ON A NOMINALISTIC ANALYSIS OF NON-EXTENSIONAL CONTEXTS

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§ 1. *Introduction*

In The *Diversity of Meaning,* (¹) L. J. Cohen proposes a "theory of operators" as a kind of nominalistic analysis of non-extensional contexts; this theory is contrasted with Frege-Church semantics, in order to throw light on the explanatory advantages of Cohen's proposal. The object of this paper is to point out some defects of Cohen's formulations; at some points I shall indulge in a comparison of both theories.

According to the author, what is needed for the solution of the problems raised by non-extensionality is "a general theory of statements (i.e., true-or-false saying)- forming operators on sayings" (p. 201). For our purposes here it will be enough to say that a "saying" is just "a sentence-token described under a determinable" (p. 166) and it is intended to represent "what a man can repeat to himself, communicate to others, or treat now as a premise, now as a conclusion" (p. 163). The author takes "assertive sayings" (= statements, in his terminology) to be the objects of propositional attitudes, which fits well with his rejection of subsistent entities in semantics; the words or phrase tokens which compose a saying are called "terms" (p. 162) and are obtained by the same process of successive abstractions by which the author gets the concept of saying. (²) A statement-forming operator is simply an expression governing a subordinate clause, like "It is necessary that," "It is a causal law that", etc.

The theory advanced by Cohen presents two different levels of analysis to which I will refer as Analysis 1 and Analysis 2. According to the first one, "all non-extensionality may conveniently be taken to arise from the implicit or explicit occurrence of statement-forming operators on sayings" (p. 201). In order to maintain this general diagnosis of the origin of non-extensionality he is compelled to introduce some meaning equivalences which are supposed to make explicit the hidden operators required by the theory. Thus the sentence

(1) The characteristic of being a featherless biped is not to be identified with the characteristic of being a rational animal

is assumed to be "equivalent in meaning" (p. 204) to

(2) It is not analytic that all and only featherless bipeds are rational animals,

where (2) brings to light the wanted operator "it is not analytic that" which was "hidden" in (1). It is contended that, under this approach, the failure of extensionality is easily explained as resulting from the fact that the substitution may change the identity of the saying (Cf. § 3, below).

But "the reduction of all non-extensional contexts to statements containing statement-forming operators on sayings [Analysis 1] is only a half way stage. There is a logically simpler form of statement that is equivalent to this one. Any such statement turns out to have an equivalent in meaning that predicates something of either some or all tokens of its constituent saying" (p. 207). This "logically simpler form" of statement is essentially a modified version of Scheffler analysis of statements of assertion and belief, (3) and according to it a sentence like

(3) A cretan says that it is raining

is transformed into

(4) There is a token x such that (x is a statement by a cretan and x is a statement that it is raining) and (x is true if and only if it is raining).

This example shows the import of Analysis 2. We obviate the further transformation that (2) would have to suffer under this second analysis, as we won't be concerned here with this type of statements.

An important feature of the theory of operators is that it does not allow for that change of the normal meaning and reference of an expression in non-extensional contexts which characterizes Frege's semantics, and by which he sought to save the law of extensionality:

> "If we suppose that, explicitly or implicitly, all informal non-extensionality arises from the addition of certain operators to sayings, we must suppose that the identity of the sayings is unaffected by this addition. The same saying occurs, now in an extensional context, now in a non-extensional one, and since the sayings are the same so too are their meanings and the meaning of their terms" (p. 206).

As is to be expected, no attempt is made to save the law of extensionality:

> "Rather than maintain a law of extensionality for unformalized statements by modifying their normal interpretation [as in Frege's semantics], the theory maintains instead the normal interpretation of these statements and it is content to accept that the law of extensionality does not apply to them even in its weaker version" (p. 207).

The referred weaker version of the law of extensionality — that is, the one saved under Frege's analysis — is presumably a contextual version of Leibniz law; it does not say that if '$a = b$' is true, a and b are interchangeable *salva veritate* in every context, but rather that they are interchangeable if they have the same reference *within* the contexts in which the substitution is carried out.

By contrast and in a way of hopeful compensation, the author announces that

> "when comes to formalization the theory of operators

permits us to be more extensional than Frege. The general logic of statement-forming operators on sayings turns out to admit of representation in a formal system for which the law of extensionality is provable in its stronger version" (p. 207).

Unfortunately, the strong extensionality allowed by Cohen's formal system results simply of an *ad hoc* restriction over the substitution-instances of the predicate variables and is unclarifying for the analysis of informal statements (cf. § 4, below).

§ 2. *On some stubborn statements*

A curious point arises in connection with a type of sentences which Quine presents in *Word and Object* (1960, p. 215) as a "peculiar difficulty" for Scheffler's analysis. Consider:

(1) Paul believes something that Elmer does not,

which on Fregean platonistic lines could be symbolized as

(2) $(\exists p)[\mathrm{B}(x,p) \,.\, {\sim}\mathrm{B}(y,p)]$,

where "$\mathrm{B}(x,p)$" stands for "x believes that p".

As Quine observes, however, for Scheffler "it will not do to say that Paul believes-true" (4) some utterance that Elmer does not believes-true, for it may happen that no such utterance exists or ever will; believing does not, like saying, produce utterances".

Yet Cohen proposes (p. 228) the following nominalistic reading of (1):

(3) $(\exists F)((w)(Fw \equiv Sw) \,.\, ((x)(Fx \supset \mathrm{B}xy) \,.\, {\sim}(\exists x)(Fx \,.\, \mathrm{B}xz)))$,

where "S" stands for "a certain single statement" (i.e., certain single assertive utterance-token), "y" for "Paul", "z" for "Elmer" and "Bxy" for "x is believed by y".

It may be felt that the very presence of "($\exists F$)" in (3) is a rather scandalous beginning for a nominalistic analysis; but I will not dwell on this now. A more specific complain against (3) is that it could be vacuously true even if there were no utterance-token in the universe and F were an empty property; in that case "Fw" and "Sw" would be always false and therefore "$(w)(Fw \equiv Sw)$", "$(x)(Fx \supset Bxy)$" and "$\sim(\exists x)(Fx . Bxz)$" would be true; it is obvious however that the lack of utterances shouldn't be enough for the truth of (1) under an analysis which conceives of believing as a relation between persons and utterances.

It could be replied, however, that I have misunderstood "($\exists F$)" and that the meaning explicitly given to it by the author prevents (3) from being vacuously true (in the sense explained); according to Cohen, "quantification over properties need not be regarded as anything mysterious or puzzling in this connection. We may conveniently suppose that a property exists if and only if an instance of it exists" (p. 224). Following the author's suggestion, then, we might perhaps take it that he understands "($\exists F$)" as an abbreviation for "$(\exists x)Fx$"; but if we try to interpret (3) in this way we obtain

(4) $(\exists x)Fx . ((w)(Fw \equiv Sw) . ((x)(Fx \supset Bxy) . \sim(\exists x)(Fx . Bxz)))$,

which (because "F" is neither interpreted nor bound by any quantifier) is not a statement but only a formal scheme, and so cannot be an *analysis* of (1). This shows that Cohen's non platonic claim concerning his use of "($\exists F$)" is not well supported.

On the other hand, it may be suggested that (3) should be changed into

(5) $(\exists F)((w)(Fw \equiv Sw) . (\exists x)(Fx . Bxy) . \sim(\exists x)(Fx . Bxz))$.

But then it would be apparent that the difficulty raised by Quine is fully applicable to (5); for its falsity is perfectly compatible with the truth of (1), in view of the fact that the utterance whose existence is claimed is not exhibited. The adequacy of a nominalistic analysis requires at least that the correspondence in truth-values between analysandum and analysans be assured by the existence of the required utterances; if the claim that they exist cannot be substantiated in some way, the factual posibility of a divergence in truth-values will remain open. We may conclude, then, than neither (3) nor (5) are fitted to express the content of (1).

§ 3. *Substitutivity and logical form*

As we already know, Mr Cohen contends that the theory of operators can easily explain "why there are so many kinds of degree of non-extensionality in informal discourse". The explanation is simple:

> "Different predicates apply to things in virtue of differents kind of features in them, just as 'hot', 'cold', etc., apply to an object in virtue of its temperature, and 'red', 'yellow', etc. in virtue of its color. Change an object's temperature beyond a certain point and it may no longer be truly described as hot: change a sentence's wording in any other than trivial respects and may no longer be truly described as the saying uttered by a person on a certain occasion" (p. 211).

I will not deny the intuitive plausibility of this general diagnosis, which can be easily reformulated in Fregean terms. Considering that in a Fregian semantics a belief or assertion statement is viewed as establishing a relation between a person and a proposition (named by the 'that'-clause), it could also be said, paraphrasing Mr. Cohen, but from a Fregean point of view: "Different predicates apply to propositions in virtue of different kinds of features in them... Change a sentence's wording in any other than trivial respects, and it may no longer be truly described as expressing the proposition asserted (or believed, etc.) by a person on a certain occasion". And as in Church's intensional system modal operators are simply predicates applicable to propositions, a similar account is available for the failure of substitutivity in modal contexts.

But I will argue that within Cohen's approach the above referred plausibility is only an ephimere flame, because of the lack of further theoretical support. In this section I will raise a difficulty connected with the principle of indiscernibility of identicals; another one will be dealt with at § 4.

Consider the true statements:

(1) George IV doubted that Scott = The author of Waverley

and

(2) Scott = The author of Waverley.

Yet the substitution of "The author of Waverley" by "Scott" produces the falsity:

(3) George IV doubted that Scott = Scott.

According to the theory of operators this change in truth-value is not surprising, because (a token of) "Scott = the author of Waverley" is not the same saying as (a token of) "Scott = Scott", so that the predicate "George IV doubted that" is applied respectively to different sayings in (1) and (3). So long as we consider a saying (and a proposition) as a non analysed unity, there is here a superficial paralelism with Frege.

But suppose someone says: "In virtue of the logical principle of indiscernibility of identicals, it holds that $[(x = y) . \varphi x] \supset \varphi y$; so, the formal pattern of inference: "$x = y; \varphi x; / \therefore y$" is a valid one. Now the argument: (1);(2);/∴(3) seems to fit this formal pattern; how is then possible that the premisses are true and the conclusion false ?" This is a very fashionable question; what answer could be given — and *some* should be — from the point of view of the theory of operators ?

One possibility is to say that the true logical subjects of (1) and (3) are the subordinate clauses, so that the logical form of (1);(2)/∴(3) is really the invalid one

$$(4)\ \frac{\begin{array}{l} x = y \\ \varphi p \end{array}}{\varphi q}$$

where "x" and "y" are, as usual, individual variables, "φ" stands for a predicate of sayings, like "George IV did not know that", and "*p*" and "*q*" for different sayings. This imply that the "that"-clauses are not analyzable here; but why? After all, "George IV did not know that Scott = ..." produces a true-or-false sentence when applied to a proper name, and to this extent behaves as a genuine predicate.

Mr. Cohen says that, in opposition to Frege, the theory of operators mantains the "normal interpretation" of intensional statements, and I take this as implying that there is no change of reference, as in Frege's theory. But *the fact that "Scott" and "the author of Waverley" keep in (1) and (3) their normal reference, conjoined with the aforementioned circumstance that the application of "George IV doubted that Scott = ..." to "Scott" or "the author of Waverley" produces a true-or-false sentence, makes even more arbitrary the prohibition to consider "Scott" or "the author of Waverley" as possible logical subjects of (1) and (3)*; by the way, this does not happen, for example, with Quine's explanation, according to which the occurrences of "Scott" and "The author of Waverley" in (1) and (3) are not referential at all.

At this point Frege offers his theory of indirect reference, which allows us to show that, however we analize (1) and (3), *taking into account the inner structure of the "that"-clauses,* the logical form of (1);(2)/∴(3) will not fit into the patter of the principle of indiscernibility. This theory attempts (i) to explain the semantical behavior of the names composing the subordinate clauses, and (ii) to give the range of the intensional predicate "George IV doubted that Scott = ...", avoiding thus to reject it by an arbitrary fiat; and whatever the success of the enterprise, this is an unquestionable advantage of Frege's theory. In relation with this, Mr Cohen simply says that the law of extensionality does not hold even in its (Fregean) weaker form; this is to be expected, because the weaker form requires something like the Fregean change of reference. But such a restriction calls for an explanation, which is not offered by him.

Before going further it may be worth-while to substantiate briefly, in a perhaps too compressed way, the above claims in favor of Frege. ([5]) As is well known, on Frege's view non-extensionality is a special case of referential ambiguïty. What happens in (1) is that the subordinate clause, and therefore also its componentes names (including "="), denote there the senses they ordinarily express (direct senses) and not their usual denotations. The that-clause as a whole denotes a proposition (not, as usually, a truth-value), and its components name the senses whose combination that proposition consists in. Now, assuming that the direct sense of "Scott" is different from that of "the author of Waverley", it follows that the substitution which leads from (1) to (3) changes the denotation of the subordinate clause. Up to this point we haven't gone further than Cohen. Now let us use a bracketed name with a subscript ([6]) in order to denote its direct sense, for example "$[\text{Scott}]_1$" for the direct sense of "Scott", and the structural description "($[\text{Scott}]_1$ $[=]_1$ $[\text{the author of Waverley}]_1$)" in order to represent the proposition denoted by the subordinate clause of (1). Then the logical form of (1) could be made perspicuous by

(5) George IV doubted ($[\text{Scott}]_1$ $[=]]_1$ [the author of Waverley$]_1$),

where the obliquying prefix "that" has disappeared. Out of (5) we can get the monadic predicate

(6) George IV doubted ($[\text{Scott}]_1$ $[=]_1$ $[\quad]_1$),

where "$[\quad]_1$" indicates a place to be filled by the name of an individual sense. (6) is, in effect, a predicate applicable to individual senses, and stands unequivocally for the property represented by the natural predicate "George IV doubted that Scott = ...", which misleadingly looks as a predicate of individuals.

So (5) allows us to solve directly the problem raised by the principle of indiscernibility of identity, showing that because of obliquity (= indirect reference) we have confused a property of senses with a property of individuals. Under this treatment we can say that the logical form of (1);(2);/∴(3) is the invalid one:

$$(7) \quad \frac{\begin{array}{c} x = y \\ \varphi[x]_1 \end{array}}{\varphi[y]_1}$$

and not that authorized by the principle of indiscernibility. The important point, which makes all the difference between (4) and (7), is that on these lines we do not need to seal off the subordinate clause for purposes of logical analysis; on the contrary, the Fregean approach try to reveal the semantical behaviour of singular and predicate expressions *within* the subordinate clause of (1). I am not contending that Frege's machinery is highly credible on intuitive grounds; but as Wilfred Sellars has wittingly said in connection with a Frege-like proposal of his own: The "mechanics, if not the metaphysics, of the move, is comparatively straightforward". [7]

Yet Mr. Cohen has at his disposal a second possible strategy. According to his method of formalization (or Analysis 2) the true logical form of (1) and (2) is such that their subordinate clauses only appear as parts of a predicate term applicable to tokens; thus, (1) is transformed in something like

(8) There is an x such that x is *a saying that George IV doubted that Scott = the author of Waverley*, etc.,

where the underlined phrase stands for a property attributed to the token x. So, it would not be here any possible substitution position for "Scott" and "the author of Waverley". But the stubborn question remains: Are these predicates non-analyzable and why ? Generally, sentences can be analized in different ways: "7 > 5" can be analized as "(7) > (5)" or as "(7 >)5" or as "(> 5)7"; why is not possible to do the same in the case of sentences which contain these peculiar predicates about tokens ? Of course, Mr. Cohen may *rule* that in his formalized language such predicates are non-analyzable units, and this would be all right; but it might hardly be considered as a clarification of a problem originating from the ordinary language. This point is also relevant to Scheffler's nominalist analysis; [7] his 'that'-clause predicates, like "that-Scott-is-identical-with-the-author-of-Waverley", must be viewed as non-analizable in order to prevent a recurrence of non-extensionality. But he does not pretend to deal with failures of substitutivity; his proposal is primarily related with Church's special objection to any nominalist analysis of statements of assertion and belief. [8]

§ 4. *Something on modality*

This unsatisfactory feature of Cohen's approach is even more apparent in his treatment of modality, where the promised "stronger-than-Frege" extensionality is achieved by the exclusion of predicates containing modal operators, like "is necessarily the Morning Star". After explaining to the reader the well known troubles associated with quantification into modal contexts, Mr. Cohen says:

> "It follows that in a thoroughly extensional formalization of general logic we can never afford to make a non-extensional expression like 'is necessarily the Morning Star' the equivalent of a predicate letter" (p. 233).

To this end the author introduces a stipulation (p. 213) that has the intended effect of preventing the introduction of such a kind of predicates in the interpretation of his calculus. Referring to the example:

(48) The Morning Star is necessarily the Morning Star, [9]

he rightly explains that this stipulation "is a natural consequence" of the theory of operators, that is of

> "the general theory that all cases of non-extensionality arise [...] through the occurrence of statement-forming operators on sayings and that they therefore may be viewed as being generated by certain kinds of statements about sayings. For, since 'necessarily', even when inserted thus in the middle, operates on the whole of the saying 'The Morning Star is the Morning Star', we must treat (48) as a statement about this saying, viz. the statement that it is necessary, rather than as a statement about the Morning Star to the effect that it has the property of being necessarily the Morning Star" (pp. 233-34).

I would not object too much to this outright rejection of the *de re* modalities if Mr Cohen has not contended that his theory mantains "the normal interpretation" of these statements (that is, no change of reference !). The arguments advanced in the precedent section are also relevant here, but we may complete the case with this question: if (48) is *about* the whole saying "The Morning Star = the Morning Star", it must refer to it in some way; but which is the referential mechanism here ? In Frege's semantics the situation is clear: within non-extensional context the subordinate clause is a combination of names which denotes, by isomorphic correspondence, a combination of senses, in the way explained by the theory of indirect reference. But Mr Cohen dislikes these sort of Fregean fantasies, and sticks to the normal reference of its components names; how it happens, then, that (48) succeed in referring to the whole saying and not to the things mentioned by its components names ?

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NOTES

(1) London, Methuen and Co. Ltd., 2d ed., 1966, Pp. xii, 369.

(2) Cf. pp. 161-2 of Cohen's book. The details of this process are prescindible here.

(3) I. SCHEFFLER, "An Inscriptional Approach to Iindirect Quotation" (*Analysis*, vol. 14, N° 4 (1954), pp. 83-90) and *Anatomy of Inquiry* (New York: Alfred. A. Knopf, 1967), pp. 88-110.

(4) Following a suggestion by Quine in "Quantifiers and Propositional Attitudes" (*The Way of Paradox*, New York, Random House, 1966), Scheffler uses the predicate "believes-true" for the relation between persons and inscriptions, instead of the vernacular "believes that". Cf. also QUINE, *Word and Object*, p. 212.

(5) For more details, cf. my paper "Sobre la eliminación de los contextos oblicuos", *Critica*, vol. 1, N° 2, (1967), pp. 21-33.

(6) The need for a numerical subscript for intensional abstraction springs from the fact that in Frege's theory each name has an infinite number of indirect references, corresponding to the iteration (potencially infinite) of non-extensional operators.

(7) "Some problems about Belief", in *Synthese*, vol. 19, N° 1/2 (December 1968), p. 164.

(8) A. CHURCH, "On Carnap's Analysis of Statements of Assertion and Belief" (*Analysis*, X (1950), pp. 97 ff.).

(9) I have kept Cohen's numbering.