can also be pointed out that Hintikka's suggestion for the strong sense of knowing seems to give far more than objective criteria, he is saying that  $a$ 's knowing  $p$  in one possible world is a *sufficient* condition for his knowing  $p$  in an alternative world. The good grounds for  $a$ 's knowing  $p$  in  $M$  might not even exist in  $N$ , unless the only grounds to be counted as good



are logical truths or laws of nature or such. But all of this was set out as being rather tentative anyway.

Hintikka places far more weight on an argument which insists upon the *conclusiveness* of knowledge. In section V of KKR he writes:

"One way of interpreting this requirement is to take it to say that no further information will make any difference to one's acceptance, that 'further inquiry has lost its point.' Now in a sense one has not reached this conclusiveness if there are possibilities admitted by one's knowledge which are such that, if they should turn out to be realized, they would logically imply that one does not know what one could in a weaker sense truly say that one knows. If such possibilities are left open, our knowledge is not conclusive, ..." (24)

Since Hintikka takes ' $P_ap$ ' to be read as 'It is possible, for all that  $a$  knows, that  $p$ ', and also takes ' $\sim K_ap$ ' to be virtually equivalent to ' $P_a \sim p$ ' he is able to argue as follows. — If we should add to  $K_ap$  in  $\mu$ ,  $\sim K_aq$ , then in some possible world  $\sim q$  is the case. But if  $q$  follows from  $p$ , and hence from  $K_ap$ , then in the alternative possible world  $K_ap$  is not the case, and so  $K_ap$  cannot be conclusive because it is possible for it to be false.

So, in a deductive logic we would need the rule

$$\frac{K_ap \supset q}{K_ap \supset K_aq}$$

which is virtually the same as having the KK-thesis.

Three things need to be said about this 'conclusiveness argument'. In the first place Hintikka is taking " $K_ap$  is conclusive" to mean " $a$  must know  $p$ ", rather than " $K_ap$  and hence nothing will falsify  $p$ ." The former is too strong. Even though we might expect  $a$  to know  $p$  and though we know he has all the grounds and so on it does not follow that he *must* know  $p$ .

So we need to weaken Hintikka's argument by making the conclusion: if  $q$  follows from  $p$  then in the alternative world  $p$  is not the case, so  $p$  is not conclusive and can be falsified.

This modification will at least weaken HS4 to HT. But HT still preserves the notion that what is known is conclusive.

In the second place, Hintikka asks us, somewhat reluctantly, to accept the notion that  $a$  is "deductively omniscient".<sup>(25)</sup> We will return to this notion later.

In the third place there is the problem of the virtual equivalence of  $\sim K_a \sim p$  and  $P_a p$  (i.e.  $\sim K_a \sim p \equiv P_a p$ ). This equivalence is vital to Hintikka's conclusiveness argument and if we can show the equivalence to be unsound the conclusiveness argument will crumble.

Hintikka at times has said that problems of translation from ordinary language to symbolism, and the dictionary for the translation are not crucial problems. But this is clearly a very real problem here. We are told that ' $\sim K_a \sim p$ ' is to be taken as equivalent to 'It is possible, for all  $a$  knows, that  $p$ ' and are, on this ground, given a rule which asserts that there must be a possible world where  $p$  is true. But such a rule cannot be justified if we take ' $\sim K_a \sim p$ ' as 'It is not the case that  $a$  knows not  $p$ '. This latter is patently equivalent to ' $a$  does not know not  $p$ '. An instance of this would be ' $a$  does not know  $\sim(p \& \sim p)$ '. Certainly we could affirm this but deny that it is possible for all that  $a$  knows that  $p \& \sim p$ .

Conversely, if we begin with the equivalence  $K_a p \equiv \sim P_a \sim p$ , we can see how Hintikka's readings have misled him.  $K_a p$  is not virtually equivalent to 'It is not possible for all  $a$  knows that not  $p$ '. All that we can really maintain is  $P_a p \supset \sim K_a \sim p$ , if we are to hold to the readings given to the operators. This would mean abandoning the rules (C.  $\sim K$ ) and (C.  $\sim P$ ) and introducing a rule (C.  $K \sim$ ) as follows:

(C.  $K \sim$ ) If " $K_a \sim p$ "  $\in \mu$ , then " $\sim P_a p$ "  $\in \mu$ .

In other words, if  $a$  knows that  $\sim p$  then it is not possible, for all that  $a$  knows, that  $p$ . But the converse is not true.

Having once shown that  $K_a p$  and  $\sim P_a \sim p$  cannot be taken as equivalent, we are forced to take note of a feature of HS4

which is somewhat glossed over when Hintikka refers to the alternates to a given  $M$  as "possible worlds". The fact is that the alternates to  $M$  are possible worlds consistent with what is logically possible for all that  $a$  knows in  $M$ . And when we take  $\sim K_a q$ , not as " $a$  does not know  $q$ ", but as "It is possible, for all  $a$  knows, that  $\sim q$ ", then we see that Hintikka's conclusiveness argument is empty.

So, there seems to be every good reason for accepting  $(C.K\sim)$  and having another look at the general problem of finding a logic for 'knows'. At this point we can take up also the problem of deductive omniscience or what Hilpinen calls 'logical omniscience'.<sup>(20)</sup>

#### IV

This problem of deductive omniscience is closely related to the questions raised above. One way of putting this problem is to point out that in HS4 (and HT also) it is indefensible to say of anyone that he does not know any tautology of propositional calculus.

This feature of HS4 and HT is parallel to that feature of both S4 and T whereby, if  $T$  is a tautology of propositional calculus,  $\Box T$  is a theorem of both S4 and T. One of the rules for S4 and T is R3 which includes the rule:

$$\vdash_{PC} A \rightarrow \vdash_{S4 \text{ or } T} \Box A$$

In order to be rid of R3, and even the weaker R2, we have to go to very weak calculi indeed. But, if one such calculus more truly reflects the meaning of the concept, why should we not use its semantics as a logic for the concept of knowledge.

Consider then the system E0.5 defined by  $\{A1-A5; R1\} = E0.5$ , using the axiom schemata and rules set out above. Such a system would be isomorphic with an epistemic logic whose consistency rules would be:  $(C.\sim)$ ,  $(C.\&)$ ,  $(C.\vee)$ ,  $(C.\sim\sim)$ ,  $(C.\&)$ ,  $(C.\sim\vee)$ ,  $(C.\sim P)$ ,  $(C.\sim K)$ ,  $(C.K\sim)$ ,  $(C.K)$  and  $(C.P)$ , where all the rules save  $(C.P)$  and  $(C.K\sim)$  are as in *Knowledge and Belief*, and  $(C.P)$  is:

- (C . P) If " $P_a p$ "  $\in \mu_i$  and  $\mu_i$  belongs to a modal system  $\Omega$ , then provided  $\mu_i$  is not an °alternate to any member of  $\Omega$  (with respect to  $a$ ), there is in  $\Omega$  at least one °alternative to  $\mu_i$  (with respect to  $a$ ) such as  $\mu_j$ , such that " $p$ "  $\in \mu_j$ ; but if  $\mu_i$  is an °alternate to some member of  $\Omega$  (with respect to  $a$ ), there is in  $\Omega$  at least one °alternate to  $\mu_i$  (with respect to  $a$ ) such as  $\mu_k$ , such that " $\sim p$ "  $\in \mu_k$

and (C .  $K \sim$ ) is

- (C .  $K \sim$ ) If " $K_a p$ "  $\in \mu_i$  and  $\mu_i$  belongs to a modal system  $\Omega$ , then if  $\mu_i$  is not an °alternative to any member of  $\Omega$  (with respect to  $a$ ) and  $\mu_j$  is an °alternative to  $\mu_i$  in  $\Omega$  then " $p$ "  $\in \mu_j$ ; but if  $\mu_i$  is an °alternative to some member of  $\Omega$  (with respect to  $a$ ) and  $\mu_k$  is an °alternative to  $\mu_i$  in  $\Omega$  then " $\sim p$ "  $\in \mu_k$ .

In such a system " $P_a$ " is simply read as " $\sim K_a \sim$ ". This epistemic logic could be called 'HEO.5'.

If there is a desire to preserve the logic of the complex notion of what is possible for all that one knows, then we can use HS4 for this, but the operators would be  $^L K$  and  $^L P$ . The modal rules would be:

(C .  $^L P^*$ ), (C .  $\sim ^L P$ ), (C .  $\sim ^L K$ ), (C .  $^L K$ ) and (C .  $^L K ^L K^*$ ) parallel to those in *Knowledge and Belief*.

The following rules would also seem to be called for:

- (C .  $^L PP$ ) If " $^L P_a p$ "  $\in \mu$ , then " $P_a p$ "  $\in \mu$ .  
and

- (C .  $K^L K$ ) If " $K_a p$ "  $\in \mu$ , then " $^L K_a K_a p$ "  $\in \mu$ .

Let us call this multiple system 'HK'.

In HK the quest for conclusiveness is satisfied without the consequences of HS4. For, when  $a$  knows that  $p$  in  $\mathbf{M}$  it follows that it is not *logically* possible, for all that  $a$  knows, that  $\sim q$ , if  $q$  follows logically from  $p$ . Formally:

(a)  $\{K_a p, \vdash p \supset q, {}^L P_a \sim q\}$  is not defensible because if  $N$  is a logical alternative  $M$  the partial description of  $N$  would be:

(b)  $\{p, \vdash p \supset q, \sim q\}$

But when  $a$  knows that  $p$  in  $M$  it is not impossible for  $a$  not to know that  $q$  even if  $q$  follows logically from  $p$ . Formally:

(c)  $\{K_a p, \vdash p \supset q, P_a \sim q\}$  is defensible.

(c) is defensible because the rules of HK propose, in effect, that as well as the *logically* possible alternatives to  $M$ , there should be *open* alternatives to  $M$ . Hence, if  $N'$  is the open alternative to  $M$ , whose partial description is (c), then the partial description of  $N'$  can be either:

(d)  $\{p, \vdash p \supset q, \sim q\}$  or,

(e)  $\{\sim p, \vdash p \supset q, q\}$

One of these is defensible.

So what (C.P) is in effect allowing is that if  $a$  does not know  $p$  then  $p$  might be true or might be false, in the quite general sense that when  $a$  knows no logical reason from which it follows that  $p$ , the truth of  $p$  is an open question for  $a$ .

This last point can also be put formally in terms of a contrast between (c) and

(f)  $\{K_a p, K_a(p \supset q), \sim K_a q\}$ .

(c) is defensible but (f) is not. (cf. Axiom A4)

So, it is defensible under HK for someone not to know a tautology. In this sense we seem to have avoided the problem of deductive omniscience.

The logic at which we have arrived seems to be a better explanatory model for ordinary language than HS4, yet the logical conclusiveness of the concept of knowledge is not sacrificed. So, from either viewpoint there is a convergence towards HK.

#### CONCLUDING PHILOSOPHICAL REMARKS.

It is worth noting that if what has been said above is true then the typically standard analysis of the concept of knowledge will not do. By a standard analysis I mean one of the form:  $a$  knows that  $p$  if, and only if, (1)  $a$  believes that  $p$ ,

(2)  $p$  is true, and (3)  $a$  possesses adequate evidence for  $p$ .

In as much as this kind of analysis sets out *sufficient* conditions for knowledge it runs counter to our explanatory model. Except for certain particular cases our model does not allow for sufficient conditions for what someone in fact knows. The exceptions fall into two classes. The one is exemplified by  $p \ \& \ \sim p \ . \supset \ . \ K_a q$ , and is clearly irrelevant. The other is where,  $B \supset K_a p$ ,  $B$  contains  $K_a$ . This latter is exemplified by  $K_a(p \supset q) \ \& \ K_a p \ . \supset \ . \ K_a q$ . Cases of this latter kind also have no bearing on our point. In our model, HK, formulae of the form  $K_a A$  are strictly contingent unless  $A$  is a contradiction.

This is not to say that there is no possibility of justifying a knowledge claim, but we cannot prove, deductively, that a statement that  $a$  knows that  $p$  is true from premises of belief, evidence, and factual truth.

This also means that to criticise the concept of knowledge on the basis of there being no sufficient conditions for knowledge is to criticise vacuously. We allow that a justifiable knowledge claim is not one with sufficient reasons, and so it seems irrelevant to say that the concept is valueless because it is how it is.

Knowledge claims can be falsified. That is in the structure of the concept and explained by  $K_a p \supset p$ , and the other axioms and rules. But, as to what actually justifies knowledge claims, that may best be left to those concerned for the sociology of knowledge.

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## FOOTNOTES

(1) J. HINTIKKA: *Knowledge and Belief* (Cornell University Press, Ithaca, N.Y.) 1962. I shall refer to the system of logic there set out as 'HS4': pp. 40, 41, 43, 47 especially.

(2) This thesis is embodied in the rule (C.KK\*)

(3) cf. the symposium in *Synthese*, Vol. 21 no. 2. June 1970.

(4) J. HINTIKKA, "Epistemic Logic and the Methods of Philosophical Analysis". *Australasian Journal of Philosophy*, Vol. 46. No. 1. May 1968 (abbreviated in what follows as 'ELMPA').

(5) J. HINTIKKA, "Semantics for propositional attitudes" in *Philosophical Logic*. Eds. J. W. Davis et al. D. Reidel, Dordrecht Holland. 1969 (abbreviated below as 'SPA').

(6) J. HINTIKKA, "'Knowing that one knows' reviewed". *Synthese*, Vol. 21. No. 2. June 1970 (abbreviated below as 'KKR').

(7) *Op. cit.*, p. 39.

(8) *Op. cit.*, p. 40 \* my emphasis.

(9) *Op. cit.*, p. 41. n.

(10) *Op. cit.*, p. 50.

(11) ELMPA — p. 43.

(12) R. HILPINEN: "Knowing that one knows and the Classical definition of Knowledge", *Synthese*, Vol. 21, No. 2. June 1970, p. 109.

(13) cf. E. J. LEMMON, "Algebraic Semantics for Modal Logics I and II", *The Journal of Symbolic Logic*, Vol. 31, p. 47 ff.

(14) *Op. cit.*, pp. 40-41, 43.

(15) ELMPA, p. 45.

(16) cf. ROBINSON, R., "The Concept of Knowledge", *Mind* LXXX, 1971, p. 22.

(17) KKR, pp. 141-161.

(18) *Ibid.*, p. 141.

(19) cf. in *Knowledge and Belief*, p. 18 ff.

(20) SPA, p. 24 ff.

(21) KKR, p. 143.

(22) KKR, p. 144.

(23) KKR, p. 144.

(24) KKR, p. 145.

(25) KKR, p. 142.

(26) *Op. cit.*, p. 121.