THE RELATION BETWEEN NEGATION IN LINGUISTICS, LOGIC, AND PSYCHOLOGY A PROVISIONAL CONCLUSION

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I. *Negation and Linguistics*

A. *Introduction.*

In recent years Klima (1), Kraak (2) and Jackendoff (3) have presented analyses of negation in natural language.

Klima's attempt tried to be uniquely syntactic, Jackendoff introduced semantic features and Kraak although he did not acknowledge this clearly went in the same direction as Jackendoff.

Our own analysis has a double aim a) we want to show the relevance of these linguistic data for logical research and b) we want to indicate that only a pragmatic analysis of negation can give the final solution.

B. *Sentence negation in logic and natural language.*

To start let us begin with an easy type of expressions:

(1) Albert is not well
(2) It is false that Albert is well
(3) It is not true that Albert is well
(4) It is not the case that Albert is well
(5) It is true that Albert is not well
(6) Albert is ill
(7) It is not false that Albert is not well (unwell).

These expressions have not been selected haphazardly. If we look at symbolic logic we see that every negation is an operator on a sentence. Even if these sentences include individual constants or belong to the functional calculus P(a) or (Ax)(fx)), the negation written in front of the expression

(—P(a)) or —((Ax)(fx))) is an operator applying to the whole of the sentence and if by chance it is recognised that —((Ax) (fx)) is equivalent to (Ex) — (fx), then we can again recognise that by virtue of the derivation rules of functional calculus this sentence is equivalent to —(f(a)), for some a, where once more the negation applies to the whole sentence.

This simple state of affairs cannot be upheld if we look at everyday language.

This has been clearly recognized by Henry Hiz (4) when considering sets of sentences or texts G_1 G_2 ... G_n in a model M of a natural language, he introduces a negation for these sets on his p. 442 by means of the familiar property that O_i(b) can never be in a G_i when b is in it, and conversely. He explicitly stresses "we can take different operations 0_i and O_j for the different $_i$ and $_j$. A negation «by no means» is usable for some subsets of English but not for all kinds of English. ... About the negation, we do not assume that it is iterative English ... does not allow negation to be reapplied" (p. 442). We are not sure that he is clearly aware of the extent and the way in and to which linguistic negation deviates from our usual logical negation but we wanted to present this quotation to the reader to convince him that we are not alone, neither in denying that logical negation is not English negation, nor in believing that these diferences can be formally expressed.

The following remarks will show very clearly our intention.

a) The sentence (1) is not equivalent to the sentence (5): in (5) the eventuality that Albert was well has been considered and has been discarded. This may mean at least two things: a) it is presupposed that someone asserted Albert's being well and this has been denied in a dialogue b) there were indications as to his being well, but these indications were not strong enough and other ones, pointing to an illness have overcome them. In both interpretations (5) is a stronger assertion than (1): more is needed to overcome either dialogical or monological resistance than to yield a simple observation.

b) But there is even more: "Albert is ill' (6) and it "it is false that Albert is well' (2) are even stronger than (5). Here indeed in the second case the possibility of the truth of the sentence has no longer even been considered, and in (6) the possibility of health has not even been indicated: the pointers towards illness were overwhelming.

If finally we compare 6 to 2, then 6 is stronger than 2, because the predicate "well" is even completely absent from 6 while it is still present in 2.

As a result of these remarks, if we construct a scale going from weak to strong assertion of illness, our sentences would be situated as follows:

7 3 4 1 5 2 6

c) we consider 3 to be a weaker assertion than 4 because it denies a relationship between a sentence and a state of affairs while 4 denies the existence of the state of affairs itself.

The double negation in 7 appears to be the weakest statement because of the fact that attention is drawn a) both to Albert's being well and b) to the possibility of its being false that Albert might be not unwell. The reader will already observe that our gradation mixes linguistic with psychological remarks.

The scale we just constructed is obviously a pragmatic scale drawing upon the context and the presuppositions of the assertions in question (5).

We ask ourselves in what circumstances such sentences might be produced, to what questions they might be answers, what claims they might be the denials or confirmations of, and having asked those questions we come to the order proposed.

Let us now give a transcription of the seven sentences under study:

1. — P(a)
2. F(P(a))
3. — T(P(a))
4. — [(E!x) (x = P(a))]
5. T(—P(a))

6. Q(a)
7. — F(— P(a))

We give some explanations: T (true) and F (false) belong to semantic metalanguages and we may, if we accept Tarski's analysis of truth, assert both a) p equivalent to T(p) and b) T(p) equivalent to —F(p).

Q is a predicate that we might consider as identical to "ill". The E! quantifier has been introduced by Leonard and designates strong existence (not merely an existential quantifier). We must consider the variable "x" in assertion 4 as ranging over a peculiar set of "states of affairs".

We leave it to the reader to verify that, once this transcription is executed, the shades of meaning that made possible the ordering of these expressions on a scale, measuring their strength, have disappeared (even if their syntactical form is different indeed).

Yet, we do not see in the logic we have at our disposal better translations, preserving the relationships that are really present in our everyday language.

This brings us to the easy conclusion that only a *pragmatic analysis of negation will do justice to the properties of the operator used in the vernacular.*

C. *Adaptation of formal logic to natural language.*

We accept this conclusion and it compells us to try to execute the following task: let us examine the facts brought together by Klima, Jackendoff and others (6); let us, as completely as we can translate them into formal language, and let us then see what additional formal structure we have to add in order to do justice to the distinctions everyday language desires us to make.

It will be easier for the reader if we announce the main features of our results from the beginning:

a) we shall have to introduce higher order functional logic to express certain meaning differences in everyday language;

b) we shall have to introduce a variety of negation operators, and we shall not be able to satisfy ourselves only with a sentence negation.

c) we shall have to introduce a variety of context and presupposition relations (and here we shall be able to make use of a generalisation of Harris' approach). In general we shall start off with members of the Chomsky school and their critics but we shall finally consider Harris' approach as more helpfull.

These three points of view will be the specific features of our analysis, but we wish to announce a more general point of view: language is communication. We can not reach a theory of negation if we cannot relate it to the function in the communication as such. This part of our article cannot be completely separated from the other part that studies the function of negation in thinking and problem solving. The bridge between the two is the process of communication.

Having now stated our intentions, we may procede to their execution.

D. *The analysis of negation in everyday language makes necessary a reference to higher order functional logic.*

"John is not going to New York with his motor car to day" (8). The well known problem, whenever one finds oneself confronted with a sentence of this type is to determine the scope of the negation.

The sentence can obviously be understood as follows:
"It is not John, but Peter that is going" (9).
"It is not to New York but to Boston that John is going" (10)
"It is not with his motor car that John is going to New York" (11).
"It is not today that John is going to New York" (12).

The interpretations (9) till (12) all make the negation bear up on a part of the sentence. There is still a central negation that relates "not" to "going" (13).

Let us represent John by J, New York by N, the motor car by M to day by T and "to go" by R. The global negation will be $-R(J,N,M,T)$.

The local negations — and this is our proposal — will have to be represented by paraphrases similar to the following one: "The person that is driving today to New York is not John" as follows $[(ix)R(x,N,M,T)] \neq J$ (14)
Using the same way of rewriting we can consider the following sentences:

$[(iy)R(J,y,M,T)] \neq N$ (15)
$[(iz)R(J,N,z,T)] \neq M$ (16)
$[(iu)R(J,N,M,z)] \neq T$ (17)
$[(iv)V(J,N,M,T)] \neq R$ (18).

The i-operator is well known in the literature: it represents a function mapping a function upon (and indeed we meet here in the formula 14 till 18 a series of functions) upon a unique individual object. In everyday language we can translate it by "the one and only one ...".

We must add that even (18) is different from the general negation $-R(J,N,M,T)$ but that all members of the set 14-18 imply that the general negation is true while the general negation implies the disjunction of the members of the set.

The problem remains open to know what type of rewriting our negation will impose? There are obviously two solutions a) either we find a rule that for all given circumstances will yield one of the series (or a subset of the series) b) or we indicate further circumstances that will help us to pick out which one we have to select.

The easiest alternative is the second one; it can be decomposed into two alternatives: either the selection indices to be added are part of the sentence itself or they come from the outside. We give one example of each case:

a) intonation can stress any of the five parts of the sentence and we have a general rule "the negation is applied to the part of the sentence most heavily stressed".

b) or we can have questions in front of the sentence under study. If the question was "Where does John go with his motor car to day? and the answer is the sentence (8), then it is clear that the negation has to be attached to the place name.

These two rather obvious cases (by no means a complete list of the possible ones) are not always applicable; the necessary indicators might be missing.

In that case — and only in that case — we propose the following semantic rule:

A sentence expresses an action; the action has as first component the actor, as second component the aim, as third component the means and as fourth component the conditions under which the action is to be performed. We accept the general *pragmatic* rule: every sentence must be interpreted as having maximal content. And the content is maximally preserved if the leading parts of the action are untouched by the negation operator. If a negative sentence is presented it must thus be interpreted as having maximal semantical content. This implies that if there are conditions mentioned the sentence must be interpreted as follows "John drives to New York with his motor car (but not to day)". If no circumstances are mentioned, the sentence has to be interpreted as follows "John drives to New York (but not in his motor car) asf. This proposal derives from a generally accepted rule that might have exceptions but that is generally obeyed.

As a summary a) if internal indicators are present in the sentence, these indicators determine the scope of the negation b) if external indicators are present, and no internal ones, they determine the selection c) if neither internal nor external indicators are given, a pragmatico-semantical rule determines the selection (for those sentences that present the elements mentioned in the hierarchy just presented) d) if there is a conflict between external or internal indicators or the semantico-pragmatic rule then there is a strict hierarchy: internal dominates external, the obvious dominates the inferential.

We might here perhaps mention Kraak's proposal to make the negation bear upon the most specific part of the sentence. We do not think that the degree of specificity can easily and without doubt be determined, and we want to stress that our own proposal, in its last subsidiary part, is by no means the same as his. Our proposal has been inspired by Charles J. Fillmore's article on "Types of Lexical Information" (p. 116) (7), when he mentions that the case notions that are most relevant to the subclassification of verb types are agent (A), Counter agent (C), Object (O), Result (R), Instrument (I), Source (S), Experimenter (E). Fillmore's classification of verb-indicators is much more complete than ours but we can for his list with equal ease indicate a hierarchy that will again, in the absence of internal and external *obvious* indicators, give information as to the scope of the negation sign.

Before finishing this development let us indicate that the assertion according to which we need higher order functional logic to interpret negation is clearly confirmed by formula 18. If however the reader is willing to consider first order functional logic with the iota operator added to it already as higher order functional logic then all 5 formula's (15-18) bear out our claim. That this should be the attitude to be taken is shown by the fact that the iota operator is indeed a function of functions (mapping functions upon individuals).

As a conclusion, we remark that *the scope rules for negation in sentences can be very clearly explained by means of a rewriting in functional logic and by means of a semantico-pragmatic rule*. But while here logic as it traditionally stands (even thought it has to use the underdevelopped pragmatics) is sufficient, this is not at all the case in our first list of sentences where logic would bring us to throw them together while linguistic intuition puts them into a clear hierarchy.

In the following paragraph we shall be compelled to modify our logic not too abandon our linguistic intuition.

E. *The analysis of negation in everyday language makes it necessary to introduce a sequence of specific negations.*

Edward S. Klima believes, contrary to our point of view that all negative phrases can be built up out of positive phrases to which a negative "neg" is added. His article consists of a complicated set of rules that *places* this neg operator in various positions and combines it with various parts of speech.

Our project is the following one: *we are going to take his main examples and transcribe them in a language that is an extension of the calculus of functions.*

When this is done we shall try to argue that our transcription is to be preferred to his rules.

We shall begin by skimming through his work, giving the main examples:

(19) The students did not believe that it had happened.
(20) The students never believed that it had happened.
(21) The students hardly believed that it had happened.
(22) The students rarely believed that it had happened.
(23) None of the students believed that it had happened.
(24) Few students believed that it had happened
(25) The students were unable to believe that it had happened
(26) The students were too intelligent to believe that it had happened.
(27) The students doubted that it had happened.
(28) The writers will never, scarcely, hardly, rarely, seldom, barely, little believe the boy.
(29) Do no be sitting there when I arrive !
(30) Nowhere did the writers believe the boy.
(31) Close the door, won't you !
(32) They closed the door, did'nt they ?
(33) Publishers will (usually, always, not) reject suggestions, and writers will (not, scarcely, hardly, never, seldom, rarely) accept them either.
(34) Writers will never accept suggestions, and neither will publishers.
(35) Writers will never accept suggestions, and publishers will always, surely, usually, commonly reject them either.
(36) Writers will (never, not rarely, seldom) accept suggestions, not even reasonable ones.
(37) Publishers will reject suggestions, will they not (won't they)?
(38) Writers will unintentionally reject suggestions.

(39) Not much rain fell and not much snow fell either.
(40) Not much rain fell and neither did much snow.
(41) Not much food had been left by them.
(42) They had not left much food.
(43) Scarcely anybody rejects suggestions, or: Scarcely anybody was hit by anyone.
(44) Not much was given to anyone by anybody.
(45) Nobody was hit by anyone.
(46) Not even then did any snow fall anywhere else.
(47) No one thinks any rain fell anywhere else.
(48) There won't ever be another party.
(49) He didn't get there any too soon; he got there none too soon.
(50) He didn't know that anything had happened.
(51) He knew that nothing had happened.
(52) I will force you to mary no one.
(53) Writers need not accept suggestions, not even good ones.
(54) He is unable to find any time for that.
(55) It is impossible for him to do any more.
(56) He disliked doing any more than necessary
(57) They doubt that I need ever consider the problem.
(58) He forbids her to have a child for several years.
(59) She's too weak to speak to him for several days.

It is clear, as Yehoshua Bar-Hillel pointed out in his review of Klima's article (9) that no treatment of negation can be considered up to date if it does not take into account Klima's material. This we are going to try to do, in what follows, taking a point of view that is radically different from his own (and even from the one defended by Jackendoff, overcoming Klima's syntactic bias and presenting a semantic theory of negation).

The main point we are in disagreement with is clearly stated p. 295 (Klima, op. cit.) "To account for the similarities between the preverbal particle neg, and its various combined forms (nobody, nothing) on the one hand and the constituants negative inherently (doubt) or by affixation (unable) on the other hand, it will be assumed that the same constituent (neg) is assumed in both, that is that "unable" is "neg-able", "impossible" is "neg + possible" and "doubt" is "neg + believe". The word "not" in sentences like "The writers will not accept suggestions" is "neg P constituent P" representing the constituent "preverbal particle" neg in the sentence in which it occurs" (p. 295).

We to the contrary are of the opinion that various types and forms of negation are to be distinguished, and that, once this distinction is made, it is possible to give positive postulates for the so called "negative" particles. Our final problem will be to show in what ways the so called "negative terms" belong to one and the same family. Here the reader will recognize the intent of our other article in this series.

F. *Negative non standard quantifiers.*

We begin with the following particles: nobody, nothing, never, nowhere. In our analysis these particles are so called negative quantifiers. Indeed if our strategy aims to remain as close as possible to the linguistic facts, then the introduction of the ghost particle "neg" is simply the projection upon everyday language of the propositional calculus in which indeed (as in classical functional calculus) one unique negative particle "not" serves everywhere to express negation. *This is not the case in the surface structure of natural language.* We do not deny that sometimes it is useful to construct a deep structure having very different features, but as far as possible it seems to us necessary to remain close to the observable facts.

F_1.

As a first evident feature we consider that natural language presents many different types of variables: variables for persons (everybody, somebody, nobody), variables for moments or places (everywhere, nowhere, somewhere, always, never, sometime) and still others we do not need at the moment. We know that many sorted functional logic has often been considered (among others by Hao Wang and Arnold Schmidt (10). It does not offer particular difficulties; we can present it as the union of n isomorphic applied functional logics, taking care that a quantifier over one type of variable never influences another type of variable. Let us then consider *a many sorted applied functional logic* and call it *FLm.*

Let us now consider the various types of quantifiers. In his book "Topics in Philosophical Logic" (11). Nicholas Rescher considers, in chapter X "Non standard quantificational logic". He starts out by pointing out the systems using many types of variables (11), but then he comes in his paragraph 5 to "Plurality quantification", studying on pp. 170-172 concepts like "most".

The reader can consult himself the properties of this interesting non standard quantifier. We however want to point out that *if "most" can receive a positive characterisation by means of postulates, the same can be done for "few" and for "none".*

We shall write "A few x satisfy fx" as follows "(Fx)(fx)" and "No x satisfy fx" as "(Nx)(fx)."

In the same style that yields some interesting properties for all, some and most we are now going to look for properties applying to F and N.

We consider the following statements:

1. $[(Fx)(fx)] \rightarrow [(Ex)(fx)]$: if few x satisfy x at least one x satisfies x (the inverse does not hold).
2. We cannot assert $(Ax)(fx) \rightarrow (Fx)(fx)$ because of the fact that we are formalising not "a few" but "few". If we take care of this fact we must assert to the contrary $[(Ax)(fx)] \rightarrow [-\{(Fx)(fx)]\}$
3. $[(Fx)(Px \rightarrow Qx) \wedge (Ax)(Px)] \rightarrow [(Fx)(Qx)]$ (a formula similar to Rescher, p. 171, fmla 4).
4. $[(Fx)(fx)] \rightarrow [-(Mx)(fx)]$ and $[(Mx)(fx)] \rightarrow [(Fx) - (fx)]$.
5. $[(Fx)(Px \wedge Qx)] \rightarrow [(Mx)(Px \wedge - Qx) \vee (- Px \wedge Qx)]$.
6. $\{[(Ax)(Px \rightarrow Qx)] \wedge [(Fx)(Qx)]\} \rightarrow \{(Fx)(Px)\}$.
7. $(Fx)(Px \vee Qx) \rightarrow [(Fx)(Px) \wedge (Fx)(Qx)]$.
8. As for the quantifier F, we cannot assert the possibility of permutation: (Fx) (Fy)R(xy) is not equivalent to (Fy) (Fx)R(xy)

(few men love few women is certainly not equivalent to few women love few men).
9. There is a clear diference between F (few) and M (most) because in syllogistic inference no conclusion can be drawn from two F premises, nor even of a combination of an F and an M premise (only from the combination of an F and an A premise).

The reader will by now perhaps be convinced that so called "negative quantifiers" can receive independent postulational definitions. He will then be prepared to study the complete negative quantifier N (this quantifier applied to various types of variables gives us nobody, never, nowhere, nothing).

F_2

1. $(Nx)(fx \vee gx) \rightarrow [(Nx)(fx) \wedge (Nx)(gx)]$.
2. $(Nx)(fx \wedge gx) \rightarrow \{ (Ax)[(fx \wedge - gx) \vee (- fx \wedge gx) \vee (- fx \wedge - gx)] \}$.
3. $(Nx)(fx \rightarrow gx) \rightarrow [(Ax)(fx \wedge - gx)]$.

Let us again stress the fact that even though in some formula's the negation sign is used, peculiar and specific properties of the negative universal quantifier can be defined in a purely positive way.

The introduction of these two negative quantifiers "few" and "none" (applied to various categories of variables) allow us to formalise the following parts of the list of examples 19-57:

sentence 20: never
sentence 23: none
sentence 24: few
sentence 28, part I "never"
sentence 30: nowhere
sentence 34 and 35, part I: never
sentence 36: part I
sentence 45: nobody
sentence 47: no one, anywhere
sentence 51: nothing.

This list shows clearly that we made some progress but that much remains to be done.

Among the negative quantifiers "none, nobody, few" we find also "not much". We certainly are obliged to give an interpretation for this quantifier also: We offer the following proposal (not without doubt). "Not much" would be close to "few", but it would only be applicable to wholes, and not to sets "there was not much milk in the bottle" clearly implies that the liquid was a totality, while "not many bullets were left" clearly refers to a set. We would surmise that an operation akin to "few" applied to a totality could not be expressed in the same way as "few" because one has to start out with the whole and one is not able to draw together the few units one desires to speak about because of the fact that there simply are no units.

The proposal is thus that one adds to mereology (the science of the relations between wholes and parts) an operator having the properties of "few" and considers this operator to be the interpretation of "not much".

Continuing our analysis of quantifiers, we meet the quantifier "many" and its negative counterpart "not many". "Many of the arrows hit the target, but many did not". The concept of "many" is certainly weaker than the concept of most and stronger than the concept of "few". The dificulty in its analysis resides in the following fact:

If we compare "many" to "most' using as reference again p. 170-1 of Rescher then we see that quite a few laws valid for "most" disappear for "many":

1. "$(Ax)(Px) \rightarrow (Mx)(Px)$" is untrue for "many" while valid for "most": all arrows can hit the target and still only a few arrows can do so (if not many have been aimed at it).
2. The second law is all right: $(Ma\ x)(Px) \rightarrow (Ex)Px$.
3. The third law is neither more nor less doubtfull for most than for many.
3a. If Most x are P and most are Q then there is at least one x both P and Q.

3b. If many x are P and many x are Q then there is at least one x both P and Q.

Perhaps it might even be said that for "many" the law is more probable than for "most" (because of this indefinite reference to quantity present in many and absent in most).

Law 4 is clearly valid for most and invalid for many: If most x are Px then indeed it is not the case that most x are not Px. But if many x are Px, many x can still be non Px. Our sentence example expresses exactly that fact.

Law 5 and 6 are equally valid for many and for most. Our difficulty arises from the following fact: have we enough formal properties for "many" in order to establish it as an acceptable quantifier? We have a few less properties than for most but when we add the hierarchical line going from all to most perhaps the reader will be satisfied with the facts we have.

When we have arrived thus far it will be easy to present sentences like the following one:

$$\text{"}-[(Max)(Ax)\wedge H(x,t)], \text{ and } (Max)H(x,t)\text{"}.$$

Let us, after having introduced the general idea of positively defined negative quantifiers over a many sorted language, introduce another fundamental idea: topological cover or vicinity.

The reader will certainly have been puzzled by expressions like "hardly anybody came" or "they did barely believe him".

These expressions, according to us can only be analysed by using concepts like direction, or "neighbourhood". For the moment we neglect the important distinction between "somebody" and "anybody" and we concentrate upon the expression (ungrammatical) "hardly somebody" "scarcely anybody". "hardly a few" "scarcely a few".

We introduce the concept of a directed neighbour, of a quantifier: "nearly everybody" we would represent as "$(\overrightarrow{Ax})$", "nearly nobody" as "$(\overrightarrow{Nx})$". The direction however can be an internal or an external direction; in order to express this distinction we need a pair of quantifiers and a directional indicator.

Let us consider the well known topological concept "being in the neighbourhood of" and call it C. The properties of C are also well known a) Co = o b) CCa = Ca c) C(a ∪ b) = (Ca ∪ Cb) (where ∪ indicates union and o indicates the empty class. As only innovation let us apply the C operator on the syntactical level. We then can speak about the neighbourhood of a proposition p, Cp, or C[(Ex)(fx)] and so forth.

"Nearly nobody came" can now easily be expressed as follows: "C[(Nx)(fx)]" by using the earlier introduced *negative quantifier* and introducing the *topological operator.*

In Klima's list the topological operator can take care of sentence 21: hardly believed (where the C has to be added to the belief operator).
sentence 28, partly: scarcely, hardly, barely.
sentence 33: scarcely, hardly.
Sentence 35: usually, commonly (the C has to be added to the M quantifier of Rescher).

The concept of "neighbourhood" has however less advantages than the concept of direction.

F_4

Let us define both on the syntactical and on the semantical level a concept of direction.

On the syntactical level:

d + ((Ax)(fx)) = (Ax)(fx), d — ((Ax))((fx)) = Mx(fx),
[d — ((Fx)(fx)] = (Ex)(fx)].
d + ((Mx)(fx)) = (Ax)(fx), d — ((Mx)(fx)) = Fx(fx).

Better still would be that we define the d + or d — as relations (presupposing these relations to be strictly anti-symmetrical). The same type of syntactical operator can equally be defined for certain propositional sequences satisfying the property that p implies q, implies r implies s asf (and all the inverses being false). For such sequences that are isomorphic to the non standard quantifier sequences just introduced one can equally use the d + and the d — operator.

The sentence "Hardly anybody came" would then receive the following transcription (d — (Ex)(fx)) (in order to make this expression understandable we must add to our former series that we use the proposal to make the d operators the names of antisymmetrical relations between expressions.

The directional feature (similar to a non numerical vector) can however also be introduced in a semantical fashion.

Few writers have directed their attention towards this topic; notable exceptions are Rogowski and Sesic (12).

These writers however refer to time. We must refer to meaning. For us "$\overrightarrow{p}$" means "a proposition is true that comes close to the proposition p", and the inverse "$\overleftarrow{p}$" means "a proposition is true that is far away from the proposition p".

The sentence "he could hardly speak" would naturally be of the form "$\overleftarrow{p}$" and most of the negative propositions would be characterised by such inverse arrows.

We only mention this possible formalism because of the fact that once again we can write down positive postulates for these expressions:

a) $(\overrightarrow{p} \wedge \overleftarrow{q}) \rightarrow \overleftarrow{(p \wedge q)}$

b) $\overleftarrow{(p \wedge q)} \rightarrow [(\overrightarrow{p} \wedge \overleftarrow{q}) \vee (\overleftarrow{p} \wedge \overrightarrow{q}) \vee (\overleftarrow{p} \wedge \overrightarrow{q})]$

c) $\overleftarrow{(p \vee q)} \rightarrow \overleftarrow{p} \wedge \overleftarrow{q}$

d) $-\overrightarrow{p} \leftrightarrows \overleftarrow{p}$ (important)

e) $\overrightarrow{p} \leftrightarrows (Eq)(q \wedge \overrightarrow{p})$

f) $(\overrightarrow{p} \rightarrow \overrightarrow{\overrightarrow{p}}) \wedge \sim (\overrightarrow{\overrightarrow{p}} \rightarrow \overrightarrow{p})$ (can be generalised).

The logician among our readers will perhaps be interested by the new concepts suggested by everyday language. But the linguist will most probably be unsatisfied because he misses until now the main points that Klima makes in his article. We are briefly summarising them in what follows with the purpose to rewrite them as we did before in the language of logic.

G. *Klima's tests for sentence negation.*

The following sentences show Klima's tests for distinguishing negatives from positives.

1. "He did not like John and she did *neither*" is opposed to "He liked John and *so* did she" (in French: "Il n'aimait pas Jean, et elle *non plus*" is opposed to "Il aimait Jean, et elle *aussi*").
2. "He did not like students, *not even* good ones" is opposed to "he liked students and *certainly* good ones".
3. "He *dis*liked her, *did* he?" is opposed to "He liked her, did*n't* he?"
4. He did not see *any*body he knew" is opposed to "he saw somebody he knew".

These four examples put to us the following problems:

1. The logical meaning of "neither", is to be analysed and opposed to "so" and "either".
2. The logical meaning of "not even" is to be opposed to the meaning of "even".
3. The negative question tag, *"did'nt he"* added to a positive sentence has to be opposed to a positive question tag, added to a negative sentence.
4. The transformation of some into any, in all negative phrases has to be explained.

We are aware of the fact that these four analyses, even carried through to the mutual satisfaction of the logician and the linguist (an act difficult to perform) do not yet give the complete solution of the problem: we still should have to see *how the placement rules of the negation signs can be derived from either our logical analyses alone, or from these analyses and pragmatic coding and decoding principles.*

Let us however start already with our analyses:

1. The "neither" tag seems to us again to be analysable in higher functional logic. Indeed, what does it mean? It tells us that the following is true "$[(R(ab) \wedge S(cb)) \wedge (R = S)]$ and moreover either Neg (R) or Neg (S) (where we indicate by Neg(R) or Neg(S): the relation in question is a negative relation. In our other paper in this series we give a general definition of negative and positive (13). The essential point of the either or neither seems to be a cross reference to the relation expressed in the sentence that came just before and a classification of this sentence as either positive or negative.

This last classification does not compell us to have a neg element in the sentence at all; one could infer the negative character of the sentence from the presence of certain types of quantifiers that we studied before. The fact that identical relations have identical properties would seem to be the main point to be made here.

2. The analysis of "not even" is more difficult. The meaning is obvious: it is the case for most persons, or it ought to be the case for most persons that they like good students (more generally: that they stand in a certain relation to a class K — and we come to the conclusion that R(ab) and not R(cb) while most x realise R(xb) or most x ought to present R(xb). The "even" seems to be analysed by means of the non-standard quantifier "most" and by means of a deontic operator. But unhappily enough, while it is our plan to remain as close as possible to the observable facts of language itself and thus not to introduce "pour les besoins de la cause" non visible negation signs, we did not follow it here. We thus cannot be satisfied with this analysis. Our only way out will be the following one: we must place students on a scale, and relations with students equally on a scale, and introduce two different types of relations R and S, liking and disliking (two positive relations). Then we must state that the more one advances on the student scale, the more one advances on the R scale while suddenly for a given person a, and a student quite high up on the student

scale instead of aRx, we encounter aSx for this student and all higher ups. The word "even" would indicate that this change from R to S is unexpected, improbable and, if one wants again to speak about obligations, contrary to the norms supposed to be accepted by those concerned with the two orders.

3. The third case concerns questions. The opposition confronts typically "He is ill; is he not ?" to "He isn't ill; is he ?" That Klima considers the positive tag as an indicator for the negative character of the base sentence is very simply understood. Let us consider the simplest of questions, demanding a "yes" or "no" answer. The examples we study here concern precisely such questions, with this added characteristic that the two possible answers are explicitly mentioned in the questions themselves.

It is obvious that ?(pOp) is pointless (as is (— pO— p) ? It is necessary to ask either (pO — p) ? or (—pOp) ? For the obvious reason that a question is a request for information, it is necessary to create an opposition between the two alternatives explicitly mentioned. This opposition can be created by means of an explicit "not" or to the contrary by means of any other verb, adverb or adjective that shows an opposition with the meaning of the first sentence mentioned in the question.

It is however necessary to mention here that Gerhard Stickel in his paper on "Ja" und 'Nein" als Kontroll und Korrektur-signale" (Linguistische Berichte 1972, pp. 12-17) interprets no (nein) as a pragmatic signal, used to correct previous utterances, to whom it refers. "Nein is in allen diesen Fällen eine reaktive Ausserung" p. 16). This point of view that finds strong confirmation in our psychological part shows the negative clearly as a metalinguistic operator.

4. Again we must introduce a new type of quantifier to understand our fourth case. Strangely enough the distinction between "somebody" and "anybody" creates a rather deep problem. We meet it also outside of the range of negative propositions. When I say "somebody can do it" and when I contrast this sentence to "anybody can do it", I have the obvious French translation "Quelqu'un pourrait le faire" et "N'importe qui

pourrait le faire". This opposition is again visible in French between "He did see somebody" (Il voyait quelqu'un") and "He did not see anybody (Il ne voyait personne") or in German ("Er hat jemand gesehen" "Er hat niemand gesehen").

The quantifier "anybody" is not well translated by the usual sentence "(Ex)(fx)" because there is a difference between "everybody did not do it" and "there was not anybody who did it". The concept "anything, anybody, anywhere" as distinct from "somebody, something, somewhere" is, we suggest, again of a higher order: it is a quantifier over selections in a set. Saying "He did not see anybody" we say that for all possible selections in the set of relevant persons, they were invisible (a positive characteristic).

We are aware that these short remarks cannot be said to give a complete theory of the negative phrases we are commenting about but we hope that our four remarks show that it is misguided to remain on the purely syntactical plane on which Klima moves; that the facts he mentions can be naturally explained not in the classical first order functional logic but in higher order extension of functional logic. The reader will be able to inter from our four analyses that they are indeed tests for the negativity of the sentences, as Klima claims they should be. We however give reasons.

H. *Negative verbs, adverbs, adjectives.*

We now finally procede to that part of our task that is perhaps the most difficult one: negative verbs, adverbs and adjectives.

Let us consider the sentences 54 and 55 containing the words "unable" "impossible" (we could consider similar ones containing untrue, unreal asf). If we follow our usual strategy, instead of constructing them out of the positive predicate combined with "not", exactly as we introduced negative quantifiers, positively defined, we will now do the same for the predicates.

Let us call Pn a negative predicate and Pp a positive predicate i) For (Ax)Pp(x) is valid that it implies Pp(a); for (Ax)Pn(x) is equally valid that it implies Pn(a). ii) Robert Blanché in his interesting article "Opposition et Négation" (14) has stressed that for predicates (all his examples refer to predicates) the traditional aristotelean logical square is not sufficiently discriminative: between possible and impossible, able and unable lie many intermediaries: the excluded third is valid for "a is able or not able" and equally for "a is unable or not unable" but it is not the case that we are allowed to say that" a is able or unable" (this by the way shows how we need really negative predicates: "unable" is not identical to "not able"; quite to the contrary we seem to have a hierarchy: able-not unable-not able-unable. The typically characteristic property of the negative predicates seems to lie partially in this hierarchy.

This fact can direct our thinking in two different directions: a) either we go even further than Blanché who places between the two extremes able-unable, skillfull-awkward, possible-impossible, a third eventuality lying outside the dimension of which the extremes are the polar opposites and we consider the introduction of complex many valued logic. We call complex many valid logic a many valued logic in which a given proposition does not only have one truth value, but to the contrary many. The number of truth values is simply related to the number of different dimensions on which the predicates are considered. b) or we consider the operator embedded in the adjective-predicate as an operator on the meaning of the positive adjective "Unabled" would then read as follows "the meaning opposite to the meaning of able is applied to the object a".

The meaning can be introduced by using the lambda operator of Church: (λPx) (a)). For the rules concerning this lambda operator we have to consult the "The Calculi of Lambda Conversion" (Annals of Mathematics Studies, n. 6). The relation of opposition is to be definied axiomatically: 1. Nothing is opposed to itself. II. If something is opposed to something else, the second is also opposed to the first. III. If some a is opposed to some b and some b is allied to some c, this same a is opposed to this same c. IV. Transitivity obviously does not hold (rather anti-transitivity in most cases).

We must give some reason for this introduction of an intensional way of speaking; the reason is the following one: "unable" or "impossible" indicate, if (as is our custom) we follow as closely as possible the linguistic facts, an operation on "ability" or "possibility". This operation can only be an intensional one and for this reason we think that we may introduce these difficult concepts.

Let us moreover add that the introduction of *complex many valued logic* and of *intensional operators* in order to give an account of negative adjectives is by no means incompatible: it seems quite to the contrary, that both measures are needed. It is true that we have many intermediaries between the opposites on the one hand and moreover it is equally true that we need an intensional operator, both measures should be combined.

I. *Negative Verbs*

We think that the analysis of negative verbs will throw even more light on the analysis of negative adjectives, but we do not wish to leave the latter before mentioning what we consider to be the meaining of an expression. For us (and this opinion has been expressed in our publications of the CIEG and in books and articles of F. Vandamme (15), a meaning is a program, an ordered sequence of imperatives and any operation on a meaning is either a permutation or a deviation or a reversal or a deletion of these operations. It was necessary to stress this point to make more understandable what went before.

Let us now look into the even more difficult matter of so called negative verbs.

accept-reject
ascend-descend
lighten-darken
robe-disrobe
accept-refuse.

Again we can clearly see that any anlysis (significantly, Klima did not attempt to give even such an analysis) by means of a negative particle, would be wrong

1. Not to accept does not signify to reject; not to reject does not signify to accept.
2. Not to ascend does not signify to descend and not to descend does not signify to ascend.
3. Not to lighten is different from darkening, not to darken is different from lightening.

The type of verbs we mention all indicate actions and if we wish to do so we can always represent formally actions by relations between their initial and final states.

The calculus of relations presents indeed negative relations: — R meaning "the relation that holds when R does not hold and inversely" But — as we just pointed out — it is not the case that these complementary relations can be used to express the relations between the verbs mentioned. Neither can the inverse (or converse) of the relation be used. One might think so when looking at ascend or descend, lighten or darken, but certainly not when analysing accept-refuse, accept reject. It is not the case there that, instead of going from an initial to a final state, one goes from the same final to the same initial state.

And it seems desirable to us to analyse all the members of the same set in the same way. If this is to be the case, we are of the opinion that exactly as we have to introduce negative quantifiers and negative predicates, we also have to introduce negative predicates, we also have to introduce negative relations.

Rn(ab) will certainly imply — R(ab) but the inverse will not be the case. [Rn(ab)$\wedge$Rn(cb)] implies Rn((a + c) . b is here an operator that makes from two individuals a totality. In goodman's "The Structure of Appearance' this operator is analysed.
b) Rp(ab) implies — Rn(ab) (special case of the first rule).

J. *Negative adverbs*

We did neglect the adverbs. There the situation is even more complicated, because in our analysis adverbs would indicate predicates or relations of predicates or relations.

"He runs very fast" or "He speaks slowly" would have the form F(R(a)). If the reader accepts our strategy of looking for positive postulates introducing negative predicates or relations in the lower functional calculus, he will do the same in the higher functional calculus. No new complications or interesting facts seem to arise there.

K. *Conclusion.*

We have presented here an analysis of the negative sentences mentioned by Klima that compelled us to introduce quite a number of innovations in our formal logic. *Our analysis is a projection of the linguistic superstructure upon the logical basis.* It is moreover in complete disagreement with Klima on the following point, stated very clearly by him on pp. 290 "Implicit in the analysis presented so far is the rejection of a basically different interpretation of negation. In the present study negation — at least to the extent that sentence negation is involved — has been treated as unitary (that is attributable to the optional presence of a single constituent, the preverbal particle neg) and as mobile (that is the element to which negation is attributed may ultimately appear and be incorporated in and fused with other constituents (p. 290). Our own analysis of negation is multiple (we avoid the particle neg as systematically as we can) and not mobile (our negative indicators remain close to their sources).

L. *Functional logic Analysis and Transformational Analysis.*

We must try to give a justification for this fundamental difference between Klima's treatment and our own.

Let us begin with a concession. It is true that the extensions of functional logic we did use are a priori structures that are themselves under attack. It is also true that we cannot easily find a criterion for the adequacy of the translations presented.

Our defense will thus ly in our attack (and in one positive remark).

1. Let us try to define the difference between deep structures and surface structure in Chomsky's system (16). When all is said and done, the deep structure is the most economic set of core sentences out of which accepted operations can produce the surface structure we are interested in. This means that our selection of core sentences is solidary with our election of production operations (and of their combinations). This procedure introduces even more a priori and conventional elements than our reference to functional logic. Certainly it is a fact that we have to extend functional logic to make it perform as we wish, but the extensions are in a sense "natural" extensions for which (cfr. most and many, asf) analogous though not identical extensions have already been proposed before. Moreover we must stress that functional logic is able to express a large part of mathematics and that, if we can make it moreover express a large part of natural language, its advantages over a system that is only able to do one of these things becomes clear. In order to compare our way of handling negative expressions to that of recent transformationalists we must give logical equivalents for their method of analysing sentences:

a) a parsing tree has according to us the following logical representation: an arbitrary element of the class of sentences S is identical to the concatenation in a given order of an arbitrary element of the class of X, an arbitrary element of the class of Z and an arbitrary element of the class of Y. The formal representation of "an arbitrary element of a class" can be "any x, such that it is an element of the class Z". If no concatenative order is intended we may preserve the features but then we must speak about a node at whatever height as an unordered class of classes. The important relation, both for Klima and Jackendoff of "being in construction with" can be expressed by means of the concept of "domination" that is in our transcription equivalent to "being an element to the n-th degree of an ordered or unordered class of which b is an element". Our transcription equivalent to "being an element to the n-th degree of Z" is: there are n — 1 A, B, C, X such that $a \in A$, $A \in B$ ...... $X \in Z$.

This interpretation makes it possible to say that a sentence is composed of a noun phrase and a verb phrase. It also makes possible to say that somewhere in the series an individual element (like "not") is to be found but it makes *impossible* to say that the element neg (not a class of elements nor an individual element) is to be found in the tree.

Kraak is completely right where he says that for the sentence negation an assertion operation or a denial operation must be introduced but he is wrong when he claims that they are constituents like the others. They are *operators on the whole* tree. And the denial operator (even though we can give easily an axiomatic characterisation for it) is not symmetrical to the assertion operator.

The reason of the asymmetry is simple: the following rule holds for every x and for every p, Den x p implies the following two sentences: a) ASDenxp and b) DenAs xp but by no means can it be said that when x asserts a sentence a similar situation holds for denials. As means here "Asserts" (see N. Rescher, Topics in Philosophical logic), Den means "Denies". We claim that whoever denies, denies an assertion, and asserts a denial, while we reject that whoever asserts denies by this very same thing anything at all (he might do so in particular cases, but in general he does not). The following minimal axiom system for denial, is a mirror image of the axiom system for assertion (Rescher, p. 251, ch. XIV).

1) (Ax) (Ep) Den (x, p) (non vacuousness).
2) (Den $xp \wedge$ Den xq) $\rightarrow$ Den $x(p \wedge q)$ (conj).
3) Denx$(p \wedge \sim p)$ Consistency.
4) If $p \vdash q$, then Den $xq \vdash$ Den xp. (we would prefer 4'). If Asx $(p \vdash q)$, then Den $xq \vdash$ Den xp. (4' is obviously weaker than 4).

It is important to stress the general fact that the only reasonable interpretation of the analysis tree for a sentence excludes the idea that assertion or denial could be elements in it. They are operators upon the whole sentence. And this is one of the reasons why we try so hard to analyse intra-sentence negative features in various different ways not referring to sentence negation (*and why we define denial positively*).

Jackendoff when discussing the question of the scope of negation tells us "traditional symbolic logic is of no help in this respect, because it deals only with negations of propositions (sentences). It does not talk about the negation of predicates, constants or variables" (pp. 16-17). The author is right in voicing this reproach; in what came before we did our best however to show that traditional symbolic logic can be extended by adding negative quantifiers and other devices so that also in its own context we can make ourselves free from the overwhelming dominance of sentence negation.

If we do this, some expressions that are the result of a long series of transformations in the tree representation of Klima become to the contrary quite simple results of a few operations.

Let us discuss the sentence "No one ever gave John anything". Jackendoff gives the following derivation 1) neg someone once gave John something. 2. indefinite incorporation: neg anyone ever gave John anything. (the "some" becomes "any"). 3. the neg moves in front of the verb (auxiliary verb if there is any) and obligatorily incorporates into anyone: No one ever gave John anything. We to the contrary start out with the sentence "Someone gave sometime John something" and we replace "someone" by "no one" (applying the negative quantifier we have introduced). 2° we replace sometime (a traditional existential quantifier applied to time points) by never (a negative quantifier again this time applied to time variables). We read ever as not ever and suppose that the negative quantifier has here been unusually expressed to avoid contamination with the first negative quantifier.

4. Finally we replace "something' 'by "nothing" and again write "anything" to avoid ambiguity (the anything can also be introduced as an "any" unorthodox quantifier (quantifier over selections, as we said before).

Our derivation has to be compared with the sentence "No one never gave John nothing" and "No one never gave John anything": the first sentence could mean "everybody gave always John something". We accept that we need two pragmatic rules to provide for the replacement of positive by negative quantifiers, but we claim that our sequence of operations is quite clearly understandable; while a) no one can justify why the neg element of Klima and Jackendoff moves as it does and b) no one can understand why it is incorporated (as they call the operation) and still remains active after incorporation, on other parts of the sentence. The derivation we present is not in the same sense a derivation as Klima's or Jackendoff's are. We have simple paradigms in which we replace positive by negative quantifiers guided by pragmatic rules for the avoidance of ambiguity.

The fact that when we put the sentence in the passive "Nothing was ever given to John" our presupposed negative quantifier "nothing" appears, or that in another transformation "Never was anything given to John by anyone" the other negative quantifier "never" appears substantiates according to us our interpretation.

If we cannot accept Klima's interpretation by means of the rules we just proposed to replace, we cannot accept Jackendoff's interpretation either.

This becomes most clear when we analyse a case of non-sentence negation studied by Klima and Jackendoff.

We cannot understand how Jackendoff can believe that the sentence "They are fighting about nothing" has as a paraphrase "It is not so that they are fighting about anything". For us this last sentence denies the fight and the first sentence asserts it. Again between the sentences "I will force you to marry no one" and the sentence "It is not so that I will force you to marry anyone" there are meaning differences that prevent the paraphrase relation that we can logically transcribe as a synonymy relation to hold. These disagreements between two readings of the same sentences (related by the way to the very first linguistic remarks we made where we made a strong diference between "P is not A" and "it is false that P is A) show the difficulty to use the synonymy relation as a means to analyse negations. This example proves how difficult the test of synonymy is to handle, exactly as our analysis of the earlier derivation proved how unexplained Klima's rules are. Our translation in functional logic seems to have quite serious advantages.

2) Even though it is perhaps not necessary we now procede to an attack against transformational grammar itself and against generative rules. We certainly admire immensely the work done and think that linguistics has only now reached a scientific status. But the tree representation of sentences proposed by Chomsky seems to us to rigid: and too classical: classical categories are simply adjoined to nodes of the trees (without receiving any more deeply structural or functional definition) and moreover without taking into account the possible modifications of trees implied by contexts (pre or post contexts of verbal nature, or even socio-psychological contexts of verbal nature, or even socio-psychological contexts). One tree is associated with one sentence.

3) An even more pertinent remark is the following one: given any core set, and no restrictions whatever upon the transformations that are allowed (and nobody has linguistically produced such restrictions) we can produce any sentence whatever as a grammatical sentence.

For these reasons we do not follow the popular transformational trend, but we want to go back to its origin the work of Zellig Harris. If our projection of negative sentences has any validity whatever it must be confirmed by his distributional methods. We want to close our linguistic remarks by introducing a distributional negation.

M. *Context-Negation as Empirical Confirmation of the FL-interpretation.*

The reader who will look up Zellig Harris "Papers in Structural and Transformational Linguistics" (Reidel 1970) (17) will not find much material on negation, and the few remarks he will find will be close to Klima: once more Harris seems to consider negations as products of the various displacements of operators expressed by "is not" or "is non". On first sight it thus seems to be the case that we cannot hope to confirm our pluralistic theory of negatives by means of Harris' ideas.

We are of the opinion however that in his work, beginning with "Discourse analysis", in his co-occurrence theory and his transformation theory we can quite to the contrary find an empirical technique that can confirm or disconfirm certain features of our own more theoretical proposal.

We hope using this procedure to remain faithfull to the methodology of logical empiricism: on the one hand we propose a theory and on the other hand we propose a technique of observation. If we are successfull in this endeavour it seems to us that we are closer to empirical methodology than is the rationalism of Chomsky, mirrored in Klima's treatment of negation.

From the observational point of view, one feature is essential: *no sentence can be called negative or positive except with reference to its verbal and non verbal context.* This implies that from the linguistic point of view also, the concept of a negative sentence, or expression is a pragmatic concept and not a syntactic or semantical one.

Let us give some examples: in a circle of leftist politicians, to the question "Is he honest ? an answer may be given as follows "He is a member of the Conservative party". The answer is certainly a negative one (nobody will deny that in this psychosocial context, a negation is implied, but we are even going further and say that not only the negation is implied but that the sole purpose of giving the information is to make the auditor believe in the negative information). Let us now ask the same question in a circle of right wing politicians "Is he honest? And let us receive exactly the same answer "He is a member of the Conservative party". Obviously, now a positive answer to the question is implied and we do recognize the distinction between formal implication and pragmatic implication, but we assert that in both cases these sentences, completely devoid from any negative word or form have in the first case a negative function, and in the second case a positive function. We go even further: a proposition showing clearly negative features may be strongly positive. Let us again set up a small language game: a lady enters the room; conversation ceases and then the remark is made "she is not ugly". It seems probable that the sentence means "she is far from ugly, in fact, perhaps: she is stunningly beautifull'. We must apologize to the reader for these seemingly trite remarks. They have been made to convince him that at least in many cases the non linguistic context is a necessary element in the understanding of the sentence as positive or negative. Now, if the non linguistic context is needed in order to distinguish negative from positive sentences, it is clear that we shall have to admit that the linguistic context is also needed. And Zellig Harris' discourse analysis is precisely the clearest attempt to analyse the structure of a text as a whole, not limiting itself to the boundaries of the sentence but taking into account the relaions between sentences.

We shall limit ourselves here, to verbal context and end with a few remarks about the use of contexts in the verification of the interpretation of negatives.

The verbal context of an expression might be any expression to the left or to the right of that expression. Various restrictions can be imposed: a) either one can make restrictions as to the type of the context elements (words, word groups, sentences) b) or as to the length of the context elements (seven words before, seven words after) c) or to the distance of the context elements (continuous elements, or elements at distance d — minimal or maximal) asf.

This enumeration shows us already that the concept of verbal context is by no means a unique and clear one.

The same remark applies obviously to the concept of context in general.

One could state that the context of a phrase is the set of all objects (phrases, things, actions or situations) the knowledge of which is necessary to understand completely the meaning of the phrase.

But what do we understand when we say "understand completely"? We could ask less and only stipulate that we should understand partially (in the sense that we should know a certain number of implicates of the sentence and also know a certain number of sentences that exclude it).

Let us try to give a formal definition of this general context:

Let Co mean "context" Co(p) is the context of an occurrence of p, Co (q) idem asf. (the concept of concept is thus defined for any expressions whatever, sentences, sequences of sentences, or sentence parts). The fundamental definition however is the definition of the context of a sentence.

An element e belongs to Co (p) when *many* (non standard quantifier) q's that imply p or are implied by p, or are incompatible with p, imply a fact about e.

We could be more subjective and only speak about knowing (the knowing of many q's having such properties implies a fact about e) and we could be more severe and ask that *all* implying or implied sentences, or excluded ones ought to imply not only one, but many (!) facts about e.

Let us write down the 3 definitions:

1. $(Ax)\ (x \in Co(p)) =_{Def} (Ma\ q)\ (Ef)\ [\{(q \rightarrow p) \vee (p \rightarrow q) \vee (q \rightarrow \sim p) \vee (p \rightarrow \sim q)\} \rightarrow f\ (x)]$.
2. $(Ax)\ (x \in Co\ (p)) =_{def} (Az)K(z,\ A) \rightarrow K(z,\ f\ (x))$ (where A is the sentence between { } in (1).
3. 1 or 2 but (Ef) replaced by (Maf).

If we now want to define Co (x) where x is *not* a sentence then we stipulate that for *many* (!) sentences either elements of x, or of which x is an element, the object to be defined must belong to their context.

The verbal context is then simply the set of such x that are moreover words in the language of the persons z who are referred to as knowing, or in the language in which the definition of Co is written, or in any given specified language.

The non verbal context is the complement of the verbal context. The expression Ma refers to the non standard quantifier "many' that we introduced in former paragraphs.

This definition of context is however only a first attempt. Indeed it could be argued that not many consequences or incompatibles should be known in order to understand the sentence but that certain *crucial* and *special* ones should be known.

In order to avoid this controversy (that could again introduce context dependent arguments because the word "understanding" is itself context dependent) we could try to give an independent axiomatisation of the concept of understanding. This task would however be a major one and would exceed the limits of this article. We should have to define what it means to understand a sentence, a logical operator, a predicate, a relation and linguistically a verb, an adjective, an adverb. It is by no means clear that the same pattern could be used in all those cases.

Even if we do not attempt to give a general definition of understanding, we stil have to define contexts for non sentences or for groups of sentences.

Let us now look if we can come to context negations (K-negations):

Ki: p an q have the same pre and postcontext (we limit our selves for facility to verbal contexts) when never in the pre- or post-context, of p, q can be found, nor in the pre- or post-context of q, p can be found.

This remark might however be too weak as for instance other grammatical transformations could approximate to the same situation: questions and imperatives concerning the same matters of fact can be also never be found together with assertions about the same topics and while it is not a rule, they can have the same pre- and post-contexts. We thus must eliminate explicitly these cases. Much seems to depend upon the definition of context. We call p and q each other's alternates.

If K1 and K2 are the pre- and post-contexts of p then K1 and K2 can be either atomic or complex. These two cases can be distinguished.

The alternates, defined by contexts (themselves defined by reference to the difficult concept of understanding formalised as we proposed), can suggest also a definition for distributional implication.

"p implies contextually q" if the following is the case (the definitions become more and more weak as we go on):

a) every context of the form K1pK2 in the text T is also a context of the form K1pKiqK3 where K3 is a final string of K2 and where Ki is an intermediary string of K2.

b) Within a distance not larger than n from every (or from most) occurrences of K1pK2 in the text T there occurs a context K3qK4.

c) we introduce (b) but instead of "not larger than n" we propose "smaller than".

d) we introduce b but instead of leaving K3 and K4 arbitrary, we make them functions of K1 and K2 (these functions can in their most stringent form be identity functions).

We did introduce contextual implication next to contextual negation because of the fact that we want to point out 1) that not every sentence has an alternate 2) "p contextually implies *the* alternate of q" has a clear meaning if only we accept the convention that the many alternates one identical sentence might have are rewritten as a sequence. This sequence would then be the strong alternate and this sequence would be meant in the implication we just introduced. But the sentence "p implies contextually q" has no clear alternate; it does not belong to the text. It is a metalinguistic sentence about the text, and while in classical logic the sentence "— (p → q)" and "p → — q" are equivalent, here one of them finds a meaningfull reinterpretation while the other one, quite to the contrary, becomes meaningless in the object language. (the first is meaningless, the second is clear).

The alternate will also have for very simple reasons other properties than classical negation: it is certainly not true that for every pair of contexts K1K2 either p or its alternate occurs; sentences that are neither p, nor any alternate of p can occur. alternate does certainly not need to be p.

"X is the alternate of Y" is asymmetrical negation. It Moreover as p can have many alternates, the alternate of an remains so if we introduce the alternate in the second sence namely the element that can never be found in the same pre- and post-context. Partial alternates can equally be introduced. If we want to have the relation "being an alternate of" a-symmetrical, we may make use of the fact that the relation "being the context of" is asymmetrical. It may very well be the case that in order to understand p, facts about q have to be known and not conversely. We can now define an assymmetrical alternate as follows: x is an element of the asymmetrical alternate y, if it can only be found in contexts of y, when y or its contexts are not in the contexts of those strings that are the contexts of x. These facts being taken notice of, one could have reason to ask: is really the alternate in any sense at all the distributional equivalent of the negation ?

In order to answer this question we have to present in somewhat more detail Harris' work, always naturally (as he does not) bringing it to bear upon the concept of negation.

p. 315 he stresses a) that the distribution of sentences is not a feature of the language as such but a feature of a text and b) that there is a clear relationship between the distribution of sentences and the properties of the social situation. If we can establish a relationship between discourse analysis and the analysis of negation this makes our negation concept a) function of texts not of languages and b) relates it to social situations. This is as we want it to be.

If we find the sequences AM and AN in our text, we say that M is equivalent to N, or that M and N occur in the same environment and for this reason can be called equivalent M = N. If we then also find in the text BM and CN we say that B and C are secondarily equivalent because of the fact that they occur in primary equivalent environments. Finer distinctions that Harris does not draw would be the following ones: if M and N have one context in common $M = {}_{1}N$; if M and N have two contexts in common $M = {}_{2}N$; if M and N are secondary equivalent on the basis of contexts that have n contexts in common $M = {}^{2}_{n}N$ and so forth. We now want to stress that exactly as we can build up a positive structure of that type, we can also build up a negative structure.

If M and N have no context in common, we can write M/N; if the y have n contexts not in common we can write M/nN. If M and N have two contexts in common that have no context in common we can say that they are secondary complementary (or exclusive). It is obviously more interesting to consider stretches that have 1) contexts in common 2) than to consider stretches that have no contexts in common (this would lead us rather back to a more positive relationship).

The degrees of equivalence are one feature that may help us in setting up various types and degrees of negations, but we still have equally distributional distinction between sentence and sentence part. The separation of the negation of sentences from the negation of sentence parts can easily be drawn: a sentence can be distributionally a whole text; no sentence part can perform such a function. The partisan of Klima and Kraak or Jackendoff will now ask us if by means of the negative transformation of Harris' discourse analysis we can draw as fine distinctions and give as good an account of natural language as he does. We can already answer the following: we can distinguish in a sentence an initial, a final and a middle part and set up their exclusion classes. This gives us three types of negations. We can have as many types of negation as there are degrees and strengths of exclusions. And finally we can make both the hierarchies interact by considering equivalent sentences that have or do not have equivalent parts (negatively: by mutually excluding sentences that have no mutually excluding parts).

The distributional technique Harris uses can be helped a) by identifying elements that make each other necessary (that are dependent upon each other) and b) by introducing special rules for identification (f.i. A is C can always be replaced by AC) asf.

When sequences of equivalence classes are substitued for the initial text (or sequences of exclusion classes, or mixed sequences of equivalence or exclusion classes), then we can still apply the same technique and try to find patterns in these sequences.

We are naturally most interested in the negative use of this method. We can try to find sequences that *never* occur before or after other sequences. We can also try to define sequences of the equivalence classes that never occurr in the given text. We think that these are the only exclusion operators on sequences of equivalence classes we can get hold of.

Finally transformations can be introduced. Transformations are operations that do not limit themselves to the text itself but are relevant for the whole language. If the same morphemes occur in another order (and perhaps accompanied by other morphemes) in the same language, Harris is willing to call them equivalent and to introduce a transformation mapping the one upon the other (p. 334). We should prefer not to be as broad minded (even though we recognize with Harris that the concept is by no means trivial) and again use the environment technique: if we find the transformed sentence in a same or similar environment in the language we shall declare that there exists in the language a transformation.

If we want to relate this transformation technique to negation, given the examples on pp. 336-337 the only thing we can do is to find transformations in the language that map expressions on expressions containing elements excluding the first ones (ofr instance: "The brave fighter David vanquished Goliath" can be mapped upon "The certainly not cowardly David vanquished Goliath). If we apply this technique however we shall need to make use of the more refined equivalence relation distinguishing strengths and types of equivalence (as we proposed before) and not of the general equivalence relation Harris uses. It is however obvious that one owes the more detailed distinctions completely to his inspiration.

One could try to use Harris "Report and Paraphrase" and "Cooccurrence and Transformation" in order to reach the aim he himself formulates so clearly in his nota 6 on p. 347 "mathematics, and to a greater extent logic, have already set up particular sentence orders which are equivalent to each other. *The equivalence can be rediscovered linguistically by finding that the distribution of each sequence is equivalent to that of the others.* Our interest here however is to discover other equivalences than those which we already know to have been built in a system". In this part of our paper, we did precisely that proposing new logical structures, and our definitions of context negation and of distributional exclusion classes have as only purpose to propose an instrument of verification. We could not carry out the verification here, because, within the limits and scope of one contribution we must not only relate linguistics to logic with reference to negation but also psychology to logic, with reference to that same negation.

But we must be aware of the fact that in this part of our paper we cannot do the same as in the earlier part, namely start from concrete negative examples and give equivalents for them, because of the very feature that is the value of Harris method: it is so fundamental that only by a long series of applications, the particular examples we started out with can be reached.

II. *Negation and Psychology.*

A. *Introduction.*

In the earlier part of this paper we saw that the linguistic analysis of negation had a clear and fecund relationship with functional logic.

Now we want to point out that the same fact occurs when we study the psychology of negation.

Our study will consist of two different but related parts:

1) in the first part we shall remain as close as possible to the psychological facts and selecting, following P. C. Wason (19) advice, Herbert H. Clark's (18) model "How to understand negation?". We are going to point out that a specific model of logical negation grows out of this psychological study.
2) afterwards we are in fact relating negation to problem solving. Indeed thinking and problem solving are for us identical and if we can understand the function of negation in problem solving, we think that we have reached our purpose. Following Herbert Simon, we define a problem as the demand to trans-

form a state SI characterised by means of variables vI ... vj. into a state Sr characterised by variables uI ... uk. This task has to be performed by means of a sequence of operators 0 comprehending elements 0_1 ... 0_n. The introduction of negation would have to occur by a) negative variables in the initial or in the final state b) by negative operators in the sequence of operators. This description is however not at all clear yet, as what is to be called "negative" in an action situation (and problem solving is certainly an action situation) has to be defined.

Even this simple description of the relation between problem solving and negation has to be corrected immediately: we must consider not only negative aims or starting points, but also negative strategies (do anything, except this or that) and the negative operations have to be classified in different classes ("stopping an action" is something different from "erasing the results of an action"). We are confronted both by the multiplicity of negative states and moves and by the general problem of their evaluation as efficient or inefficient when inserted in positive or negative strategies.

Finally all these considerations must lead us to our relation with logic. A statement must be defined as a move in the development of a problem solving situation. What type of move can be called an assertion or a denial?

If however our problem solving representation of thinking is successful, we should be able to insert the psycho-logical inquiry into the problem solving inquiry.

B. *Herbert Clark's model of negation understanding as a reduction of negation to difference.*

The two basic concepts of Clark's model for negation are the concepts of a) *presupposition* and b) of *congruence*. The reason why we believe it important to analyse his facts lies in these two basic concepts (and more specially in the concept of congruence that we construct as a complex form of identity).

Indeed, even when noticing that he starts out distinguishing four types of negation, we only later briefly comment upon these distinctions, proceeding immediately to the model of negation he himself proposes .

The task given is a very simple one: a picture on the left and a sentence on the right are presented. The picture consists of a star called A and a cross called B. The star is drawn above the cross. With this simple material we can form four sentences:

1. A true positive: "A is above B"
2. A false positive: "B is above A"
3. A false negative: "A is not above B"
4. A true negative: "B is not above A".

The psychologist is not able to observe directly what happens in the heads of his subjects but the following observable can certainly be used: a) length of recall, b) fastness of answer to a yes or no question, c) correctness of answer, d) fastness of evocation.

We, not being psychologists, are not in a position to judge about the correctness of the model presented. *We simply want to bring out its logical features and to show that exactly as in linguistics we are led to a positive interpretation of negation, we are equally brought to it in psychology.*

For the processing of the sentences, two different models one main model and one so called conversation model are found in Clark's work.

The results are that positives have smaller latencies than negatives, and among negatives, the true negative has the greatest latency. These results have to be explained by a hypothetical model.

Four stages are presupposed 1) these relational sentences (one object above another) are first encoded as we have represented them. 2) In a second stage it is assumed that the experiment is so chosen that the observed situation is always encoded as follows (A above B) 3) Stage 3 is a crucial stage: the coded image is compared to the coded sentence.

The exact process of comparison (match or mismatch) is precisely what interests us. 4) In a fourth stage, as a result of the comparison process some response value is then given.

This model becomes only clear when it is applied to an example. We take case 4 (the TN, the most difficult case).

a) as a first operation, the picture is encoded as [A above B].
b) as a second operation the TN is encoded as "false [B above A]".
c) the comparison operation would take the two operations in brackets and verify the fact that there is a mismatch: logically this means the verification of the following sentences: "A left in sentence 1 $\neq$ B left in 2 and B right in 1 $\neq$ A right in 2". We want to draw the attention of the reader that no negations are needed to express this positive operation of mismatch (18).
d) as a fourth operation one would then change the initial True-indicator to a "False' indicator. (again, the fact that we use True or False has no special significance: we must have two arbitrary non identical elements).
e) as a fifth operation, the "False" standing in front of the encoded sentence is matched as identical to the "False" resulting from the earlier operation and finally.
f) as a sixth operation, this identity yields as a result "True" (as it should be in the case of a TN).

If we now only presuppose that these operations are not executed in parallel but in sequence (a), that they all take the same time (b) that their times are additive (c) while finally codings that comprehend more elements are consuming more time (d) then the observed order of latencies is a consequence.

From the logical point of view that is ours we want to point out again the following features: a) if this model of negation processing is accepted, then negation is fundamentally defined psychologically by identity and diference b) this was already Plato's point of view and can be thoroughly found worked out in negationless logic (20).

It is certain that the postulates are typically first approximation postulates but we can not do better at the present stage.

It is important for us to compare the results of the psychological study of negation with those of the linguistic study of negation. We think that we see one major coincidence: we do not examine negations whose scopes are not complete sentences and for that reason present quite different characteristics (certainly to be true, on pp. 10 Clark makes the distinction between full negations (present-absent) or quantifier negations (many-few), distinctions pertaining to concepts we have already encountered and analysed as not referring to the sentence as a whole. But this distinction, important as we saw it to be does not contribute to the development of his model).

It might be interesting to note that the dialogical feature (upon which we and Frank Van Dun commented in our remarks upon Lorenzen (21)) is not absent from Clark's work when he distinguishes explicit negation (the denial of the presupposal of the interlocutor) from implicit negation (confirmation of the interlocutor's negative presupposal). This brings us to the larger question: as Wason implies, is it not necessary to add to the operations of comparison of an object and a sentence, the setting up of presuppositions? We are sure this is necessary both linguistically and psychologically but it seems to us (we can only suggest this as a proposal) that encoding a comparison with reference to proposals can occur according to the same pattern as the one suggested here. There is however still a second model of negation-processing that is found in Clark's subjects: the "conversion model of negation". The conversion model shows the *strong tendency of our intelligence to transform negative information into positive information.* When a sentence "A is not above B" is presented pictorially, the subject transforms it immediately into "B is above A". This transformation, a well known error in elementary logic, where contrarities are often confused with contradictories, shows up in the experimental results as follows: true sentences are faster verified than false sentences but the negative-positive distinction has no importance at all as to response time.

We can only make a hypothesis: we are not going to find any strong confirmation for our point of view of predominance in intellectual conduct of the positive over the negative (point of view shared by so many that it has only slight interest) but we shall confirm that we may and can define the negative by means of the positive, by means of the fact that indeed the conversion-strategy is such a definition of the negative by means of the positive. We want to ask what are the reasons for this conversion? We propose a conjecture (we see no other one explaining the facts): here as it where in the mind of the subject a scale was constructed, with two extremes and the "*not*" operator has the effect to transport from one extreme of the scale to the other. *This certainly is a positive operation.*

A mixture in the experimental work of subjects that use the first mismatch or the second conversion model gives quite a good explanation for the variation in results about latency times.

We thus can claim that the psychological evidence is a confirmation for our point of view (and even if it is not judged to be so by the reader, he will no longer deny that *psychological evidence is essential for logical problems*).

We do not have to repeat what everybody is able to read on pp. 19-22 of Clark's paper, where a large list of references is given. But our reader will be even more interested in the psychological study of negation, so we hope, if he equally can be made to see that his strategies do not follow Klima's procedure (who reduces all negation types to one sentence negation that is then deplaced and incorporated in various sentence parts).

Who could have thought that the introduction of a non classical negative quantifier in functional logic could be partly defended on the basis of psychological facts?

And yet thus seems to be the state of affairs. Let everybody judge for himself.

The time taken by the subject to verify "A is absent' and "A is not present" is not (we repeat: *not*) the same (even though both are verified more slowly than equivalent positive sentences). This, stated by Clark on p. 25 of his paper means to him that different presuppositions of different degrees of complexity were present in the listener: if the "not" is used then it seems to him that the listener ought to have been convinced of the truth of A's presence while when "absent" is used such a presupposition is not entailed. As he states himself on p. 26 we can only make hypotheses at this stage, but one of these hypotheses, at the present moment seems to be a) we should not analyse all negative expressions (also lexical ones) by means of one single neg operator and b) (this we do elsewhere when we draw attention to Lorenzen's dialogical logic) one should not formalize the negation operator in such a way that we do not take into account the relationship between sender and receiver of the message. The same difference between instruction makes the instruction much less easier to verify than "except" (measured in latency time). There are even more subtle differences: it is more difficult to verify sentences using "most, hardly, any, many or scarcely a few" (well known expressions for our readers) than to verify "the majority of A's were B" or "the minority of" or "a large proportion" or "a small proportion" asf. We ourselves did not present any logical analysis of these expressions (though this could easily be done: more elements or less elements of set S are in a subset Ss or not, and this can be easily explained by using a relational mapping that can not be biunique), but we think that when again we see that the so called positive-negative opposition behaves otherwise when incorporated in other words, this substantiates our procedure to give ad hoc analyses without an attempt towards reduction by means of neg (in a first move; the reader will meet in this volume our general schema that will take into account to a certain extent all these negations, but this schema will be of a general biological and neither of linguistic nor of psychological nature). The attempted explanation of the experimenters themselves, seems to us needlessly complicated and unsubstantiated by the facts; indeed they hypothesise that for "majority' or "minority" statements the subjects both encode "it is true that most elements of sub A are B" and equally as "it is false that members of sub A are few". In fact they use a partial reduction of minority-majority to many or few (a reduction that is not even correct: the numerical reference is lost). We think we can discard this proposal in presence of a very much simpler correct one (indicated in our conjecture).

We can conclude this part of our paper with one strong and one weak statement. It seems to be clear that at least one model of negation process has been developed in psychology that reduces negation to difference or to a positive mapping of one extreme to another on a scale. This strengthens the attemps to follow a similar strategy in logic.

Moreover — and this is also important, for the peculiar purpose of this paper — it also seems to be the case that this very same model conjectures different processes for negation of different sentence parts (even though this last conclusion is stated on weaker grounds than the first one).

Given the fact that this material is presented to a public of logicians, we hope that these considerations will lead them to consider psychological material as relevant. We hope also that it shall lead linguists to consider psychological material as relevant.

But now we must try to find the place of negation in thought in general.

II. C. *Negation and Problem Solving.*

The paragraph that follows takes as its starting point two of the few existing treatises (22) on the topic of artificial intelligence. We are indeed convinced that thinking is problem solving and that in as far as we wish to study the function of negation in thinking, we must study its function in problem solving. Problem solving itself is only analysed with any degree of accuracy in its computer simulation, known under the name of artificial intelligence. Only Slaglé's and Nillssons's work (22) give an over all view, at the present moment.

We hope that the reader will be able to see at the end of our analysis of the negation-function, in relation to problem-solving, that we did not waste his time by bringing to bear upon the problem yet another type of approach.

The two fundamental concepts in problem solving are those of states and of operators. A state is any configuration of objects the problem is asked to organize in a particular fashion, an operator is any means to transform one state into another. (Nillsson, p. 4). As a first question may arise: a) what is a partially or completely negative state? b) what is a partially or completely negative operator ? c) what is a partially or completely negative problem (the aim indeed has not necessarily to be to transform an initial state into a final state but to prevent an initial state to be transformed into any element of a set of final states or to transform any element outside of a set of initial states into a given finite state.) These questions however are only terminological questions. It must be our purpose to define when negative states, negative operators and negative problems are efficient means in the general problem solution task.

This can only be done if we study the insertion of problems into other problems. In order to see the different ways in which this can be done, let us with Nillsson distinguish two fundamentally different problem solving methods a) the state space approach method on the one hand and b) the problem reduction method on the other hand.

The state space approach takes the initial state and the initial operators and begins to apply these operators obtaining new states until the desired state is obtained. These methods of selection of the applications are the problem solving methods in this state space approach.

The problem reduction approach will generate and solve subproblems, starting out with the main problem and subdividing it.

Chapter 2 and 3 of Nillsson's book are dedicated to the state space approach; chapter 4 and 5 to the problem reduction approach.

In both approaches we have to analyse first the definition and second the efficiency of negatives states, operators, problems, subproblems and searches.

We still have a third approach that we must mention and where our reference to negation becomes particularly complicated. Functional calculus that we used in our transcription of Klima can be used to ask questions and to provide answers. The solution of a problem can be transformed in the searching for a proof of a theorem stating that the initial state can be transformed into the final state if certain conditions are realised. The form of the theorem to be proven will depend upon the problem asked but will in general include quantifiers (existential and/or universal). Classical negation is used througout in this Robinson resolution method (22). The method is essentially negative: one tries to refute the sentence under study and its proof is essentially the refutation of the possibility of its refutation. As we said however the problem-solving aspect of this highly interesting method is not dependent upon the classical negations used in it. One can thus be satisfied in mentioning the crucial role of classical negation in the *object* of the search and understand that the type of search present in the resolution method belongs to one of the two earlier classes.

We, in this research are not interested in the underlying structure on which the theorem proving search is executed; we are interested in the operations of this search and there it might or might not be the case that classical negation has the same structure as one of the negative operators.

In the two topics that we are going to comment upon: state search, and problem reduction approach, we meet initial states, operators and final or goal states. If we want to make clear both to the logician and the practitioner of artifical intelligence the fact that logic is not only there to be applied in the theory of problem solving but that this last theory is indeed one of the *judges* (next to linguistics) of the adequacy of any logic, then we must be able to relate the *propositions* logic is talking about to one of the fundamental features of the problem as we described them. If not we shall not succeed in our endeavour to bring logic and problem solving together.

II. D. 1. *State space representations and state space search methods.*

Nilsson starts out by making the remark that states can be represented in many ways: by one dimensional sequences of signs (strings), by n dimensional sequences of signs (arrays, in the two dimensional case); by sequences of n dimensional sequences (lists) and finally by graphs (sets of nodes or points, related to each other by arcs). Our problem of negation is here to ask ourselves how completely or partly negative states will be represented. Let us first consider the graph representation. We can either classify nodes as positive or negative (in a many valued classification we can introduce degrees of positivity or negativity), we can classify arcs positive or negative or as presenting a degree of positivity or negativity, or we can do both. Going over then to the sequential representations we can classify signs, or sequences of signs or blocks as positive or negative.

This remark is only made for the record. It is obvious that the only important problem here is: how can we develop criteria for these classifications and what will be the properties of so classified nodes, arces, signs or sequences?

The reader must be aware of the fact that when we are going to speak about operator representation he will again see that operators are representable by arcs. These arcs however relate one graph to another graph and are thus from a fundamentally different type.

When operators are considered as functions mapping sets of states upon sets of states, we can distinguish such functions as positive or negative in the following way a) either by classifying them in various types of functions b) or by considering operations on functions.

We can consider: inverse functions (undo the transformation a in b), prohibitive functions (do not transform a into b), two types of elementary functions (transform an y state different from a into b, or transform a in any state different from b) and then we can consider functions mapping elements of sets, or sets that are not large, in sets disjoint from them that are larger (the reader is asked to refer to our general model for negativity where he will precisely find described as negative such operations (24); it is remarkable that in Nillsson's list of examples on pp. 20 none of these cases is to be found, while precisely strong positive rules are frequently present.

Finally: next to states and operators, we must check if a state is a goal state. This check can take many forms (from the simplest matching procedure to a much more complex verification if certain properties are satisfied by the state under scrutiny), and naturally such properties may be partly negative, partly positive. The question, if we want to interest the logician is, what type of negation would be efficient in checking goal states. We have considered the three indispensable parts of a problem. We can now consider the problem in its totality. In graph notation a problem has the general form: "Find a path leading from a point s_i belonging to a class S_n to a point t_i belonging to a class T_r" (both in the final and in the initial situation, the sets can be collapsed into an element). It may be possible that we desire the path to possess certain privileged properties moreover. If they were to be expressed uniquely in graph theorethical terms these extra properties would be "the path has to meet also the following points" (but in general, this optimal situation is not the case and we shall have to refer to general properties of the path that can not only be expressed by reference to local properties but must also be expressed by global properties).

A problem can also be put in a negative fashion (and the ways in which this can occur are identical to the ways in which an operator can be declared negative): the problem can be "find a path leading away from a region R" or "find a path avoiding a node n" or "find a path starting from outside R and leading to the node n".

In any problem a path has a given cost c(p). These costs can be zero, positive or, normally negative. Having now all these possibilities of positiveness and negativeness at our disposal we can finally begin to make some statements.

It will be clear to the reader that any problem solution can be expressed in terms of a flow chart. To select only the simplest possible one: Start! Put the program variable y equal to x; procede and put the program variable y equal to some successor of x (a member of the range of the G-function or successor function).

It can happen many times that the order is not a categorical order but only a hypothetical one and often then the selection conditions will have to be negative ones. In fact: we can state here a general rule: *there will be negative selection conditions in the flow chart when the elements to be selected upon can be in a large number of states and when only under the condition that they did not yet reach a given state a given operator must be applied.*

This explains also why in Nillsson's problem pp. 26 so many negative selection conditions are applied. In general we shall observe that in initial fragments of a program (where as is to be expected the uncertainty as to the elements is the largest), the number of negative conditions will be maximal.

The question remains open: what properties do these negatives that occurr in the selection conditions have ? Certainly, we can make some remarks: iterated negation has no point in the selection conditions and will never occur. This is of some importance but the remark is rather trite.

We certainly see that in problem solving the following procedures of a negative nature (that cannot have the classical properties) can occur: a) it can be necessary in order to avoid overflow of the information in the memory to *wipe out* this information (a type of forgetting operation) b) it can be necessary when it is possible to apply selection criteria to partially constructed elements to stop the action or operation, because the criteria are clearly not met: this *stopping* operation is again a negative one c) we can introduce the concept of subpath of a path and the concept of "class of subpaths". It may be the case that a certain subpath must be *closed.* Indeed: graphs can contain cycles and these cycles can bring the search back to points where certain directions should not be taken (because of the fact that they have proved in the past without promise). This creation of a *barrier* may be assimilated to a *disconnection* of two points.

It is however clear that these operations of *wiping out*, *stopping* and *disconnecting* can only be handled formally when we have at our disposal two graphs, the one exploring the other under certain circumstances. We can not yet do more than show the possibility of this study at the present moment. We are now only exploring the possibilities of expression yielded by the state space representation and we found already quite a number of diferent types of negativities that we can introduce in the expression of the states (1) in the expression of the operators (2) and in the expression of the problem statement (3) We must clearly stress that these forms of negation are really different (an operator is not a state, a problem is neither of them and a criterium is yet a fourth entity).

As in linguistics we see confirmation for the fact that many negation types are used: the monolithic theory of classical negation seems to be refuted for many reasons.

The reader must already have asked himself at the present moment however in how far our strategy of giving specific and positive equivalents of negative expressions in language can also be upheld here. It is certainly true that the non classical brands of negatives just mentioned will yield to a positive description, but is this also the case for all the other forms of negations mentioned (some of them are really reversals, some others are inhibitions, but some others use also quite clearly the complement of a region (and this is quite a classical notion). There would be no harm in that. It would signify only that a mixture of negation types has to be used. We want however to warn against the wrong identification of a flip-flop mechanism and a negation.

Let us take the famous and easy problem of the monkey and the banana: the monkey can only reach the banana when it has moved a wooden box under the banana and stands up on this wooden box. In the beginning of the situation the box and the animal have arbitrary positions. This problem (Nilsson p. 36) shows clearly four variables: two vector valued variables: the position of the box and the position of the monkey, and two bivalued variables: x equal to 1 or 0 according to x standing or not standing on the box; z equal to 0 or to 1 acording to the monkey having or not having the banana. We claim that the two bivalued variables, even though they are defined by means of negative expressions, are not really negations but simply selections or mappings of the situations on two different arbitrary objects. This claim is altogether traditional in the semantics of formal systems and is here in the problem solving situation quite evidently true in virtue of this situation itself.

We can finally come to the more important topic: namely state space search methods.

One point of a graph is selected as the starting point. A special operator is applied calculating all sucessors of a given node. Then pointers are set up from each successor to its source (to make possible the retracing of any path). Finally a check is made to see if any sucessor is a goal node. The negatives of any of these operations exist in some cases a) we can wipe out pointers already created b) we can destroy expansions, or not expand or only partially expand c) we can reject certain nodes as goal nodes and we can do this either bivalently or in a many valued fashion (the way in which the whole is many valued would have to depend upon the way in which later on new descendents are be created by expansion).

Now we are going to consider the search as a whole. Two types of search are fundamentally to be distinguished: the breadth first methods and the depth first methods. Both are blind methods in this sense that all nodes will finally be expanded, but the first method expands first all nodes of a given rank, the second method selects each time the last developed node of a given rank and develops it untill it arrives at a goal (which ends the procedure) or at a stop which makes the development turn back and redevelop other earlier nodes untill another stop.

We would understand something about the relationship between problem solving and negation if we could see how in the dept — first methods and in the breadth first methods the negative steps play a different role and have a different function.

In order to do that we are going to analyse the algorithm for breadth first search methods and for depth first search methods and underline the negative features present there (with reference to the various negative operations mentioned some time before).

In the breadth search procedures we meet various negative actions that we shall indicate by small letters in brackets: 1.Put the start node on a list called "open'. 2. If this list is empty (a), leave the program (b) and register failure (c) 3. If there exists a node in "open", remove the first one on this list (d), put it in on a list called "closed" and give it a name n. 4. Expand then the node n completely, generating all its successors. If there are no successors (e), return to step 2. If there are successors, put them at the end of the list called goal node, leave the program (f) and retrace the steps; if this is not the case (g), return to 2.

We have enumerated not less than 7 negative elements in this program for breadth first search. But it is obvious that these negative elements (we call them negative in the sense of our general definition of negative) are not of the same type. We distinguish the following types:

1. Negative imperatives: cases b, c, d, f.
2. Negative conditions of operation: a, g.

The imperatives are called negatives because in terms of our general definition the initial state is much more definite than the final state: executing a program, is more definite than leaving it, or registering failure; the presence of an item on a list is much more definite than the state resulting from its removal. We acknowledge however that certain of these negative commands become positive when combined with other commands (remove n and then replace it) but this is no objection against the classification presented. The negative commands can be distinguished in action commands, production commands and object commands: the order to leave is an action order, the order to register failure is a production order and the order to remove an element is an object order. The negative conditions of operation are either object conditions (lists being empty) or fact conditions (states not being the case). We can only state that there is a distinct logic for each of these types of negatives: any negative object command, implies a negative production command and a negative action command but the inverse series of implications does not hold. If we receive simultaneously negative action and production commands (run and do not shout loudly) then either they concern the same objects or not (as is the case in the example); if they concern the same object they are specification commands (do not shout loudly and do not speak French); if they do not then if they can be satisfied, they are commutative. We cannot at the present time write a chapter in the logic of commands but we wish to stress that it is possible in an axiomatic fashion to circumscribe the specificity of the negative commands that are to be found in the breadth first search program.

In the expansion of nodes we can take into account the cost of searching arcs of the tree; if this is done we expand first the node such that the path leading to it from the starting point has the smallest cost. This reference to "the smallest cost" includes if we analyse it a classical negation. We keep in essence the program just defined but one more negation of a classical nature has to be added. In the depth first method again the whole set of possible paths is swept through but this time we introduce the concept of depth recursively (the depth of the starting point is zero and the depth of each node is one plus depth of its parent). The base program is kept and instead of condition 4 of the base program we now have a condition that compares the depth of a node to the maximal depth (this concept needs exactly as did the concept of smallest cost a negative of a classical nature in its definition) obtained until the present moment.

Searching arbitrary graphs is more difficult than searching trees (after all they are very simple types of graphs).

In order to come to a deeper understanding of the relation between negative operations and problem solving strategies it would be a good question *to ask if the actions of wiping out,* *stopping, creating barriers, disconnecting have another function in the BF or in the DF. We cannot solve this problem at the given moment, any more than we can solve the general problem of the evaluation of negative actions, of the combination of negative actions with each other and of the combination of negative with positive actions.*

We can only make some conjectures: the taking away of a node or of an arc in a DF changes the relative depths of the nodes; the taking away of the same in a BF has only a local effect: a certain expansion has not as large a multiplicity as before. We thus conjecture — (but we repeat we conjecture), — that one could prove that the classical negative operations have a deeper effect on DF searches than on BF searches. We do not want however to dwell too long upon this topic because we do study PS only as a simulation of thinking. And only PS using heuristic methods can be considered as a valid simulation of thinking.

We think that in this paragraph on problem solving we have shown a new way to tackle the problem of negation; we are however very much aware of the fact that the basic questions have been asked, but have not been answered. As we could not do more in our research on operation-search, we are not going to undertake it. We are returning to logic (to many valued logic) in what follows in order to see that already now we can relate problem solving to formal systems.

II.E. *Many Valued logic and Problem Solving.*

The impatient reader who doubts the possibility to find a foundation of the theory of negation in problem solving can perhaps be convinced by an attempt to relate problem solving with negation in many valued logic. We use "Many Valued Logic" by Nicholas Rescher (25) and remind ourselves that a problem solving strategy is a sequence of application on initial states yielding final states until goal states are achieved.

The many valued negations studied in Rescher, pp. 122-129 are functions of propositions. We can decide to examine the hypothesis that a proposition may correspond in problem solving to an arbitrary action or action sequence; or we can consider that a proposition corresponds to a specific type of action. It can be made to correspond to an action that is a necessary or/and sufficient condition of a set of actions (a) or to an action closing a problem solving sequence (c) or to an action whose failure or success determines different types of other actions (c). We want to simplify and for that reason we consider the correspondence between any proposition and any action (mapping of an initial I on a final state F).

Rescher calls a function N a negation; if

a) it can never be true that both p and Np are true and
b) it can never be true that both are false.

If now our p is not a proposition but a problem solving action then we must consider the following cases a) it can never be true that both I on F and G on H can be successfull or b) it can never be true that both I on F and G on H can be executed (either because I and G can not coexist, or because any change of I in the direction of F cannot be accompanied by any change of G in the direction of H). In order to understand clearly what we mean by "an action that makes I approach F" it is obviously needed that we arrange I and F on an ordering scale, so that it makes sense to state that in a series of sequences of states the elements approach to the state X).

We have — this is a side remark — the impression that the first Rescher requirement according to which p and Np cannot both be true is more essential and general than the second one asking that both cannot be false (in action translation it is much more generally encountered that two actions cannot both be executed or cannot both be successfull than to meet the case of two actions one of which must necessarily be performed or must necessarily be successfull).

The negation of an action (if only the first demand is taken into account) will then be a positive action that satisfies the modified version of Rescher's first stipulation. One of the ways in which this can occur is that either the initial state or the final state or the operation are on the ordering scales of states and operations at the opposite pole of the one of the first action. But these three cases are only extreme ones. Two actions cannot be both executed or both be succesfull even if their elementary constituents are not as far apart as that. We thus see that we are in the presence of execution incompatibility or success incompatibility and this on the basis of extremity opposition between one or more of the three constituents or on the basis of non extremity opposition of one or more of the three constituents.

Let us now consider the following cases of negation:

p	N_1p	N_2p	N_3p	N_4p	N_5p	N_6p	N_7p	N_8p	N_9p
T	F	F	I	I	F	I	I	I	F
I	F	T	T	I	T	I	F	F	I
F	T	T	I	T	I	I	I	T	T

We think that it is natural to bring problem solving and many valued logic together because: performing an operation, not performing the operation and performing the opposite operation are so clearly distinct. In fact the multiplicity of incompatibilities we noticed before drawing upon the list of Rescher's negation types in three valued logic, would have led us already to this conclusion.

Let us now try to give an interpretation of these various negations, taking into account the following facts. An action has the value F if it is positively prevented, and its results made impossible. It has the value T if it is executed and successful. It has the value I if it is not executed.

1) The first type of negation expresses a relation between two actions A and B so that B is executed only when A is prevented, and prevented if either A is executed or if it is not positively prevented.

In a problem solving search this means "do B whenever it is not certain that A has failed, and in all other circumstances prevent B!".

2) The second negation would be expressed by the following

command "prevent B whenever A was positively executed, and in all other circumstances execute B".

3) The third negation is interesting and easy: whenever A is done, or has been prevented do with B anything you chose (neither F nor T are imposed) but when A is undecided, do B. It is obvious that this third negation is the fundamental search negation, in the same way that N7 is the fundamental anti search negation: there when the state of A is undecided, B must be prevented.

In a problem search N7 will have no function, N3 will be the opening move of a search strategy, and the first two negations are needed in different bifurcation situations: both are responses to either A or B, the first preventing B nearly always (except when A is certainly executed), the second prescribing B nearly always (except when A is certainly executed). Such nearly universal execution and prevention commands are needed in problem solving strategies, but obviously only at bifurcation points.

4) The fourth negation is in a sense similar to the third: B is prescribed in one case, the case in which A is prevented.

The reader will be able to interpret for himself the other different types of negation. Perhaps case N6 is somewhat special: there action B is left free in all possible situations of action A, neither imposed nor prevented. It is doubtfull (as it was for N7) if such negations are needed in problem solving sequences.

It is obvious however that a many valued negation that would be more than three valued would do more justice to the situation.

We see the following stages for any operation:

1) O is applied to I and F is obtained, 2) O is applied to I and F is obtained, 3) O is applied to I and F is obtained, 4) O is applied to I and F is obtained, 5) 6) 7) combinations of 23, 24, 34, 8) combination 234, 9) even if we order as seems reasonable the 8 cases as we just presented them, the two orders on the set of states and on the set of operations would cross the order just presented. For this reason we would believe that the many valued logic most adapted to problem solving would

be a product of three many valued logics one of which has 8 values, the two others having as many values as one might wish to introduce distinctions among states and operators.

We want before closing this digression that must help our analysis of the negative moves in problem solving situations, do two more things a) ask ourselves if various formulations of the principle of non contradiction are more or less adapted to the problem solving situation and b) ask ourselves if we cannot find a link between linguistics and logic on the topic of negation by looking at this many valued series.

The principles of contradiction are many in many valued logic and their strength is extremely variable.

We shall note the principle and immediately try to formulate the correspondents in action language.

1. At least one member of the pair p, Np must necessarily be false = at least one action of the pair A, B must necessarily either fail or not be performed.

2. At most one member of the pair p, Np must be allowed to be true = at most one member of the pair is allowed to be successfull or to be performed.

3. a proposition cannot both be false and true = an action cannot both be performed and not be performed; both be successfull and fail.

4. A proposition can assume only one truth value = an action can assume only one performance value or success value.

5. The principle "pand not p" must be rejected = the principle "A is performed and B is performed" must be rejected.

6. The principle "not (p and not p)" must be accepted = the principle" not "A is performed and B is performed" must be accepted. (In 5 and 6, B is incompatible with A).

Let us now, given the multiple ways in which an operation of problem solving can be denied, select one of those many negations. Let B be the action that must fail if A is executed.

In the first sense the principle is false: both actions can remain unexecuted. In the second sense it is true: at most one of them can succeed. In the third sense, it is true if our criteria of success and failure can be summarised in one; it is false if the multiple criteria cannot be unified as such. Then indeed it would be possible for an action to fail in one sense and succeed in another. The same remarks hold for the fourth version. 5 and 6 are only the metalogical versions of 3 and 4 and the same discussion remark must be made on their behalf.

But — we repeat it — these considerations do not at all exhaust the topic. We have presented the discussion of the various versions of the principle of non contradiction only for one of the many versions of negations on operations we announced.

II.F. *Problem solving and linguistics.*

We have now given an affirmative answer to the first question we had asked. Let us analyse the second question, and go back to our first list of examples referring to negation in linguistics (see page 205). It is obvious that, if we take seriously the remarks made there, at least a seven-valued logic is presupposed. We do not consider very important the presence here of the magical number seven. But it is obvious *that our language exactly as our problem solving action seems to require for its minute shades of meaning a many valued logic.*

This remark being made we might come to the crucial question: if we find a relation between logic and linguistics on the one hand, between logic and problem solving on the other hand, can we also in a systematic fashion find a relationship between problem solving and linguistics.

We must answer in a very general fashion going back to first principles. We have seen that problem solving is formally the growing of a graph (in simple but frequent cases: of a tree). A communication act we now choose to analyse as an interaction between two or more problem solving graphs. This communication act, following the fundamental ideas of Roman Jakobson and Fernand Vandamme (15) is an encoding and a decoding act. The decoding act is again the growing of a graph (various elements of the sentences, considered as strings of symbols must by syntactical analysis, be grouped in clusters, subclusters and so forth and these structures must then again be projected upon semantic graphs and structures classifying the syntactic elements as different lexical items). This happens in a different way in the encoding process where we start from the meaning to go to the expression to go to the meaning. The graphs are not the same but they must finally have a common layer.

Communication considered in this way is a very peculiar form of problem solving: it is the problem to map part of a problem solving graph (sometimes a minute part: one peculiar piece of information) upon another problem solving graph. We could ask for what specific types of the problem solving graph communication is called for, is caused and on the other hand what are its many consequences. We do not have the time to do this here. We only want to make the following point: there must be a relationship between linguistics and cognitive psychology (as it is formalised here in the theory of problem solving) for the very simple reason that the object itself of linguistics is, as was said above, the growing and connecting of two graphs in no way intrinsically different from the growing of other graphs in general.

We hope that this digression will have convinced the reader who had the feeling that our long analysis of problem solving led u astray that we can indeed find relations, first and foremost with fundamental properties of negation in logic (many valued logic here) and second, that our last remark will also have convinced him that there is no abyss between problem solving and language.

We want still to insist some more upon the relationship between the linguistic negation mentioned in the beginning of this paragraph and the problem solving intervention of negation.

In the sentences "It is neither red nor brown" or "it is not red' we express the conclusion of a problem solving process; in our problem solving actions we execute actions having the intention to reach such a conclusion. Even if in both cases as L. De Ryck and M. De Mey (26) pointed out we are proceeding to the selection among a multiplicity of alternatives, and if the only non classical feature of the "not" is that it is not the complement or the bivalued negation but either, according to context, an opposite lying far beyond any neighbourhood of the negated element ("she is not beautiful" means in general "she is definitely ugly") or to the contrary, given another context, means that we want to designate a place definitely in the *neighbourhood* of the negated element, this is also the case in the search negatives and conclusion negatives remain complete.

Much of what has been said here has been anticipated in the excellent article by Josiah Royce in Scribner's "Encyclopedia of Religion and Ethics". We want to stress a few of the points he made there and that we find again on the basis of contemporary research:

1. In distinction from the logical "not" Royce recognizes (p. 265) the negation of courses of action and courses of actions that are negations, the negation of objects and of intensional concepts of any degree of generality: all of these operations are or can be expressed by means of the "not" relation and yet they are basically different from the bivalued negation that can only be a function of a proposition or of a sentence.

2. Royce in his second paragraph recognizes that the usual negation is a completely symmetrical relationship: every assertion has a negation and every assertion is the negation of its contradictory: so there not a real distinction in this sense between affirmative and negative sentences.

3. In his third paragraph he recognizes, as we should do when we study in linguistics and psychology the negation operation that *the negation operation is an asymmetrical relation in everyday use, associated with privation, opposition, attitudes of will and modes of knowledge.* This asymmetrical relation is called by him simply "associated" with the symmetrical classical logical negation (he could not yet know the intuitionistic calculi and their many successors) but this association is so regular that we simply may be convinced that the associated elements are parts of the meaning of the negation particle.

4. In his fourth paragraph Royce in an intuitive fashion (having

not yet the theory of problem solving at his disposal, and relating too consistently the construction-destruction opposition with good and evil) studies the function of negation in thought and life. We consider this part of his article as related to our study of negation in problem solving.

In these paragraphs we simply wanted to show that even the "not" is linguistically and semantically different from the Boolean negation and that there is a natural relationship between the use of the "not" to express search results and to express search operations (the one more studied by linguistics, the other more studied by problem solving theory).

There is thus indeed no unique meaning, not even for the simple word "not". But its multiple meanings have many features in common (the trip on the semantic memory model) and these features can be formally described. The selection among the set of features is made by the context. The big problem of how the context does it and how the context (verbal and non verbal) must be described remains however open.

In the preceding pages we tried to show that many specific negations exist, that could be defined without reference to the "not" particle (or the "neg" constituent), transformational grammarians like Klima or even Jackendoff are prone to use when they analyse negative expressions even when their scope is not a complete sentence. We must however even go farther and it is in discussion with Prof. Dr. Fryda that this became clear to our group. Even the usual "not" is different from the logical negation, but (and this was not completely subscribed to by Dr. Fryda) *the way in which this "not" is different can again be described in a logical* way.

Let us consider the example "The stone is not red". The fact that this sentence is asserted (not perhaps the content of the sentence itself) implies that there was reason to suppose that the stone was red, or in another case that the stone has a colour not quite identical to red but quite close to it. (as in the sentence "The stone is not red and not brown" implying in fact that its colour lies between red and brown but that the exact colour word is lacking in our memory to designate the precise shade).

If we try out the disjunction interpretation of negation 'not p' signifying "either q or r or s or t..." then it seems to be the case that the logical negation designates the disjunction of all alternatives, while the linguistic or psychological negation means only the disjunction of a few alternatives lying in some sense "close' to the negated sentence. A clearly topological negation is thus presupposed, but the topological negation is not a unique one. If "not q" means linguistically "incompatible with q but in the neighbourhood of q" this is still different from "in the neighbourhood of q and in the neighbourhood of s, but incompatible with q". Many other combinations with neighbourhoods can in this way be introduced. (the interested reader must be referred to a standard introduction to topology for the explanation of "neighbourhood").

Let us now ask how the memory model of the psychologist studying negation is structured. Most probably it will be a semantic information net in the sense of Raphael (27) wherein a multiplicity of information units stand in dyadic relations to each other. Typical for the negation operator will be that it performs the following action a) the scanner focussing on unit a, a standing in various dyadic relations to units b, c, d asf. is moved to scan a region around a, at least having a radius r, and not having a radius greater than s, b) and within this region the scanner is moved to fix itself upon a particular point x and to grow all line segments (in the graph model) or to form all relations having as first element this very x. If the memory model is not a semantic information net including eventually cycles asf but a rigidly classified list of lists then the negation can still be represented as moving the focussing point of attention not more than r steps, and not less then s steps upward or downward a classification tree, and having as second operation the complete development of all successors of the so attained points untill a distance d.

It is obvious that if information is expressed by dyadic relations (if the nodes or elementary units are essentially of the form xRy) then the general form a psychological negation (even of the so classical sentence negation) will be an operation performing a mapping of one relational dyad into a set of other relational dyads. This we can call either a n adic relation (and so the memory model must have its disposal such structures) or to the contrary a relation between relations (a meta-relation). We find here again confirmation for the general model, namely that negative acts are mappings of small domains on large codomains.

It remains a moot question if the essentially topological notion of "neighbourhood" is needed to express the meaning of negation or if instead the quantitative indications mentioned above (*deplace the working unit successively on n units having in the graph not more than distance d and not less than distance s from the initial position and develop these positions untill a degree r*) can supersede the topological concept of "neighbourhood".

Obviously these remarks presuppose a semantical model for the linguistic structures in which negation is studied. We have here either presupposed a semantic information net or a classification tree, the most irregular or the most regular memory model we know of and in both it seems that everyday "not" *is distinct from the classical logical negation but expressible by means of meta-operators on series of dyads.*

Let us now for a moment leave the linguistic plane and analyse the problem solving activity. Once more we can be placed before very specific tasks (solve this equation) and relatively unstructured tasks (write a paper on this subject different from the one John has written). These tasks can be of small or large magnitude (write a paper different but very slightly different or write the most different paper you can think of, stil preserving some properties (either the style or the content or some other essential feature). In the most general fashion, a task is a searching task: find an object o in a space s (this can be a material object in a space. We call a negative strategy the strategy that uses commands as follows" do not look there" in the search space, or "realise an object that has the most opposite characteristics to the one presented" or "realise an object that has all characteristics of the one presented except (negative feature) this one".

Our conjecture is, that positive operations will prevail when the initial distribution of the object over the searching space is very concentrated (it is very probable to find it in one place and not in another) while negative operations will be used when the initial distribution of the probability is very flat and slight differences have to be used to transform it in a more concentrated one (in fact we find here ourselves in the design task in which we are asked to find an object having an essential feature in common but maximally different from the given one). In fact, if we have to select a subregion out of a region we can always do it from below or from above: we can always have a series of decisions "this object is certainly an element of the region" or a series "this object is certainly not an element of the series" or a mixed strategy "this object is certainly an element and this other not". The mixed strategy may be a concatenated strategy (this object is not an element, because this other object is) or to the contrary it may be a disrupted one (the commands follow each other without any relationship). At the end of both types of strategies we have to have a stop "These are all the objects in the region: there are no others" or "these are all the objects not in the region: there are no others". So at the end of both strategies a negative command has to be issued. Our conjectures are a) that for design tasks (unstructured tasks) demanding the construction of objects very close but not identical or very far away but preserving an essential feature, negative strategies will be used (a generalisation of a conjecture of Dr. Fryda).

If however the task lies in between, we conjecture that the proportion of negative moves will strongly decrease. Only empirical research that has still to be done can come to a decision on these points. If we now select the more general account a) the extension of the region or its complement: one might conjecture that if we have reason to believe that the region is large, we better begin by elimination; if we have reason to believe that the region is small, the inverse holds b) if we have reason to believe that the region is the intersection of a list of regions for which the negative strategy holds we should follow the negative strategy.

These remarks are by no means a complete answer to the problems under discussion but seem to open a way to get more information. One might certainly make the remark that the negative strategy is a risky strategy: neglecting to search a subregion of the search space on the basis of uncertain hypothesis makes for not finding the object looked for. But on the other hand it is a speeding device: by eliminating we concentrate ourselves on the region not eliminated having in general an easier structure than the one originally presented.

Conclusion

We hope to have shown some connections between the theory of negation in logic, linguistics and psychology. This was the aim of this paper.

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