A NEW MODAL MODEL

Lou Goble

I introduce here a new semantical interpretation, QM, for quantified modal logic with identity. QM is similar in many ways to the standard accounts of, for example, Kripke (e.g. [3]), but it is significantly different in its treatment of quantifiers. I shall assume the reader is familiar with the standard approach. The present account grows out of consideration of some ideas of Leibniz (expressed, for example, in his correspondence with Arnauld). Although developed independently, QM is also somewhat similar in approach to interpretations of other authors, notably Kanger [2] and Thomason (his Q2 of [4]).

1. As in the Kripkean semantics, QM begins with a quantified model structure $\mu = \langle G, K, R, \psi \rangle$, where K is a non-empty set of possible worlds; G, the 'home world', is in K; and R is a binary relation on K for alternativeness or accessibility (appropriate properties for R are assumed). ψ is a function assigning non-empty sets, D_H is the set of individuals inhabiting H.

Let $D = \bigcup_{H \in K} D_H$. (1)

Given $\mu = \langle G, K, R, \psi \rangle$,

(i) let v be an assignment function determining values for individual variables for worlds $H \in K$, so that $v(x,H) \in D_H$ for every individual variable x. x has a value in every world, but that value may be different in different worlds; i.e. $v(x,H)$ may be distinct from $v(x,H')$.

(ii) let s be an 'extension function' determining extensions in worlds for (a) all individual constants, a, so that $s(a,H) \in D_H$, and (b) all simple n-adic predicates, F^n, such that $s(F^n,H)$ is a set of ordered n-tuples $\langle d_1, \ldots, d_n \rangle$ of members of D_H. (2)

(iii) Rules of truth are as usual. Given μ, v, and s, if $d_1,\ldots,d_n$ are the values or extensions in H of the singular terms, $t_1,\ldots,t_n$ respectively, the atomic formula $F^n(t_1\ldots t_n)$ is true in H if and only if $d_1,\ldots,d_n$ stand in the relation which is the extension of F^n in H. That is to say, letting δ_v be a denotation function such that $\overset{s}{\delta}_v(t,H) = v(t,H)$ when t is a variable and $s(t,H)$ when t is a constant, if $\overset{s}{\Phi}_v$ is our evaluation function:

(a) $\Phi_v(F^n(t_1\ldots t_n),H) = T$ if and only if

$$<\overset{s}{\delta}_v(t_1,H),\ldots\overset{s}{\delta}_v(t_n,H)> \in s(F^n,H);$$

especially,

(b) $\overset{s}{\Phi}_v(t_1 = t_2,H) = T$ if and only if $\overset{s}{\delta}_v(t_1,H) = \overset{s}{\delta}_v(t_2,H)$.

Conjunction, negation, and necessity are as usual, so that, for example,

(c) $\overset{s}{\Phi}_v(NA,H) = T$ iff $\overset{s}{\Phi}_v(A,H') = T$, for every $H' \in K$ such that $H\,R\,H'$.

For the quantifiers QM requires

(d) $\overset{s}{\Phi}_v((x)A,H) = T$ iff $\overset{s}{\Phi}_{v'}(A,H) = T$, for every assignment function v' just like v in H except, perhaps, at x.

We say that v and v' are *alike in H except, perhaps, at* x, if for every individual variable, y, different from x, $v(y,H) = v'(y,H)$. This allows that v and v' may disagree anywhere in in other worlds H'. Similarly,

(e) $\overset{s}{\Phi}_v((Ex)A,H) = T$ iff there is a v' just like v in H except, perhaps, at x such that $\overset{s}{\Phi}_{v'}(A,H) = T$.

(iv) A formula, A, is *true* for s in μ iff for every v, $\overset{s}{\Phi}_v(A,G) =$

T. A is *valid* in μ iff A is true for every s in μ. A is *valid*, simpliciter, iff A is valid in every μ. B is a *consequence* of a set of formulas, Γ, iff for every μ, s, and v such that all the members of Γ are true on v for s in μ, B is also true on v for s in μ. Given μ and s a member of D, d, *satisfies* Ax in H (where x is the only variable free in Ax) iff there is a v such that

d = v(x,H) and $\overset{s}{\Phi}_v(\text{Ax,H}) = \text{T}$; Ax is *true of* d in H iff d satisfies Ax in H. (Similarly for sequences of objects satisfying formulas with more free variables, and for formulas being true of sequences.)

2. The principle and obvious difference between this interpretation for quantified modal logic and the standard accounts lies, of course, in (i). On those models variables are assigned values irrespective of worlds; i.e. their basic locution is 'v(x) = d' rather than 'v(x,H) = d'. At the same time definite descriptions are treated by the standard accounts as they are here, taking different extensions in different worlds. On those accounts proper names are sometimes treated as are the variables, always having the same denotation regardless of world, sometimes they are treated more like descriptions.

The approach of QM has the theoretical (or aesthetic) advantage of providing a simple, unified way of interpreting all singular terms in the language of the logic. We do not consider variables as fundamentally different from constants (except, of course, for their being *variables*, whence the distinction between v and s). All have their denotations defined relative to worlds. This turns out to have far reaching consequences. Moreover, all singular terms are also treated in much the same way as are predicates, whose extensions are also relativized to worlds, thus giving even greater unity to the theory.

Practically, this approach has the advantage of providing a simple explanation for the failure of inferences involving substitution of identically referring terms in modal contexts, for neither Leibniz' Law

(L) $(x)(y)(x = y \supset . Ax \supset Ay)$

nor its instances

(L') $a = b \supset . Aa \supset Ab$

are valid when Ax contains x free within the scope of a modal operator. Hence inferences from $a = b$ and NFa to NFb are not generally valid (though they would be valid if the appropriate instance of (L) or (L') were introduced as an additional premiss). Although (L) does generally fail to be valid, it is still fair to understand '=' as a genuine identity predicate (contra Quine); for truth conditions for formulas $a = b$ (in worlds) are exactly what one would want for statements of identity, viz. that a and b denote exactly the same thing (in those worlds).

The modification required of the logic of identity, expressed by (L), is offset by the fact that QM preserves all of the principles of classical quantification theory. In contrast, the standard, Kripkean, interpretation preserves (L) but at the cost of the quantifier rules. There although (L) is valid, (L') is not; in this way that approach succeeds in blocking the paradoxical inferences. This illustrates how

(UE) $(x)Ax \supset At$

is, on that approach, not generally valid, even when 't' denotes successfully in every world. (UE) is valid in QM without restriction (so long as 't' denotes and is free for x in Ax).

It has been argued (e.g. by Hughes and Cresswell [1] p. 199n.) against the sort of interpretation offered here that if the language being interpreted contains predicate variables, the rule of substitution for them does not preserve validity. This may be seen from the fact that (L) is not valid even though it would come by such a rule from

$$(x)(y)(x = y \supset . Fx \supset Fy)$$

which is valid in QM, as is any instance of (L) where A contains no modality. This observation is correct, but it should not bother us too much. So long as one does not have predicate quantifiers one may as well regard all simple predicates as constants (this is implicit in our (iib)) and so one would not

want any rule of substitution for them. In higher order languages this might, however, pose a problem. But at least the same problem applies to any of the standard semantics.
As we have seen, there (UE) ie not generally valid, since quantification is only over members of D_H (for formulas being evaluated in H) and the denotation of 't' might be outside of D_H. But (UE) would follow by such a rule of substitution from

$$(x)Fx \supset Ft$$

which is valid (so long as 't' denotes), as is any instance of (UE) in which A contains no modality. Hence, substitution for predicate variables does not preserve validity on this account either. Moreover, since

(UE') $(x)Ax \supset Ay$

is standardly valid (when 'y' is a variable free for x in Ax), substitution must fail for individual variables as well. I take this to be a more serious difficulty; it does not arise in QM. ([3])

On the theory given here both the Barcan Formula

(1) $(x)NAx \supset N(x)Ax$

and its converse

(2) $N(x)Ax \supset (x)NAx$

are valid. Neither is valid on Kripke's semantics unless special assumptions are made, viz. for (1) that (i) $D_{H'} \subseteq D_H$ whenever $H\,R\,H'$, and for (2) that (ii) $D_H \subseteq D_{H'}$ whenever $H\,R\,H'$. No such assumptions are necesary in QM. Objection is sometimes made to one or the other of these formulas. As these objections generally are directed against (i) or (ii) and not (1) or (2), I shall take the presence of these formulas to be harmless in the present system.

More serious, however, is the fact that not only is

(3) $(Ex)NAx \supset N(Ex)Ax$

valid in QM, but so is

(4) $N(Ex)Ax \supset (Ex)NAx$.

The validity of (3) is generally unchallenged; it follows on the standard accounts even without (i) or (ii), whereas on those models (4)'s validity comes with no plausible assumptions.

Against (4) it is argued that it might be that in every world, H, there is a member of the domain D satisfying Ax (so N(Ex)Ax would be true) but no member of the domain which satisfies Ax in every world and so no member satisfying NAx (so (Ex)NAx would be false). For example, considering throws of a fair die, while there must be some face turning up, there is no one face which itself must turn up.

This argument, while persuasive, does not strike against the validity of (4) on the present interpretation. According to the rules for evaluating formulas while for (Ex)NAx to be true (in our world G) some member of D must satisfy NAx, this does not mean that any individual in D must satisfy Ax in every world. Indeed, inspection of any proof of the validity of (4) in QM shows how harmless the formula is, for it shows that generally it is different individuals which do satisfy Ax in different worlds, which is what our intuitions tell us is correct. Thus (4), according to QM, says no more than we want it to. Is there a face of the die which must turn up ? Indeed there is, unless the die melts into a little plastic puddle (in which case the antecedent of (4) is false as wel). Which face ? The face that turns up. (Suppose that on the G-throw it is the 6 which faces up; does this mean that the 6 is the face which turns up on every throw ? Not necessarily. In QM, $x = y \supset N(x = y)$ is not valid.)

The objections which are raised are not really raised against the formula (4) (as interpreted in the theory). They are raised against statements in English (or other natural languages) which are supposed to be counterparts of (4). E.g. 'Necessarily there is some face which turns up, so there exists some one face which itself must always turn up.' That this may be taken to be false does not count against the validity of (4), only against the attampted modelling of the sentence in (4). We should not claim that QM provides the only way in which to interpret statements containing quantifiers and modalities, for clearly it does not. We maintain only that it presents one important interpretation. The theory, by way of (4), offers another way of reading the English statement according to which it is not objectionable at all. This reading is captured by paraphrases, e.g. 'necessarily some face turns up, so some face must turn up', which sound as trivially true as (4) itself.

Hughes and Cresswell ([1] p. 197) claim that reading the quantifiers in the way of QM commits one to the view that the values of individual variables and the referents of constants are intensional objects. 'The face that turns up' must, they argue, in this way refer to an intension or individual concept rather than to one of the (plastic) sides of the (plastic) die.

In reply, however, it should be observed that the members of the domains according to which formulas are evaluated are no more intensional than are the members of the domains on the standard interpretations, and it is only members of D that are values of variables and referents of constants. Hence this part of the semantics carries no special commitment to intensions. It is suggested, however, that for this theory the real 'objects' which are the denotations of singular terms are 'strings' or 'chains' of members of D; each link representing the extension of the term in a world. If one wants intensions, this is perhaps the way to do it; or equivalently one may take these intensions to be functions mapping members of K to members of D.

It is a mistake, however, to think that this is what QM does. Such 'strings' are not the denotations of any terms. In fact, such a pure notion of reference independent of worlds has no place in this theory. One cannot say simply that an expression, 'the face that turns up', refers (or does not refer) either to some member of the domain or to such an intensional object. All that one can say is that the expression refers to a member of the domain *in a world* (e.g. the 6-face in the G-throw). For this there are no intensions, only ordinary members of D. Similarly, one cannot say that the value of x is d simpliciter; only that the value of x *in H* is d. This is, after all, the distinctive feature of this semantics; all the basic semantical concepts are relativized to worlds, not just some such as the denotation of a description.

Our theory would become like the standard accounts if we confined our attention to assignment functions which always agree in their assignments to Variables in all worlds; i.e. to those v such that $v(x,H) = v(x,H')$ for all variables x and all all $H \in K$; and then used only these v in our evaluation of formulas (x)A and (Ex)A. This is, in effect, to make our variables range only over what have been called *substances*, that is, those indestructible individuals in D which inhabit every possible world. (Compare Thomason's Q3 [4].)

If this is the right way to think about substance, then our interpretation might be criticized for not providing the means for the language of the quantified modal logic to express the fact that something was a substance. In the standard framework that a term, a, denotes a substance in this sense may be expressed by the formula $(Ex)N(x = a)$. On our account, however, $(Ex)N(x = y)$ is satisfied (in any H) by every member of the domain (of H). Hence QM cannot distinguish between substances and non-substances in this way. For QM everything is a substance (though this notion too must be relativized to worlds, for an individual may be a substance of one world and not a substance of another). Perhaps this is the way it should be; at least it avoids some difficult questions. One wonders, for example, what, on the standard 'substantialist' account, are those members of D which do not exist in every world. They are not substances in the sense given; they are not intensions in the sense sketched above.

Moreover, this bit of substantial metaphysics must raise the familiar problem of the identification of individuals across worlds. Either a substance is what it is independent of its having any particular properties (and so may change all its properties from world to world), or else it carries with it a set of (essential) properties by virtue of which it is what it is in all situations. In the first case one would seem to be committed to *bare* substance; in the second case one would seem to be committed to essential properties. Neither course seems desireable for a modal logic. The theory which has been offered here avoids both alternatives; the question of identification of individuals in different worlds does not even arise. This may be for QM its greatest advantage.

University of North Carolina, Chapel Hill LOU GOBLE

FOOTNOTES

(1) No assumptions are made about relations between D_H and $D_{H'}$ when $H R H'$. The logic is the same whether $D_H = D_{H'}$, are $D_H \cap D_{H'} = 0$, or whether they overlap. This is not the case with the standard interpretations.

(2) We should eventually distinguish two classes of individual constants: proper names (e.g. 'Jones') and definite descriptions (e.g. 'the man next door'). However, as I shall assume for the present discussion that all these terms, especially the descriptions, do denote successfully in all worlds (and that the individual denoted by a definite description uniquely satisfies the predicate of the description in each world), it is not necessary now to draw this distinction. I do this in order to avoid mixing problems connected with quantification into modal contexts with problems about non-designating of the first sort.
terms. These are separable issues, and I am now only concerned with those

(3) One way of avoiding this last difficulty within the standard account would be to define validity so that it could only apply to closed formulas. Formulas with apparently free variables should then be understood under the 'generality interpretation' according to which, e.g. (UE') would be just $(y)((x)Ax \supset Ay)$. (Cf. Kripke [3] p. 89.) There would then be no substitution for free variables for there would be no free variables at all. In QM this expedient is unnecessary; free variables are really free, fully amenible to substitution.

REFERENCES

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