SCEPTICISM AND MODAL LOGIC [1]

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In this paper I examine two modal arguments for perceptual scepticism. The arguments are connected; the crucial premise of one being supported by the other. I criticize both arguments, charging that the first either contains a false premise or is irrelevant to the issue, and that the second is formally invalid.

I. *Argument A1*

Sceptics typically decorate their philosophical discourse with examples about sticks in water, mirages and Shakespearean phantom daggers, and they often introduce make-believe hypotheses about malevolent deities and hallucination-generating machines. I think that often one can discern, standing beneath these trappings, the following basic sceptical argument: [2]

(A1) The falsity of any perceptual statement is a possibility.

No person knows any statement whose falsity is possible.

Therefore no person knows any perceptual statement.

Two expressions occurring in A1 ('perceptual statement' and 'possible') require clarification. (i) By 'perceptual statement', I mean a contingent statement which records an (actual or possible) observation. A typical perceptual statement will attribute some sensible characteristic(s) to some physical object(s), for example, 'Howard's mailbox is black'. By stipulating that perceptual statements are contingent we exclude certain trivial counter-examples to the first premise of A1, such as 'Howard's mailbox is either black or not black'. (ii) I shall use the term 'possible' with the sense of *logical* possibility. [3] So 'It is possible that A is B' means that the claim that A is B is non-contradictory. The justification for choosing this sense of "possibility" from among the several available alternatives is that in the present context these alternative senses should be unacceptable to the sceptical advocate of A1. The first premise of the argument is false if 'possibility' is used in the sense of "minimal probability." The sceptic cannot employ "physical possibility" as an interpretation because a claim to know what is physically possible commits one to recognizing knowledge of scientific laws. "Epistemic possibility" is ruled out since the issue A1 is intended to help resolve is the size and constitution of our body of knowledge.

In order to determine the validity of A1 we symbolize it. The argument can be symbolized in Fitch's modal quantificational logic: (4)

(A1') $(x)[Sx \,\&\, Ax \supset \Diamond Fx]$

$(x)(y)[(Px \,\&\, Sy \,\&\, \Diamond Fx) \supset \sim Kxy]$

$\therefore (x)(y)[(Px \,\&\, Sy \,\&\, Ay) \supset \sim Kxy]$

(Sx = x is a statement, Ax = x is perceptual, Fx = x is false, Px = x is a person, Kxy = x knows y.) Notice that the premises of A1' have quantifiers ranging over modal operators. This is permitted in Fitch's system, but is frowned upon by some logicians. (5) (In the remainder of the paper I refer to systems such as Fitch's which permit quantifiers to range over modal operators as "extended modal systems.") Neither of the premises of A1 can be adequately symbolized in a nonextended modal system. For example, the first premise cannot be symbolized as:

(F1) $\Diamond(x)[(Sx \,\&\, Ax) \supset Fx]$

Formula F1 is the proper symbolization for another sentence:

(S1) The falsity of *all* perceptual statements is a possibility. S1 is not equivalent to the first premise of A1; it entails that premise but is not entailed by it. That the first premise of A1 does not entail S1 is seen from the fact that there is no contradiction in the following claim:

> The falsity of *any* perceptual statement is a possibility, but (since some perceptual statements are denials of other perceptual statements) the falsity of *all* perceptual statements is impossible.

Since F1 represents S1 and S1 is not equivalent to the first premise of A1, F1 is not a suitable symbolization of that premise. The second premise of A1 cannot be symbolized as:

(F2) $\sim\Diamond(\exists x)(\exists y)(Px \;\&\; Sy \;\&\; Fy \;\&\; Kxy)$.

This formula represents sentence S2:
(S2) It is impossible for a person to know a false statement. In the second part of this paper it is shown that S2 does not entail the second premise of A1. Since F2 symbolizes S2 and S2 is not equivalent to A1's second premise, F2 does not correctly represent that premise. F1 and F2 (and other formulas logically equivalent to them) are the only plausible symbolizations of the premises of A1 in nonextended modal logic. We conclude that *if* A1 is symbolized in modal logic, it must be symbolized in an extended system.

However, it is possible to symbolize the argument nonmodally. Taking advantage of the facts that A1 utilizes "possibility" only in connection with statement falsity and that 'possibly false statement' means (in the context of A1) statement which is not logically true, we can symbolize A1 in ordinary nonmodal quantification logic:

(A1'') $(x)[(Sx \;\&\; Ax) \supset \sim Lx]$

$(x)(y)[Px \;\&\; Sy \;\&\; \sim Ly) \supset \sim Kxy]$

$\therefore (x)(y)[Px \;\&\; Sy \;\&\; Ay) \supset \sim Kxy]$

(Lx = x is logically true.) Since there are doubts about the legitimacy of extended modal logic, it is prudent to adopt A1'' as our symbolization of A1. The techniques of quantification theory establish the validity of A1", and therefore of A1.

We turn our attention to the question of the truth of A1's premises. The first premise is analytically true (given the definitions of 'perceptual statement' and 'possible' provided above). Hence A1 proves its conclusion if and only if its second premise is also true. Before we can determine the truth value of that statement we must consider the sense in which its constituent term 'knows' is being used. This is a crucial matter since there are grounds for suspecting that the sceptic uses the word with a special sense.

I think it likely that a sceptic who would advance A1 understands 'knows' in such a way that 'X knows that p' entails 'p is necessarily true'. (I call this the "strong sense" of the term.) My reason for holding this is that if the word is not taken in this strong sense then the second premise is contingent. I doubt that the sceptic will regard the statement as merely contingently true. (i) He has not arrived at the statement by examining numbers of cases in which a person claims knowledge of a contingent statement. (ii) He will not permit the statement to be refuted by counter-examples. (iii) If he claims to know the premise to be true, he cannot consistently regard it as contingent. Probably the sceptic takes the word 'knows' in this strong sense, and believes that this is the standard sense.

If 'knows' is understood in this way, then the second premise is analytically true, and A1 proves its conclusion to be true. However, the English word 'knows' does not have this meaning. So if the term is taken in the context of A1 in the strong sense, then A1 is irrelevant to the nonsceptic's contention that we know (ordinary sense) some perceptual statements. It is obvious that 'knows' does not ordinarily have this strong sense. Consider these sentences:

Howard knows some false statement.
Howard knows some true contingent statement.

Only the first sentence is inconsistent. We can disprove the contention that 'X is a chair' entails 'X has four legs' by showing that virtually every speaker of English (the well-educated and the uneducated alike) will label as "chairs" objects with fewer than or more than four legs. In the same way we disprove the contention that 'knows' has this strong sense by noting that almost every speaker of English will label as "known" some contingent statements. In fact the overwhelming majority of statements labeled "known" are contingent.

Possibly the sceptic is not employing 'knows' in A1 with the strong sense. In this case the second premise is contingent and false. It is proved false by this argument:

> I know that my mailbox is black.
> It is possible that the statement that my mailbox is black is false.
> ---
> Hence it is false that no person knows any statement whose falsity is possible.

Since the argument is valid and both its premises are true, it establishes its conclusion. Of course, the sceptic will claim that this argument begs the question against him by virtue of its first premise. But this charge of begging the question is of little value unless it can be backed up with a demonstration of the falsity of the argument's first premise. And so long as the sceptic abjures the strong sense of 'knows', there is little chance that he can provide a cogent demonstration.

I sum up my criticism of A1 with a dilemma: if 'knows' is being used in the strong sense then A1 is irrelevant to the issue of perceptual knowledge, and if it is not being used in this sense, then its second premise is false.

II. Argument A2

What could lead a philosopher to believe that the term 'knows' has this strong sense ? Restated: what could lead a philosopher to regard the second premise of A1 as a necessary truth ? I am acquainted with one (and only one) plausible argument for this position. I suspect that some sceptics employ this argument. [6]

(A2) It is impossible for a person to know a false statement.

Therefore no person knows any statement whose falsity is possible.

'Possible' must retain the sense given it in A1 (i.e., "logically possible") so that A2 will be relevant to A1. The premise of A2 is true. Because the meaning of 'prove' conflicts with the meaning of 'invalid argument', it is impossible for a person to prove an invalid argument. In a similar way, because of a conflict between the meanings of 'know' (as that term is ordinarily used) and 'false statement', it is impossible for a false statement to be known. The premise of A2 is not merely contingently true, it is necessarily true. We know its truth a priori. Since necessary truths entail only necessary truths, if A2 is valid, it establishes the necessary truth of its conclusion. If the sceptic could prove that this statement is necessarily true, he would be demonstrating that the strong sense of 'knows' is (in spite of frequent mistaken usage) its proper sense. Unfortunately for the sceptic, A2 is logically invalid.

I begin my attack on an informal plane with a refutation by logical analogy. Consider this argument:

(A3) It is impossible for a person to disprove (i.e., show to be false) a true statement.

Therefore no person disproves any statement whose truth is possible.

The conclusion of A3 is false. It is equivalent to the statement:

People disprove only contradictions.

My disproving that I have twelve fingers establishes the falsity of A3's conclusion. The premise, on the other hand, is true. It follows that A3 is invalid. But since A2 has the same logical form, it also is invalid. The sceptic might side-step this proof by claiming the conclusion of A3 to be true. Other analogous arguments are available, for example, A4:

(A4) It is impossible for a team to win a game which ends in a tie.

Therefore no team wins any game whose tied ending is possible.

The sceptic cannot credibly hold the conclusion of A4 to be true. A tied ending is at least a logical possibility for any game in a sport (such as football) whose rules permit ties. The premise of A4 is true. So A4 and A2 are invalid.

The invalidity of A2 can also be shown more formally. The argument can be symbolized in an extended modal system as follows:

(A2') $(x)(y)[(Px \,\&\, Sy) \supset \sim\Diamond(Fy \,\&\, Kxy)]$

$\therefore (x)(y)[(Px \,\&\, Sy \,\&\, \Diamond Fy) \supset \sim Kxy]$

Can we symbolize A2 without employing an extended logic? Three approaches may be considered: (i) symbolizing in nonextended modal logic; (ii) symbolizing in nonmodal logic; and (iii) combining the preceding two approaches. (i) The conclusion of A2 is not correctly represented by F2:

(F2) $\sim\Diamond(\exists x)(\exists y)(Px \,\&\, Sy \,\&\, Fy \,\&\, Kxy)$.

Formula F2 treats false statements whereas the conclusion of A2 is about statements which are *possibly* false. (F2 is an acceptable rendering of the premise of A2.) F2 is the only plausible translation of A2's conclusion into nonextended modal logic. Therefore A2 cannot be symbolized in such a logic. (ii) The method by which A1 was symbolized (in the first section of the paper) in nonmodal logic is inapplicable here because the notion of "false statement" in the premise of A2 cannot be represented with the predicate 'is logically true'. (iii) Perhaps we should symbolize the premise by method (i) and the conclusion by method (ii). This would yield:

$$\sim\Diamond(\exists x)(\exists y)(Px \;\&\; Sy \;\&\; Fy \;\&\; Kxy)$$

$$\therefore (x)(y)[(Px \;\&\; Sy \;\&\; \sim Ly) \supset \sim Kxy]$$

Because the predicate 'is false' appears only in the premise of this symbolization and the predicate 'is logically true' appears only in the conclusion, this attempt camouflages the logical structure of the inference. If A2 is to be symbolized at all, it must be symbolized in an extended modal system. If such a logic is illegitimate, then A2 is a defective argument. If, however, this type of logic is acceptable, then (as I shall show) A2 is invalid. In either case the argument lacks validity. In the following I shall give the sceptic the benefit of the doubt and assume that extended modal logic is legitimate.

A streamlining of the symbolization of A2 is possible and advisable. By employing "statements" as a universe of discourse and introducing the predicate 'is known by someone', we can delete three predicates used in A2' ('is a person', 'is a statement', and 'knows'). The new symbolization, A2", not only preserves the logical structure of A2 but makes the structure easier to grasp.

(A2") $(x)\sim\Diamond(Kx \;\&\; Fx)$

$$\therefore (x)(\Diamond Fx \supset \sim Kx)$$

(Kx = x is known by someone, Fx = x is false.)

We can prove the invalidity of A2" (and hence of A2) by the method of interpretation. Retaining the same universe of discourse and the original sense of 'Fx', we reinterpret 'Kx' to mean "x is true."

Argument A5 results:

(A5) It is impossible for a statement to be both true and false.

Therefore no statement whose falsity is possible is true.

By giving 'Kx' the meaning "x is false" and 'Fx' the meaning "x is true," we produce A6:

(A6) It is impossible for a statement to be both false and true.

Therefore no statement whose truth is possible is false.

Each of these arguments has a true premise and a false conclusion. So each is invalid; hence A2" and A2 are invalid. Doubts about the falsity of the conclusions of A5 and A6 may be allayed by noting that those statements when combined with S3 entail S4.

(S3) Every statement is either true or false.

(S4) Every statement is such that either its falsity or its truth is impossible (i.e., there are no contingent statements).

S3 is true and S4 is false. Therefore either the conclusion of A5 or the conclusion of A6 (or both) is false. So at least one of the two arguments (A5, A6) is invalid. Hence A2" and A2 are invalid.

It is a principle of logic that p entails q if and only if the negation of q entails the negation of p. So A2" is valid if and only if A7 is valid.

(A7) $\sim (x)(\Diamond Fx \supset \sim Kx)$

$\therefore \sim (x) \sim \Diamond(Kx \; \& \; Fx)$

The premise and conclusion of A7 are equivalent, respectively, to the premise and conclusion of A8.

(A8) $(\exists x)(Kx \,\&\, \Diamond Fx)$

$\therefore (\exists x)\Diamond(Kx \,\&\, Fx)$

Essentially the same logical error that is committed in A2 is found in A8, but the mistake may be more readily spotted in A8 since that argument is devoid of negations. The premise of A8 states that something exists which is K and which is possibly F. It does not follow from this that there is something which is possibly both K and F. Suppose that "K" and "F" are logically incompatible properties. The conclusion would be false, but the premise of A8 might be true nevertheless. There may be an individual to which property "F" can be ascribed without contradiction even though the individual happens to be K. I suspect that some sceptics regard this invalid inference (A8) as correct. They suppose that if one admits the existence of known statements whose falsity is possible, then one is committed to the absurdity that there are statements which are possibly both known and false. Through this mistake in modal logic they are led to a position of scepticism in epistemology.

III. *Postscript*

A sceptic could make the following rejoinder to the present paper:

"You have convinced me that the symbolization of A2 involves quantifying over modal operators. For that reason I give it up and A1 with it. I replace them with the following sceptical chain of reasoning which, you will notice, can be symbolized in truth-functional logic supplemented by modality.

> The following is impossible: Howard's mailbox is not black and someone knows that it is black.
>
> ---
>
> Therefore if Howard's mailbox is not black, then it is

impossible that someone knows that it is black.

Therefore if it is possible that Howard's mailbox is not black, then no one knows that it is black.
It is possible that Howard's mailbox is not black.

Therefore no one knows that Howard's mailbox is black.

Take any perceptual statement you please, I can (by substituting it for the statement that Howard's mailbox is black in the above series of arguments) show that no one knows it. Q. E. D."

There are two defective links in this inferential chain. As an aid to showing this we symbolize the reasoning.

(F3) $\sim\Diamond(\sim B \,\&\, K)$

(F4) $\therefore \sim B \supset \sim\Diamond K$

(F5) $\therefore \Diamond\sim B \supset \sim K$

$\Diamond\sim B$

$\therefore \sim K$

(B = Howard's mailbox is black, K = Someone knows that Howard's mailbox is black.) Consider the first inference. Formula F6, which is not a contradiction, entails both F3 and the denial of F4.

(F6) $(K \dashv B) \,\&\, \Diamond K \,\&\, \sim B$

Hence F3 does not entail F4. The second inference is no better. Formula F7, which is not contradictory, entails both F4 and the denial of F5.

(F7) $K \& B \& \Diamond \sim B$

So F4 does not entail F5. All that remains of this chain of reasoning is the third link — a modus ponens which has either a false first premise (if 'knows' is used in its ordinary sense) or a conclusion which is irrelevant to the question about perceptual knowledge (if the word is used in the sceptic's strong sense).

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(1) I am indebted to David Marans, Jahn Quinn, Edgard Schuh, Ernest Sosa, and Harold Zellner, who criticized earlier drafts of this essay.

(2) Argument A1 is evident in the sceptical reasoning exhibited and discussed by James CORNMAN and Keith LEHRER in Chapter 2 of *Philosophical Problems and Arguments: An Introduction* (New York: The Macmillan Co., 1968). I think A1 is implicit in Book I of Sextus Empiricus' *Outlines of Pyrrhonism*, Montaigne's *Apology for Raimond Sebond*, the first of Descartes' *Meditations on First Philosophy*, and C. E. M. Joad's *Guide to Modern Thought* (London: Faber and Faber, Ltd., 1933), pp. 88-93.

(3) We must guard against equivocating on 'possible' in the premises of A1. This error is seductive since it is natural to interpret the premises as truths. Thus one may take the first premise as concerned with "logical possibility" and the second as treating some stronger type of "possibility".

(4) *Symbolic Logic: An Introduction* (New York: The Ronald Press Co., 1952), pp. 164-66.

(5) See W. V. QUINE, "The Problem of Interpreting Modal Logic," *The Journal of Symbolic Logic*, XII (1947), 43-48, and "Reference and Modality," in *From a Logical Point of View* (2nd ed.; New York: Harper & Row, 1961).

(6) Argument A2 appears to lie behind the following passage by H.H. PRICE:

> ... Common usage seems to be simply muddled on this point [viz., whether 'know' means the having of some fact directly present to consciousness or the believing of some proposition which is reasonably certain]. On the one hand it includes under the head of knowing the firm belief in a reasonably certain proposition; on the other, it refuses to admit that knowing can be mistaken. And yet if the proposition is only reasonably certain, there is the possibility that we may be mistaken in believing it.

"Some Considerations About Belief," *Proceedings of the Aristotelian Society*, XXXV (1934-35), reprinted in *Knowledge and Belief*, ed. by A. Phillips GRIFFITHS (London: Oxford University Press, 1967), pp. 51-52.