ON THE MODES OF OPPOSITION IN THE FORMAL DIALOGUES OF P. LORENZEN

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Starting from material dialogues P. Lorenzen has introduced a formal procedure — a dialogical game — which he has shown to be a fruitful tool for the investigation of logical and philosophical problems. One uses the criterion of "reasonableness" in order to set up such a game. It turned out, however, that other games could be set up which are, at least prima facie, equally reasonable. This raises the question of the appropriateness of a formalization of natural dialogues. Within the context set by P. Lorenzen and K. Lorenz this remains an open problem. In fact, the lack of sufficient motivation calls for *a posteriori* justification, and introduces the possibility of arbitrary variations of the rules. In the first section this possibility is explored. Philosophically this is quite unsatisfactory, but so is the lack of motivation which opens up the field for this kind of "*Spielerei*". In the second section we try to remedy the cause of this "undecidability" of the game-structure, by generalizing the dialogue to an n-person game. Within this structure we shall state the rules for a dialogue yielding once more the intuitionistic validity-concept which Lorenzen obtained from his formalization. It will be seen that it is primarily a question of the "*Geltung*" of the elementary sentences — i.e. their mode of distribution among the members of the scientific community — which will decide about the appropriateness of a mode of dialogical opposition. The n-person-game will also allow us to use a uniform basic structure for all games in stead of the multitude of structures emerging in section I. This basic structure will be that of the simple *assertive* dialogue.

I. *Two-person games*

In this section the dialogical approach to formal truth, initiated by Prof. Lorenzen (3), and applied by him with considerable success to the field of metamathematics (4), will be followed in order to indicate some of the anthropological implications and assumptions involved. In the first place we must relate the notion of information to the dialogical approach; in the second place we must take some time out for a few remarks on the cultural and social aspects which determine to some extent the criteria for a reasonable dialogue.

To many people a dialogue is in the first place an exchange of information, a means of communicating about facts. In fact, one could imagine situations consisting of nothing but successive messages from one participant to another. Let us call such a dialogue assertive; in an assertive dialogue each statement of fact is a definitive input into the situation. How does conflict arise in such a case? The simplest case is when a statement of fact is not agreed upon by all participants, and this is usually manifest when one asserts the negation of what the other asserts: White says that P, and Black says that not — P. However, suppose one speaker asserts a conditional, such as $P \rightarrow Q$; this is an assertion involving two statements of fact. Assertion of not — $(P \rightarrow Q)$ does not contribute a new element to the dialogue, the two statements of fact still being connected. In fact, in an assertive dialogue it is precisely the investigation of each elementary statement of fact that must decide who is right and who is wrong. So, when White asserts $P \rightarrow Q$, he is in an implicit way communicating the meta-dialogical message: "if you assert P, and only if you do, I am willing to assert Q", which means that Mr. White needs the information P in order to commit himself to Q. Of course, White may prefer to assert the negation of P, for instance, when he realizes that even with Black's commitment to P he will not be able to lend any credibility to Q. If a participant asserts a conjunction he asserts all of its parts; in order to oppose it the other need only commit himself to the negation of one part, leaving his opponent no choice but to turn his attention to the part in question. Thus, if White asserts $P \wedge Q$ and Black answers with $\neg P$ White must commit himself to the defence of P. The same holds for sentences with an all-quantifier. The situation is different for disjunctive sentences, or sentences with a some-quantifier. In this case if White asserts $P \vee Q$ and Black answers with $\neg P$, White has a choice: either to give in and commit himself to Q, or to oppose Black with the assertion of P.

It should be kept in mind that each assertion of an elementary statement of fact is unconditional. The assertor is committed to it by the act of assertion itself. In the formal dialogues that will be given below, statements of fact are always represented by elementary sentences (or prime-formulae as they will be called). These are the dialogical units of information. However, not all dialogues are assertive. It might be that a participant wants to oppose a statement but does not wish to, or may not be in a position to do this in the way described above: by committing himself to an assertion, i.e. by an input of new information. In other words he is only willing to commit himself conditionally, the condition being that his opponent first takes account of the possibility by showing it is sufficient for his purposes, or by showing that the mere possibility can be refuted.

Let us call such a dialogue a modal one. Its characteristic is that contributions are made which provide only conditional information. Take e.g. W — If Hamlet had not existed, Shakespeare could not have portrayed him so splendidly. In an assertive dialogue, Black can only oppose this with the statement:

B- Hamlet has not existed.

which may then be denied by W or answered with: "Shakespeare cannot have portrayed him as well as he did." But it is rather unreasonable to expect Black to assume the responsibility for unconditionally asserting that Hamlet has not existed. In stead one would not be surprised to hear Black say

B- Well, possibly, Hamlet has not existed. Now what are your arguments for saying Shakespeare could not have portrayed him so splendidly?

If W's arguments prove worthless B may reject all commitment to his statement as to the non-existence of Hamlet.

Other types of modal dialogues can be thought of.

Dialogues have a typical cultural setting; they do not take place in empty space. Even the formal dialogues that will be discussed shortly are characterized by rules, conventions which can only be justified with reference to the task, the claims of the participants. Whereas sometimes it is a consideration about the nature of the information that is required in order to oppose a certain statement, it may also be a consideration about the social implications (legal, moral, conventional,) that leads one to a reformulation of the simple rules of the assertive dialogue. One must realize that the argument-schemes a lecturer is entitled to use may be quite unthinkable in the legal philosophy guiding the reasoning of an officer of the prosecution. A public hearing is not the same as a cross-examination in court, if for no other reason than that the consequences differ markedly. In some situations it is unethical to obtain more information than is actually required, in others no objection can be raised. Etc.

Both considerations, about the nature of information in dialogues, and of the social and cultural environment can be translated into rules for *formal* dialogues — these being dialogues where one participant has all the facts and the other all the logic, so to speak. It will be seen that different rules reflect different formal systems, i.e. different logical validity-concepts.

In the present paper we start with a formal game representing a dialogue of the assertive kind. Then a modal dialogue is introduced emphasizing in the first place the role of conditional information, in the second place the strength of a positive argument in contrast to an argument based on the defeat of an opponent's position, and thirdly a short discussion of the modes of some arguments and counterarguments.

Assertive dialogues

Following P. Lorenzen's lead (3) a formal dialogue will be set up but this time with special reference to the assertive type of dialogical opposition. The description of the dialogue will be in terms of a two-person-game.

There is a set of two players — one is called W (for White), the other B (Black). The game is asymmetrical: The conditions for making a move are not identical for both players. This is because it is intended that the players each defend different claims. The players make their moves alternatingly.

There is a set of positions: u,v,w, ... which are of two kinds: assertions and challenges. Assertions are either prime-formulae: p, q, ... or compound formulae P, Q, ... If P is a formula, so are $\sim P$; $\forall x$, Px; $\exists x$, Px; If P and Q are formulae, so are $P \wedge Q$, $P \vee Q$ and $P \supset Q$. Only closed formulae will be considered. Challenges are positions of the form (P?) — I.e. an assertion followed by a question-mark. If an assertion is challenged *in toto* we shall write (?) in stead of (P?) — unless confusion is likely to arise.

A challenge does not commit the speaker. In fact one can easily do without them. However, since most accounts of the dialogical games have them, we shall have them too. (*vide*, p. 105).

Positions are sometimes numbered in order of appearance in the dialogue; positions taken before the play begins have index zero, and are called initial positions: u_0, v_0, ... At least one player must be committed to an initial position.

In the metalanguage we shall use expressions such as (u,I) — i.e. player I takes position u. The expression (u_p,I) is used to indicate that player I commits himself to a prime-formula.

The transformation-rule T is a binary many-many relation obtaining between configurations:

$$(u,I)_n \; T \; (v,J)_m, \; I \neq J, \; n<m$$

n and m are numbers of the positions; u is called T-predecessor, and v is the T-successor. A position for which no T-successor

is defined is called an end-position. The player who succeeds in reaching an end-position wins the play.
The rule T has three parts: we shall follow W. Stegmüller's account in this matter (6). There is a *logical rule* TL, a *basic rule* TB, and a *structural rule* TS. The first part TL establishes the game as a logical tool by determining how the logical signs $\sim$, $\wedge$, $\vee$, $\supset$, $\forall$, $\exists$ function in language-games; TB makes it a formal game, and TS a formal game of some special kind.

TL is given by table I. The columns on the right and on the left contain positions held by the same player, the column in the middle a position held by his opponent. The rule is applied in this way: let

$$X \;//\; Y \;//\; Z$$

be an element of TL. Now if player I has taken a position of the form X, his opponent is allowed to take a position of the form Y, and if he actually does, I is allowed to take position Z.

Table I

TL1	p		
TL2	$\sim\alpha$	α	
TL3	$\alpha\supset\beta$	α	β
TL4	$\alpha\wedge\beta$	$(\alpha?)$ $(\beta?)$	α β
TL5	$\forall\varkappa, \alpha(\varkappa)$	$(a?)$ $(b?)$	$\alpha(a)$ $\alpha(b)$
TL6	$\alpha\vee\beta$	$(?)$	α β
TL7	$\exists\varkappa, \alpha(\varkappa)$	$(?)$	$\alpha(a)$ $\alpha(b)$

TL1 is proper to formal dialogues: prime-formulae cannot be transformed. The other parts have been discussed before, except that in TL4-TL7 opposition is expressed by a challenge rather than by assertion of the negation of a part of the assertion which is opposed.

The basic rule TB introduces the first asymmetry in the game. Since prime-formulae cannot be transformed, the player asserting one in opposition to e.g. $\sim p$, or in response to his opponent's opposition to $P \supset p$, exhausts the logical aspects of a certain line of thought: one has arrived at statements about (contingent) facts. Let there be one player, say W, who is instructed to defend the claim that either his initial position is true (valid) on formal grounds alone, or that his opponent's initial position is false on the same grounds. B, the other player, is given to defend a much weaker claim: to construct a counter-example showing that either his initial position is not necessarily false, or W's initial position not necessarily true. Therefore B must have free use of prime-formulae, whereas W should be restricted to assert only those prime-formulae which have been asserted by B — who has the obligation to back them up. This stipulation is what the basic rule is all about:

$$(u_p,B)_n\ N(u_p,W)_m,\ n<m$$

if B has not asserted p, W cannot be allowed to assert p.

Validity must be established in a finite number of steps. Therefore we must stipulate that W wins a given dialogue if he succeeds in reaching an endposition after a finite number of moves. B wins in all other cases. The justification of this rule is given by the strength of the claims of the players. However, if this rule is to be of any use, some precaution must be introduced in order to prevent B from indefinitely continuing the dialogue, at least in some cases. The structural rule TS provides a clause to this effect. Since there is no danger that W will benefit from indefinitely prolonging the dialogue, there is no need to introduce a similar restriction for W. In order to avoid fixing the length of the dialogues at some arbitrary maximum, and to allow B at least one chance of transforming a position held by W, Lorenzen selected this rule for B:

> TS(B): B is allowed to provide a T-successor to the last, and to no other, position taken by W.

W, on the other hand, is allowed to provide a T-successor to any position taken by B.

Dialogical validity-concepts can be formed if we consider the set of all possible dialogues starting from a given initial formula. In order to do this we shall use the game-theoretical concept of a strategy: "a player has a strategy if for each possible position of his opponent the T-successor is uniquely determined. A strategy is called a W-win-strategy iff, whatever position B may choose, the strategy allows W to reach an endposition favourable to him. If a W-win-strategy exists the dialogue is of finite length (if the strategy is adhered to). If there is a W-win-strategy for a formula held as an initial position by W, the formula is said to be *valid*; otherwise it is said to be rejectable; If there is a W-win-strategy for a formula held as an initial position by B, that formula is said to be *invalid*; otherwise it is said to be *satisfiable*.

Dialogues will be represented by means of diagrams. The column on the right is reserved for positions taken by W; the one on the left for positions held by B. Positions are numbered as they appear. Between brackets is the number of the position taken as T-predecessor — this number will be omitted if the position transformed is the one immediately preceding. If in a dialogue one wishes to represent strategic options, one may use the method of nested sub-diagrams, or -tableaux, as they are usually called. As an example we take:

	B	W	
	.	$p \vee \sim p$	0
1	(?)	$\sim p$	2
3	p	.	
	.	p	4 (1)

W claims $p \vee \bar{p}$ to be valid and proceeds to prove this: there is but one way for B to oppose W's initial position, TL6. W cannot defend with p because of TB, so he must assert $\bar{p}$. Again B can only respond with p. Now W has only one choice: to respond once more to (1), but this time there is no objection to chosing p (TB). (4) is obviously an endposition. Since there are no alternative strategies for B, the diagram represents a W-win-strategy for $p \vee \sim p$.

In the following case it is equally possible to find a W-win-strategy for the initial position.

	.	$\sim \forall x, Px \supset \exists x, \sim Px$	0
1	$\sim \forall x, Px$	.	
	.	$\forall x, Px$	2
3	(a?)	.	
	.	$\exists x, \sim Px$	4 (1)
5	(?)	$\sim Pa$	6
7	Pa	.	
	.	Pa	8 (3)

Finally, an example with B as the player holding the initial position.

0	$\sim (p \vee \sim p)$	.	
	.	$p \vee \sim p$	1

The dialogue continues as in our first example. One can easily verify the existence of a W-win-strategy, and thus the invalidity of $\sim (p \vee \sim p)$.

The game described is immediately seen to be K. Lorenz's classical logic game (1). However the important thing is to relate this game to the concept of an assertive dialogue. Two remarks must be made. In the first place, TL reflects the fact that opposition to a statement requires one to commit oneself to an assertion — at least where the controversial signs "$\neg$" and "$\supset$" are concerned. This implies of course that the participants in such a dialogue are willing to commit themselves that easily. If they are not it is not at all likely that there will be a great amount of conversation among them! In the second place TS provided no restrictions for W. We must now show this freedom for W to be dependent on the assumption of full commitment. The dialogue which was shown in the first diagram, provides a good example. Indeed, in (4) W was able to give a T-successor for B's move in (1), which would not have been allowed if the restriction TS(B) applying to B had applied to W as well. On the other hand, B being fully committed to p in (3), it would be rather strange to deny W the right to exploit this state of affairs. Example:

— "Either he is coming, or he is not!"
— "Well ...?"

Now, whatever choice the first speaker makes, in order to oppose him his opponent must *assert*, i.e. commit himself to, the other alternative; and since the first speaker has but committed himself to one of the propositions, without specifying which, he cannot be refused the right to defend his position on the basis of information supplied by his opponent in the course of the game. But this is precisely the point where the assertive dialogue becomes unrealistic. The rules of the game force the players to give information whenever they want to express their opposition to some statement. Is this a necessary feature of formal dialogues?

Modal dialogues

Paul Lorenzen, who was the first to introduce logical validity-concepts based on dialogues, claimed that games representing dialogues justify intuitionistic logic and no other. The question, then, is: in what sense can it be maintained that the "classical" game described above is not an adequate representation of dialogues? Lorenzen's argument may be illustrated with the following diagram.

	.	$((P \supset Q) \supset R) \supset P$	0
1	$(P \supset Q) \supset R$	.	
	.	$P \supset Q$	2
3	P	.	
	.	[P]	4 (1)

Lorenzen notes (4) that B may object that he asserted P, in (3) only in the attack on $P \supset Q$; and that B may therefore insist that W first defend himself against this attack. What Lorenzen has in mind is apparently this: if one utters a sentence in an attack on some other sentence, one is not committed to it — at least not to the same extent — until it is adequately countered, either by a defence, or by a successful counterattack. In opposing a statement one is not forced to play some information into the hands of one's opponent: instead the information is offered under the condition that the opponent first accomplishes what he set out to do: Example:

— "Either he is coming, or he is not."
— "Well ...?"
— "Apperently he is not coming."
— "But, what if he is?"

This last response can hardly be said to justify the alternative: "He is coming," since B has not given enough information for that.

In as much as Lorenzen's remark makes sense the rules of the game described in the previous section may be modified.

Lorenzen's argument applies only to B. He makes a clear distinction between an attack by B and a defence for a statement to which B is committed. This is to be understood in view of B's role as the supplier of "facts and figures". Notably where B might be called upon to make statements about infinite sets it seems only natural that the information he gives is only conditional. Thus, where B is concerned the rule TL is modified in this way:

Table II

TL2'	$\neg\alpha$	//	$?\alpha$	//	
TL3'	$\alpha \rightarrow \beta$	//	$?\alpha$	//	$!\beta$

where $?\alpha$ indicates that α is an attack, $!\beta$ that β is a defence. A challenge is not an attack; in stead we stipulate that if an attack is challenged, the response to that challenge is itself an attack, and if a defence is challenged, the response is itself a defence. The justification for this is obvious: e.g. he who attacks with a conjunction attacks with every part of it. The distinction between attacks and defences allows us to modify the rule TS in an appropriate way, so as to accommodate Lorenzen's remark. TS(B) is not changed. A rule for W is introduced:

TS(W) W is allowed to attack or challenge any position held by B; and — *provided there is no subsequent attack by B against which he has not yet defended himself* — he is allowed to answer any attack or challenge made by B.

It is known that with this rule a logical validity concept can be defined, in the way indicated in the previous section, that is co-extensive with intuitionistic validity (1,6).

In what way is the latter game more reasonable than the former? Let us state it in these words: if a position held by W is attacked this does not necessarily mean B commits himself to some statement; the dialogue may continue without new information, i.e. factual information, being added. This is a first step towards a more colloquial situation, where some statements appear as mere hypotheses, which must be answered before they may be used as arguments.

A second step towards a more natural dialogue is made when one requires that the players, and more specifically, the player whose claim is strongest, live up to their promises: e.g. that whenever they announce their willingness to defend some position, they proceed to do so. If a statement is made of the form $P \supset Q$, and the opponent grants the possibility of P, the speaker should not be allowed to prove his superiority by successfully counterattacking P. The opponent might feel cheated — and not unjustly: he might argue that he only granted the possibility of P for the sake of argument. One could think of statements such as: "If the moon is made of cheese, the square root of 2 is a rational number." How do you oppose such a statement, how do you make it clear that no advance in cheese-technology will solve the problems of mathematics (if this is what you believe)? With the present rules of the game W can easily win the dialogue, although one feels that this is one dialogue he should have lost — if you are concerned about finding a game that is a reasonable representation of *natural* dialogues. One obvious way to achieve this is to provide a *clause allowing the opponent to repeat any attack that has not been successfully answered, although a defence against the attack was, in fact, announced.* This clause, when added to the structural rule TS, applies whenever W has reached an endposition favourable for him, in a play without the clause, but has not defended himself against all attacks, although such a defence was announced.

An example: in the classical logic game

	.	$(((p \supset q) \supset p) \supset p)$	0
1	$((p \supset q) \supset p)$	.	
	.	$(p \supset q)$	2
3	p	.	
	.	p	4

is a diagram representing a W-win-strategy. If this game is played with the clause added, B may repeat his attack with P, in (3) against which W has not defended himself, although by all appearances, q was to be the natural choice for W's reply to p. The clause works well to get rid of some unnatural 'truths'. In classical logic games the clause allows us to note the difference between theorems such as $\sim(p \supset q) \supset (p \supset (q \supset r))$ and $\sim(p \supset q) \supset (p \supset \sim q)$.

From the point of view that formal dialogues must be adequate representations of natural dialogues, no objection can be raised against this clause, which is after all nothing but a corollary of Lorenzen's remark — an extension of it that is quite in line with the idea of rational dialogues containing little or no superfluous assertions.

So far we have studied three games. Each of them was seen to be correlated to some special consideration about dialogical modes of opposition. In an assertive dialogue the following characteristics were found:

1) the opponent (B) must commit himself to some statement, i.e. provide factual information, which cannot reasonably be denied to W for eventual use.

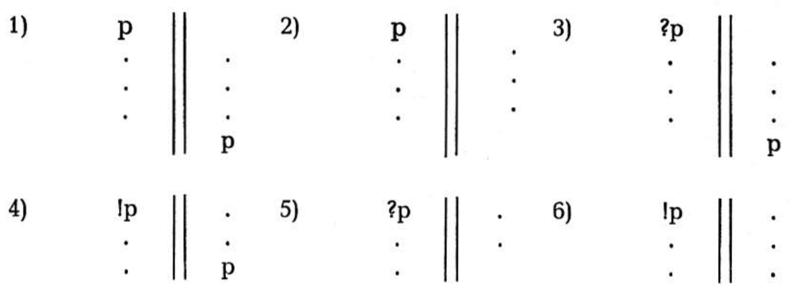
2) the proponent W is not under the obligation to provide a positive proof for his assertions: a counterattack is as good as a defence. The latter characteristic could be eliminated by means of the repetition clause for B. In a modal dialogue the opponent has the possibility of conditional commitment: only when W has given an adequate response to an assertion by B does the latter become committed to it. These characteristics were seen to be readily translatable into rules for formal games, yielding a classical validity concept in the former case, and an intuitionistic one in the latter. *A minimallistic subsystem* could be found for each of them by means of the repetition clause.

There are other situations that may be of some importance, being more of a cultural than of an informational nature. This is the case when the participant who has all the facts feels justified in insisting that all the facts with which he has supplied his opponent are reflected in the conclusions of the latter. This "humanistic" feature may be vital when one looks into the social field. E.g. in a court of law it would under certain circumstances be deemed illegal if the judge did not evaluate some information given by one of the parties and based his conclusion only on part of it. And it would be quite unethical if some agency forced one to give more information than is strictly needed for its function. In order to accommodate this remark a modification of the basic rule TB is proposed in order to express the requirement that W must in his argumentation make use of all the information, conditional or not, supplied by B. For this purpose we add a sufficiency-condition to TB:

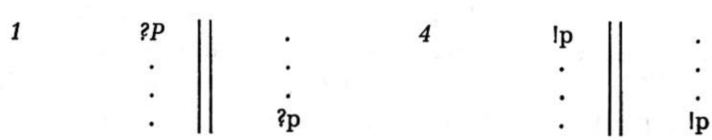
TB' (U_p, B) NS (U_p, W)

or: If and only if B asserts a prime-formula, W must also assert it. With this rule $p \supset (q \supset p)$ is no longer a theorem — although $(p \wedge q) \supset p$ still is: in the latter case W has a means — the challenge (P?) — to select the necessary information, whereas in the former case he has forced his opponent to provide unnecessary information.

Sofar we have seen two modes of assertion ?P and !P. [1] These were related (in connection with prime-formulae) to conditional and unconditional information, and restricted to B. The following configurations involving nothing but prime-formulae are therefore familiar:



Under TB' (2), (5) and (6) are of course not allowed. The case can be made for an extension of the use of ? and ! to W as well as to B. Indeed one could argue that the player holding the strongest claim need not always be required to back this claim with an unconditional assertion. If this is accepted the following *basic* configurations appear:



	Left		Right
2	?p . . .	‖	. . . !p
5	!p . . .	‖	. . . ?p
3	?p . . .	‖	. . .
6	!p . . .	‖	. . .

(3) en (6) are not allowed by TB'. The forms of these configurations constitute a set K with six elements. In any given diagram of a dialogue, all appearances of prime-formulae are reducible to one of these forms; this means that all the basic configurations (i.e. all prime-formulae in the left column, and all those in the right column, arranged so as to yield configurations of a given form) appearing in the diagram of a given dialogue can be grouped into at most six categories. The powerset P(K) has $2^6 = 64$ elements. Under TB all 64 are allowed. It is clear that, formally at least, there is an incredible number of selections that can be made among these 64 elements. There is, in other words, an immense choice of basic rules for the dialogues, and therefore a considerable number of validity concepts that can be defined with the help of this device: modification of the basic rule. And some of these rules may be related to social conditions governing some argumentations.

First one should note the difference between these dialogues:

B- What if it is raining? *
W- Then it is raining **

and

B- What if it is raining? +
W- I'm telling you: it is raining. ++

The second dialogue has clearly nothing to do with necessary truth, because W's reply is seen to be only accidentally related to B's question, whereas in the first dialogue it is clearly conditionally related to it and thereby irrefutable.

This gives us the diagram

$$\begin{array}{c||c} & p \supset p \\ ?p & !p \\ + & ++ \end{array}$$

as compared to

$$\begin{array}{c||c} & \neg p \supset \neg p \\ \neg p & \neg p \\ ?p & ?p \\ * & ** \end{array}$$

The former diagram would be regarded as not representing a W-win-strategy whereas the latter would. In order to exclude basic configurations such as

$$\begin{array}{c||c} ?p & . \\ . & . \\ . & . \\ . & !p \end{array}$$

one could adopt a basic rule such as

TB_1 $(u_p,B)\ N\ (?u_p,W)$
$(!U_p,W)\ N\ (!U_p,W)$

allowing one to reject such formulae as $P \supset P$ and $P \supset (Q \supset P)$. A extremely strong basic rule is the one eliminating

$$\begin{array}{c||c} ?p & . \\ . & . \\ . & . \\ . & ?p \end{array}$$

as an admissible basic configuration. This rule is

TB_2 $(U_p,B)\ N\ (!u_p,W)$
$(!U_p,B)\ N\ (?U_p,W)$

Almost all important theorems in classical logic are rejected with this rule.

The rule

TB$_3$ $(U_p,B)\ N\ (!U_p,\ W)$
$(?u_p,B)\ N\ (?u_p,W)$

eliminates

$$\begin{array}{c|c} !p & . \\ . & . \\ . & . \\ . & ?p \end{array}$$

as an admissible configuration. $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$, $(p \supset q) \supset (\sim q \supset \sim p)$ are among the theorems rejected.

Many other rules can be formed. Some of them may be found to fit certain dialogical situations, others may be of interest in their own right, i.e. as generating systems of formal expressions. An important rule both for formal and dialogical reasons is the symmetrical one:

TB$_5$ $(?U_p,B)\ N\ (?u_p,W)$
$(!U_p,B)\ N\ (!u_p,W)$

with which a validity concept can be defined which is neither reflexive ($(p \supset p)$ is rejected) nor transitive, relative to logical implication.

This rule may be assumed to have an important social role: one participant in a dialogue must accept the facts as they are presented to him (as hard facts or as conditional information).

For an application of these rules to real live situations it is necessary to analyze the roles of the participants, and to give clear indications of the mode of the statements exchanged. Therefore the relevance of these rules is not primarily the abundance of formal systems one can generate with their help — even if, as we did, only logical truths were considered:

Prof. Lorenzen himself has used his dialogical approach in the definition of many other kinds of truths —, it is rather the flexibility and versatility which a formal dialogue can be made to show when adapted to the requirements of special dialogical situations. The fact that within the frame-work of the approach it is possible to obtain what may be called a continuum of validity concepts, is a strong indication that for quite a few communication — and information — exchange processes a rather simple model can be found, corresponding to the empirical conditions at hand, yielding the valid forms of argumentation.

II. n-Person games

In this section I shall give an introductory and informal discussion of generalized Lorenzen-dialogues.

When Prof. Lorenzen devised his dialogical procedure for testing the logical validity of sentences, he claimed that it justified intuitionistic logic. As it happened, his rules governing the dialogical use of the logical connectives, subject to the structural rule (vide: K. Lorenz), allowed him to do just that. However, K. Lorenz's game yielding the usual classical concept of validity must be accepted as an equally reasonable one. The most disturbing about this situation is the fact that both use the same pragmatic rules for the logical connectives, thereby giving the impression that in both games the pragmatic meaning must be the same. The intuitionist will certainly deny this. However, one thing is clear: the *structural rule* which must be introduced in order to obtain one or the other concept of validity, establishes a relation between the participants in the dialogue, rather than between their assertions, but does so in a very obscure way — meaning: there is no element within the dialogical situation allowing to decide either this way or that. Nobody will, of course, be allowed to justify such a rule because it happens to yield the validity concept one wants to have. *What we have to do* is to look within the dialogical context for an indication as to the appropriate rule. *What we have to bear in mind* is the fact that dialogues happen at some time, at some place and, if they are to be fruitful, between people who have something to say to each other. It may not be easy to formalize this situation, but Prof. Lorenzen has shown it to be possible, at least in principle.

Let us take a closer look at his game. It has the characteristic of being a two-person game — which is rather curious a restriction, unless shown to be a necessary one — and it has one very striking rule, which cannot be justified either by an argument based upon the pragmatic meaning of the logical connectives, or by one based upon any characteristic of the dialogical situation. At least no such argument has been given so far. Let us therefore try to find the weak point in the discussion Lorenzen gives of a dialogue just before introducing his version of the structural rule.

Clearly he has Pierce's theorem in mind when he gives as an example of a thesis the formula $[(P \supset Q) \supset R. \supset .P]$-vide section I. His comment on the ensuing dialogue goes like this: at some point the opponent (B) may be forced to assert P in the attack on $P \supset Q$: now it would not be fair to allow the proponent (W) to take advantage of this by asserting P as a defence against the opponent's opening statement: $P \supset Q. \supset R$. According to Lorenzen, the opponent may argue that he only asserted P in the attack against $P \supset Q$, and therefore expects the proponent to defend himself against this challenge first. This is however not a conclusive argument: the other player W has an equally valid counterargument — i.e. that nothing in the situation has allowed him to find any clue about his opponent's expectations, and that indeed the latter cannot deny having asserted P. What is the problem, and what is the way out? According to Lorenzen there are some assertions made by B which are not opposable to himself, for which he is allowed to deny any responsibility — in other words: the question of whether B is fully committed to his assertions must be answered in the negative; only if W has answered all preceding attacks does B become committed to his assertions. There is no use arguing about this; the fact remains that it is a serious complication which cannot be justified by an appeal to any dialogical 'principle' enunciated so far. Therefore we were justified, as it were *by the lack of rules*, to indulge in the kind of '*Spielerei*' presented in the first section of this paper, where we assumed 'commitment' to be an unrestrictedly variable component of the game. For any philosophical purpose this is an intolerable but not, as we shall presently argue, an incurable fault. To be sure our remark at the end of section I — that such variations of the basic rule may be found to fit some empirical situation — cannot be of any help in the present context, where we are trying to find a pragmatic rule for deciding about the degree of commitment of an assertor to his assertions, or rather: for deciding when an assertion is opposable to an assertor.

Every restriction imposed on, and every freedom granted to a participant in a dialogue must be motivated with reference to a particular domain of social or cultural activities and the goals which characterize it. However clear Prof. Lorenzen's motives may be, he never states the particular domain he is interested in as a determinant factor in the organization of his dialogues. At one time he seems prepared to admit this (1969, p. 39), when he questions the law of the excluded middle in arithmetic. But he fails to indicate how being concerned with arithmetic influences one's view of a reasonable dialogue. As a matter of fact we are practically forced to accept the rules of the 'classical' version of the game because they are far more 'objective': by this, I mean that, if the pragmatic rules (i.e. the 'logical rule') are taken to be intended for overt acts — and who doubts this? — and to be used among equals — even if defending different claims — there is no way within the game-structure, but an arbitrary one, to introduce exceptions to the 'law of the dialogue' that every participant should stand by his assertions (TB). From this we must conclude that there has to be more about the dialogical situation than has been said so far. The problem emerging from this discussion is, then, one about the dialogical nature of the relation of *commitment* — i.e. the commitment of an assertor to back up his assertions — or, alternatively, of the relation of *opposibility* —

i.e. of the conditions forcing a participant to accept an assertion as opposable to him.

In a social situation the level of commitment is usually not left to the discretion of a participant: in a dialogue of the kind we are interested in this would indeed be disastrous for the institution as a whole. On the contrary commitment is usually the result of moral, contractual or even legal obligation. Indeed, most of the time one is not committing oneself — one is simply committed. By the mere fact of somehow belonging to an organized society. To be committed is to be under the obligation to answer some questions, to perform, or abstain from performing, some actions. This is so because one is not free to pretend that certain situations do or do not exist, or that certain events never happened. This is what jurists have in mind when they talk about the opposibility of certain events to a person who may, or may not, have been involved with them. The mere fact of standing in a social relation to another member of the community may lead one to be committed simply because the other person is. Society has developed means to distribute the commitment which arises in one spot over a more or less well-defined subsystem within itself. Even in a society where all social relations arise out of free association, i.e. a person voluntarily associating with another, one may expect this kind of distribution. Consider a case of 'culpa in eligendo'. Some person has to perform a task; not being able to perform all of the actions required he seeks the help of another person who consents to do it for him. Being careless or unfitted the latter causes some harm to a third party. Who has to pay? Frequently the answer will be: the man who sought the help, because society expects him to make sure the people he associates himself with are capable or socially acceptable to do the job. Free association is an important phenomenon because it does not always allows one to use the argument based on the 'culpa in eligendo'. It is a necessary condition that the chooser may be assumed to have vouched for the man of his choice.

In fact one may have this situation: a man is offered a job, but he is unwilling to perform the actions required; now he is asked to find a replacement, which he does. In this case the ensuing contract would still have to be made between the man who offered the job and the one who was asked to replace the first.

How do we relate these remarks to the dialogical game? By now it must be clear that one's commitment is a social burden, imposed by society, and governed by rules, regardless of whether one likes it, whether one is prepared to carry a heavier load of it. Whenever I take part in a dialogue, my utterances are assertions when and to the extent the community I live and work in rules them to be. The rules may be flexible, but I am not the one who is allowed to bend them.

In the formal dialogues we are concerned with, a move is really an invitation to challenge a thesis (if it is not a challenge itself). As long as we confine our attention to two-person games it is clearly an invitation to the other player. Now Lorenzen contends that the importance of the game lies in the existence of winning strategies — strategies assuring a participant a favourable outcome against every conceivable opposition — but seems to restrict this concept to two-person games. Prof. Apostel raised the possibility of generalizing this game to an n-person game as a means to recognize its philosophical status. We are now in a position to say that the game has to be generalized, because it is simply not true that in social situations we have a winning strategy by assuring a favourable outcome against every conceivable opposition *by one opponent*. Indeed we must allow the opponent to seek the help of others in his environment (e.g. we must allow him to quote from the books he has read, to call in the authorities he accepts, etc.), i.e. game-theoretically speaking, to form coalitions. Only then do we have the possibility to use the phrase 'every conceivable opposition' in an appropriate sense. It is of course out of the question to allow the opponent to form coalitions without having introduced some rules governing the formation of coalitions, if we are not to wind up with no defensible position at all. There are two questions that concern us here:
1) when is the opponent allowed to call in a coalition-partner?
2) who may be taken to be a coalition-partner of the opponent?

The first question calls for a pragmatic rule governing the formation of coalitions, and must prescribe how the players are to act if, say, the proponent invites his opponent to call in a coalition-partner, or invites a coalition-partner to challenge a certain thesis, or if the opponent issues a command to all his coalition-players, or to one of them (i.e. vouches for all or some of them).

The second question asks for a *structural rule* in the sense of a statement determining how the coalition is structured, i.e. a statement as to the form of admissible coalitions. The fact that a structural rule re-emerges at this point need not worry us: we now know exactly what it is whose structure we are talking about. Of course, some problems remain to be settled but the element of arbitrariness which characterized the structural rule in the two-person game can be eliminated by referring to the structure of the social group to which the opponent claims to belong, or the proponent claims to defy. But this cannot be decided on an a priori basis.

Let us now introduce the pragmatic rules we need in order to generalize the game in such a way as to allow for a satisfactory use of the concept 'win-strategy against every conceivable opposition'. First we remark that the proponent may wish to strengthen his claim by inviting his opponent to call in the help of a coalition-partner or co-player. If he wishes to defend the formula α in this way, he can make his intentions clear by prefixing the symbol *L* to the formula. The opponent may then select a co-player who will be asked to challenge α. This gives us the rule

$$(W_L) \qquad L\alpha \quad || \quad B \Rightarrow x : ? \quad || \quad \alpha$$

with x continuing the attack on α. Or W might want to weaken his claim in the sense that he is willing to defend α only if he be allowed to determine himself who among B's coplayers must challenge α. In order to signal this intention W shall be using *M* as a prefix. For *M* we have the rule

$$(W_M) \qquad M\alpha \quad || \quad ? \quad || \quad x(\alpha)$$

where x must be a coplayer of the opponent who has attacked $M\alpha$. This rule allows W to challenge a member of B's coalition without having to challenge B in person. Likewise we can find rules for B, who may be induced to strenghten his claim in such a way as to vouch for all his coplayers (including, of course, himself!) that they will assert α or to weaken it so as to vouch for some of them (possibly excluding himself). In the first case this amounts to allowing W to pick some coplayer out of the pack and to defy him to assert α, and in the second case this amounts to allowing B to make the choice; thus we have the rules:

(B_L) $L\alpha$ || $x(\alpha?)$ || $B \Rightarrow x : \alpha$

(B_M) $M\alpha$ || ? || $B \Rightarrow x : \alpha$

It must be kept in mind that in the rules W_M and B_L where the proponent is allowed to make the choice, the coplayer x must be chosen from among the coplayers of the opponent B; any other choice is not *opposible* to B.

Before we give some examples of dialogues which illustrate the application of the rules we recall the fact that we had to recognize the classical version of the game as reasonable. Therefore we simply add the new rules to that game. It is also self-evident that the relation 'to be a co-player of' is reflexive: every opponent is his own coplayer. The opponent who attacks the thesis in the initial position held by W will be called *o*, the others *a*, *b*, *c*, ..., eventually with primes added: *a*', *b*', ... Some examples:

	.	$Lp \supset p$	0
1	Lp	.	
3	$o \Rightarrow o : p$	$o(p?)$	2
	.	p	4

O attacks $Lp \supset p$ by asserting Lp; W cannot answer with p (basic rule); however he can make *o* ask any of his coplayers to assert p; not knowing who *o*'s coplayers are he is in this case forced to ask *o* himself; *o* asserts p and W can now assert p.

The thesis $p \supset Lp$ is not defensible as is clear from the diagram:

	.	$p \supset Lp$	0
1	p	Lp	2
3	$o \Rightarrow a : ?$	.	

W cannot use *o*'s assertion that p against *a* who has not asserted p (basic rule). The following dialogues are completely analogous to the ones just given:

	.	$p \supset Mp$	0
1	p	Mp	2
3	?	o(p)	4

	.	$Mp \supset p$	0
1	Mp	.	
3	$o \Rightarrow a : p$	?	2
		.	

It is easy to see that whenever $L\alpha$ is defensible so is $\sim M \sim \alpha$ and vice versa. Indeed under the given rules they represent the same dialogical situation: B selects a coplayer against whom W has to defend α.
Following accepted usage we shall call L and M modal operators. Others can be introduced along the same lines. We shall give the rules for $\mathbin{-\!\!3}$ and $\neg$ without argumentation.

(W$\mathbin{-\!\!3}$) $\qquad \alpha \mathbin{-\!\!3} \beta \;||\; B \Rightarrow x : \alpha \;||\; \beta$

(B$\mathbin{-\!\!3}$) $\qquad \alpha \mathbin{-\!\!3} \beta \;||\; x(\alpha) \;||\; B \Rightarrow x : \beta$

It is straightforward to show that $\alpha \mathbin{-\!\!3} \beta$ is just another way of saying $L(\alpha \supset \beta)$.

(W$\neg$) $\qquad \neg\alpha \;||\; B \Rightarrow x : \alpha \;||$

(B$\neg$) $\qquad \neg\alpha \;||\; x(\alpha) \;||$

Clearly $\neg\alpha$ is just a way to say $L \sim \alpha$ or $\sim M\alpha$. Some examples:

	.	(p ⥽ ~p) ⊃ ¬p	0
1	o : p ⥽ ~p	¬p	2
3	o ⇒ a : p	.	
5	o ⇒ a : ~p	a(p)	4
	.	p	6

	.	p ∨ ¬p	0
1	?	¬p	2
3	o ⇒ a : p	.	

¬ and ⥽ are called strict negation and strict implication, respectively. In the games discussed in the first section ~p is an alternative statement for p ⊃ F where F is some forbidden assertion (~ is then a device for avoiding the use of F even in the rules of the game). ¬p means the same as *L*(p ⊃ F).

What we have done so far amounts to this: we have opposed the proponent to a field of opponents; within this field the opponent has the possibility to choose his co-players; by the mere fact of choosing an opponent designates the man of his choice as a coplayer. This means that he accepts that the results obtained by W against the coplayer be opposible to him. This is a generalization of the basic rule of the dialogical game: B has to stand by his choices as well as by his assertions. It is a social necessity that one's own actions be opposible to oneself. On the other hand there has to be some involvement if opposibility is to be justifiably invoked. It need not be an active involvement, at least not in actual society where one can be involved *de jure*.

In the present game the opponent is involved with his coplayers. But who are they? In order to answer this question we must know what the structure is of the group to which the opponent belongs. But as one becomes a coplayer because one is selected to be one, we may restrict our attention to the act of choosing a coplayer.

It is only reasonable to call a coplayer of B anyone (B included) who has been chosen by B. If the range of the relation 'to be a coplayer of' is the set of these opponents and no others, we say that the relation is of range one. In that case we obtain a game which we shall call a I-game. Any classically valid thesis (with only $\wedge$, $\vee$, $\supset$, or $\sim$ as logical signs) is, of course, defensible in a game which is certainly a I-game. So is $Lp \supset p$; and also

	.	$L(p \supset q) \supset (Lp \supset Lq)$	0
1	$L(p \supset q)$	$(Lp \supset Lq)$	2
3	Lp	Lq	4
5	$o \Rightarrow a : ?$	.	
7	$o \Rightarrow a : p \supset q$	(a?)	6 (1)
9	$o \Rightarrow a : p$	(a?)	8 (3)
11	$a : q$	p	10 (7)
	.	q	12

Since *a* is selected by *o* himself W may regard him as a coplayer of *o*, and therefore assume that *o*, by asserting *L*p, has vouched for *a*. It is also obvious that prefixing *L* to any classically valid thesis has no consequence whatever for its defensibility: if some thesis α is defensible against *o*, $L\alpha$ is defensible against any coplayer *o* may choose. Indeed if any thesis α is defensible in the I-game, $L\alpha$ is defensible too. The analogy with T-validity in modal logic is obvious. A thesis which is not defensible in the I-game is

	.	$Lp \supset LLp$	0
1	Lp	LLp	2
3	$o \Rightarrow a : ?$	Lp	4
5	$a \Rightarrow b : ?$	.	

Since *b* is not within range one of *o*, the latter has not to vouch for *b*; therefore W cannot win. It is however sufficient that *b* be a coplayer in order that $Lp \supset LLp$ be defensible. Another thesis which is not defensible under the present game is $p \supset LMp$:

	.	$p \supset LMp$	0
1	p	LMp	2
3	$o \Rightarrow a : ?$	Mp	4
5	$a : ?$	.	

Since *a* has not chosen *o* the latter is not a coplayer of the former, although *a* is a coplayer of o: The relation 'to be a coplayer of' need not be a symmetric one.

One can go a step further and require the opponent to accept the consequences of the results obtained by the proponent against the opponent and his coplayers and also those against *their* coplayers. This yields a game of range two. In this way it is possible to define an arbitrary number of games of range *n*. In stead we may simply make the relation '... is a coplayer of - - -' a transitive relation: i.e. if y is a coplayer of x and *z* is a coplayer of y, then *z* is a coplayer of x. This means that eventually the opponent must vouch for an arbitrary number of people simply because they were selected by the people he selected. Certainly $Lp \supset LLp$ is valid in this game, which we shall call a II-game.

An alternative extension of the I-game can be obtained by making the coplayer-relation symmetrical: whenever x is a coplayer of y, y is a coplayer of x. Thus we obtain a III-game. It is now not difficult to find a win-strategy for the formula $p \supset LMp$. Readers familiar with Kripke's models for modal logic will certainly notice the analogy with S_4-validity and the validity-concept for the so-called Brouwerian system.

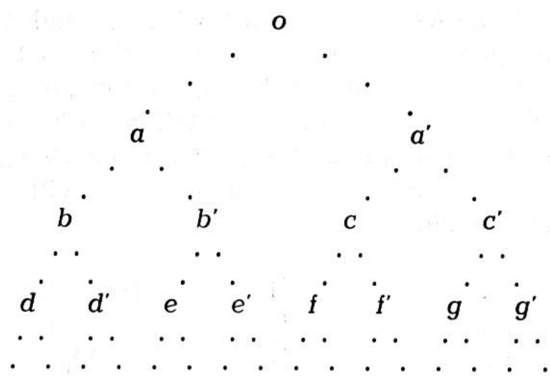
Lumping these extensions of the I-game together and adding them to it gives a game where the relation 'to be coplayer of' is reflexive, symmetrical and transitive: the IV-game. As a defensible thesis we have $Mp \supset LMp$. One will notice the analogy with S_5-validity.

	.	$Mp \supset LMp$	0
1	Mp	LMp	2
3	o ⇒ a : ?	Mp	4
5	a : ?	.	
7	o ⇒ b : p	?	6
	.	b(p)	8

In IV every opponent is a coplayer of every opponent. This amounts to the situation where people are assumed to be involved with anyone who is known to be — or have been — a member of the group. And this allows the proponent to view the group of opponents he is facing as one person.

The jurist will concede that there are indeed different kinds of partnership which are characterized by one of these distributions of responsibility. But we cannot leave it at that: indeed we have to know which game is appropriate, when it is, and why it is. In order to do that we must consider once more what is being distributed among the members of a coalition in the sense of the games under discussion. Apart from any meta-dialogical considerations, we must conclude that all formulae with an L prefixed to them are distributed over a coalition whenever the opponent generating that coalition asserts them. The difference between such a formula and a non-modal one with no L-prefix allows one to distinguish between the 'personal' and the 'official' assertions made by an opponent. How can we exploit this distinction ?

Let us take a look at a coalition in II where the coplayer-relation is transitive. We could represent one in the form of a 'tree':



Imagine for a moment that we have here a genealogical structure: o is the father (the *pater familias*), *a* and *a*' his sons etc. Assume further that ascendants do not inherit from their descendants, as is the case with genetic properties. Let us also assume that such properties are transmitted to all descendants without loss. With this in mind we introduce the concept of a 'hereditary proposition'. If a hereditary proposition is asserted by an opponent, it is asserted by all his 'descendants'. Now we shall consider a game such that all elementary propositions are hereditary. As logical connectives we allow $\wedge$, $\vee$, ⥽, and $\neg$, and no others. If we add these restrictions to the game II, we obtain a game called II'. Some examples:

	.	$\neg(p\wedge q)\wedge p$. ⥽ $\neg q$	0
1	$o \Rightarrow a : \neg(p\wedge q)\wedge p$	$\neg q$	2
3	$a \Rightarrow b : q$	.	
5	$a : \neg(p\wedge q)$	$[\neg(p\wedge q)]?$	4
	.	$b(p\wedge q)$	6

b cannot challenge q, which he has asserted himself, and he cannot challenge p which he has inherited from *a*. Compare this with:

	.	$\neg(p\wedge q)$. ⥽ . $\neg p\vee\neg q$	0
1	$o \Rightarrow a : \neg(p\wedge q)$	$\neg p\vee\neg q$	2
3	$a : ?$	$\neg p$	4
5	$a \Rightarrow b : p$	.	
	.	$\neg q$	6
7	$a \Rightarrow c : q$	.	
	.	$b(p\wedge q)$	8
9	$b : q?$	.	
	.	$c(p\wedge q)$	10
11	$c : p?$	.	

where W cannot win. A final example:

	.	$\neg\neg(p\vee\neg p)$	0
1	$o \Rightarrow a : \neg(p\vee\neg p)$	.	
	.	$p\vee\neg p$	2
3	$a : ?$	$\neg p$	4
5	$a \Rightarrow b : p$	.	
	.	$b(p\vee\neg p)$	6
7	$b : ?$	p	8

What is so curious about this game? In the first place, the fact that we have introduced a special kind of formula, or rather: a restriction on the kind of formula that may be used. In this case a formula that is distributed in a certain manner over a coalition (cfr. *L*-formulae). In the second place, the fact that it allows one to defend a thesis which is intuitionistically valid, and does not allow one to defend a thesis which is intuitionistically rejected. What has happened? We have substituted two logical signs, $\supset$ and $\sim$ by their modal counterparts ⥽ and $\neg$; according to the rules given for these signs we must recognize that they are acceptable to an intuitionist (who will take negation to be a special case of implication). The intuitionist meaning of implication (vide e.g. Gentzen) is that p implies q if there is a proof which allows one to derive q if a proof of p exists: if such a statement is to be attacked in a dialogue the opponent would have to concede that such a proof exists; if he is not willing to assert this he must be given an opportunity to find someone who is willing to take up this responsibility, but this does not commit him to the existence of that proof — it must be actually given. This the attack-defence rule for ⥽ allows him to do. On the other hand once the existence of a proof is given, the proponent has the right to assume that all the opponents that are brought into the game by the one conceding the existence of the proof, will be *au courant* of this: a proof has no use unless it is being communicated to others. If a descendant has the right to challenge his predecessor this would mean that there was in fact no proof. Also, it must be clear that an ascendant does not have to accept what is granted by a descendant: the latter may have given something which is not acceptable *as a proof* in the eyes of the former.

Now this exposition of the intuitionist view cannot be generalized to all propositions, or rather: to all sentences. It is obviously well-suited for mathematical purposes, but it is certainly not the case that mathematics is the only subject that can be talked about in a dialogue.

However, the justification of a game such as the one we just described, is not restricted to mathematical, arithmetical sentences. Indeed all sorts of sentences will do as long as they fit the scheme of an hereditary proposition of the kind described. We could say that it is the community of the users of a certain language which must come to an agreement about the rules deciding under what condition an elementary sentence, for which '*Geltung*' is claimed may be distributed, and how this distribution must take place. Until this has happened it does not make much sense to talk about *logical validity*.

The foregoing brings into the focus of our attention the methodology of the sciences, for it is precisely this activity which concentrates on the criterion for deciding about the '*Geltung*' of the elementary sentences.

The generalized Lorenzen-dialogues provide then a powerful, non-arbitrary tool for ascertaining the logical validity of argument-schemes which are applied to the field of scientific activity in question. It is therefore not at all surprising that we first had to develop validity concepts for modal logic before we could find a justifiable means for deciding about the reasonableness of some set of dialogical rules. We had to remember that dialogues take place among people who have something to say to each other and who are not indifferent as to the effect of what they say upon the group which provides the context of the dialogue.

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([1]) These modes were found to be relevant while taking account of Lorenzen's 'structural rule'. Clearly, however, they affect the application of the 'basic rule': i.e. the 'law of the dialogue' stating that every participant should be prepared to back up his assertions.

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