ON NEGATION

AN INTERDISCIPLINARY STUDY

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Summary

In Part I, we briefly reviewed some historical approaches to the negation phenomenon (Plato, Aristotle, Russell, Von Wright, Strawson). We discussed shortly a method, based on the notion of existence-free quantifiers for avoiding the orthodoxical criticism on the Aristotelian logic. This because of the importance this criticism has as a justification of the notion 'presupposition'. This notion seems to be very crucial in many approaches to linguistics and psycholinguistics to day.

From all this, it seems that historically one needs to differentiate between two types of negations: (A) the exclusion of an affirmation; and (B) the affirmation of a (positive) alternative.

In Part II, we isolated an abstract structure, which we believe to be present in all kinds of exclusion negation. We have tried to show that this structure can be found back in several phenomena, which commonly are called negative, etc. This in the pragmatically individual and social context as well as in the wants and conceptual context. In these last two fields, one can speak of an interiorization of the pragmatical negation.

In this framework, we look at the non-existence paradox and the problem about the differentiation 'true, false, not true, not false'.

In Part III, we considered the role of negation for the intelligence. This leads us to the conclusion that negative information is very important, although it is necessary to introduce techniques for minimizing negative information and this for economical reasons.

I. *SOME PRELIMINARY THOUGHTS ON NEGATION AND LOGIC*

I.A. *Introduction*

It is almost a triviality to say that in logic negation plays an important role. The importance of this notion is certainly reflected by Heyting's (1956, p. 122) statement: "The logic of negationless mathematics is difficult to formalize for several reasons. First of all, there is no calculus of propositions, because only true propositions make sense." This expression also illustrates the general trend (see also B. Russell, 1969, p. 129) to identify falsity with negation of truth (¹).

Another proof of the importance of negation in logic is that many different logical systems differ from each other by the role negation plays in each.

A typical characteristic of approaches to negation is, of course, the fact that many authors by touching on the problems of negation feel it necessary to take the Parmenidian problem (as found e.g. in Plato's dialogue 'Parmenides') into consideration about the attribution or the negation of a predicate to a non-existing element. Among these authors we can mention Aristotle (Ross, 1928), Russell (1919), Strawson (1952), Von Wright (1959), Toms (1962).
Their vision of this problem and its solution seems to determine very strongly their treatment of negation.

As a result on this phenomenon, it seems to us worthwhile to consider more closely the Aristotelian approach to the problem. Then the discussion which will follow on the more modern authors and their respective interpretations of negation, will lead us to the problem of finding some consistent negation interpretation which is more closely related to that of Aristotle. The reason we did not start with Plato's solution (which he has expanded in the Sofist) based on an interpretation of negation as 'otherness' is that it has not influenced the logical development very much (except perhaps in recent times in Grice's approach to negationless logic). Moreover the solution Plato suggested, does not seem to be at all complete (Toms, 1071; Vandamme, 1970 a).

I.B. *The negation seen by Aristotle*

I.B.I. The Aristotelian oppositions.

In his "Categoriae" Aristotle treats the negation as an element of a specific kind of opposition. He differentiates four kinds of oppositions, which are the correlative opposition, the contrary opposition, the opposition privative-positive, and the opposition affirmative-negative.

I.B.I.1. The correlative opposition is described by Aristotle (Ross, 1928, Categoriae Ch. 9, 11 b) as follows: "Such things, then, as are opposite the one to the other in the sense of being correlatives are explained by a reference of the one to the other." This opposition will not be discussed extensively here, because it is not so relevant to the problem of negation (²).

It seems, however, interesting to us to mention that from the examples Aristotle introduces, a kind of confusion originates about what kind of elements the arguments of the correlative opposition are. In any case, it seems clear that this opposition has something to do with relations. What is not clear, is whether the opposition exists, between the arguments of the relation or between the relation itself and one of its arguments.

The examples in casu are: (a) "double is a relative term for *that*, which is double is explained as the double of something", (b) "knowledge, again, is the opposite of the thing known in the same sense, and the thing known also is explained by the relation to its opposite". Example (a) illustrates the first interpretation, viz. the opposition is an opposition between the arguments of a relation, i.e. between x and y, where e.g. x is the double of y. It is, however, more difficult to interpret the example (b). Must the opposition between an 'x' and a 'y' be taken in mind here, viz 'is x knowledge of y' ? This interpretation could be taken, if one would argue that in the sequence 'knowledge of y', the element x is implicit.

If one does not accept this point of view, then on the basis of example (b), one must conclude that the correlative opposi-

tion is an opposition between the relation and its second argument.

This last interpretation can be confirmed by the following statement of Aristotle (Categoriae 10, 23-26): "Pairs of opposites which fall under the category of relation are explained by a reference of the one to the other, the reference being indicated by the preposition 'of' or some other preposition".

This remark shows that only the correlation between a relation and one of its arguments (this is only true if one is dealing with a relation with more than one argument) can be described in this way.

This manner of looking at the situation can only be understood if one considers the essential characteristics of the Aristotelian logic, where a relation together with its second (or more) argument(s) is considered as predicate of the first argument ([3]).

One could try to explain this approach on the basis of the strong influence the characteristics of the language, one is living with, had on the constructions of logic by Aristotle and the other ancient philosophers.

For, the relation between a relation and its second argument is mostly indicated by special syntactical means, e.g. a genitive or a special proposition. In fact, Aristotle has already mentioned this. The relation between a relation and its first argument at the other side is indicated in many cases by much more general means, as is illustrated by the sequences (1) and (2).

(1) John is Mike's father.
(2) John is ill.

The same syntactical means is applied for indicating the first argument of the relation as well as the argument of the predicate. This perhaps explains the tendency to treat them identically. So, '$\frac{father}{R}$ of $\frac{Mike}{B}$' is seen as a predicate, while it itself is composed of the two components R and B. An opposition is seen between R and B.

Another problem which arises about the correlative opposition is: "Is the characterization by Aristotle really clear ? Is it really true that both terms, whatever they are, of the opposition '1 L is the double of a $1/2$ L' are indeed explained by the reference to each other ?".

I.B.I.2. The second of the Aristotelian oppositions is the contrary opposition, which is introduced as follows: "Pairs of opposites which are contraries are not in anyway interdependent, but are contrary to one another. The good is not spoken of as the good of the bad ('), but as the contrary of the bad (Ross, Categoriae 1928, 11 b, 33-36)." Aristotle divides this opposition into two types.

I.B.I.2.a. The first type corresponds to: "Those contraries which are such that the subjects in which they are naturally present, or of which they are predicated, must necessarily contain either the one or the other of them, have no intermediate (Ross 1928, Categoriae 12 a, 1-3)".

I.B.I.2.b. The other type is formed out of the contraries about which no such necessity exists. They always have an intermediate (Ross 1928, Categoriae 12 a, 3-4).

Illness and health, odd and even are examples of the first type. Blackness and whiteness, badness and goodness are examples of the second type.

I.B.I.3. Now we come to the opposition between privatives and positives. The arguments of this type of opposition always refer to the same subject (Ross 1928, 12 a, 26). Privatives are concepts which indicate the absence of a faculty or possession in that in which, and at the time at which, it should naturally be present (Ross, 1928, 12 a, 28-32). Positives refer to having such faculty or possession. An opposition of this kind is 'sight and blindness' (Both have reference to the same subject 'the eye'). Aristotle also stresses that: "It is a universal rule that each of a pair of opposites of this type has reference to that to which the particular 'positive' is natural (Ross 1928, Categoriae 12 a, 28-29).

It seems interesting to us to mention here the Aristotelian arguments why the opposition between privatives and posi-

tives (I.B.I.3) cannot be reduced to the contrary opposition (I.B.I.2): "Of a pair of contraries such that they have no intermediate, one or the other must needs be present in the subject in which they naturally subsist, or of which they are predicated ... (Ross 1928, Categoriae 12 b, 26)". However, ... "In the case of 'positives' and 'privatives' on the other hand neither of the aforesaid statements holds good. For it is not necessary that a subject receptive of the qualities should always have either the one or the other; that which has not yet advanced to the state when sight is natural, is not said either to be blind or to see (Ross 1928, Categoriae 13 a, 1-5)".

Also the private-positive opposition does not belong to the contraries of the second type: "For under certain conditions it is necessary that either the one or the other should form part of the constitution of every appropriate subject. For when a thing has reached the stage, when it is by nature capable of sight, it will be said either to see or to be blind (Ross 1928, Categoriae 13 a, 7-10)". This is not true for the contraries of the second type.

I.B.I.4. The fourth Aristotelian opposition is the one between statements which are opposed as affirmation and negation (Ross 1928, Categoriae 13 b, 1-4). The characteristic which differentiates this type of opposition from the others is that: "It is necessary for the one opposite to be true and the other false (Ross 1928, Categoriae 13 b)". Aristotle emphasizes that this is not true for the contrary opposition: "At the same time, when the words which enter into opposed statements are contrary these more than any other set of opposites, would seem to claim this characteristic. 'Socrates is ill' is the contrary of 'Socrates is well' but not even of such composite expressions is it true to say that one of the pair must always be true and the other false. For if Socrates exists, one will be true and the other false, but if he does not exist, both will be false; for neither 'Socrates is ill' nor 'Socrates is well' is true, if Socrates does not exist at all (Ross 1928, Categoriae 13 b, 15-20)".

But in the case of affirmation and negation, whether the subject exists or not, one is always false and the other true.

For manifestly, if Socrates exists, one of the two propositions 'Socrates is ill', 'Socrates is not ill' is true, and the other false. This is likewise the case if he does not exist; for if he does not exist, to say that he is ill is false, to say that he is not ill is true.

I.B.II. Several roles of "not".

Aristotle differentiates in his 'Analytica Priora' (Book I, 46) between several roles for the sign 'not'. So he explains that 'to be not-white' is not the negation of 'to be white'. The negative is 'not to be white'. In other words, the use of the element 'not' does not automatically imply a negation. As such, 'to be not-equal' is not the same as 'not to be equal'. Interesting is the way Aristotle explains it: "For there is something underlying the one, viz. that which is not-equal, and this is the unequal but there is nothing underlying the other". And he subsequently states: "Therefore not everything is either equal or unequal, but everything is equal or is not equal (Ross 1928, Analytica Priora 51 b, 25-30)" ([5]).

I.C. *The roles of 'not' in modern logic.*

I.C.1. Von Wright: on the logic of negation.

Having taken into account the two functions of 'not' as exposed by Aristotle, Von Wright comes to the introduction of two kinds of negation, a weak and a strong one. The strong one is the 'not-P'. The weak one is 'merely' denial (Von Wright, 1959, p. 4).

I.C.2. It seems interesting here to look at Strawson's ideas about these problems (Strawson, P. F., 1952, p 7): "... When we notice that this function of exclusion is implicit in all descriptive uses of language, we should not find it surprising that language contains devices for rendering the function

explicit; devices of which, in English, the word 'not' is the most prominent. There are many very different kinds of occasion on which our primary concern is with the explicit exclusion of a predicate ...

What is common to such cases is that they create a need or a motive for emphasizing a difference rather than a resemblance. It is instructive to compare the use of 'not' with the use of those words which begin with negative prefixes, like 'intolerable'. These words bear their incompatibilities on their faces as surely as any phrase containing 'not', *but one would hardly say of them that they have the same function of explicitly rejecting a suggested description.* They do not point more emphatically to differences than to likenesses; *they rather serve to underline the fact that the two are complementary"* (⁶).

I.C.3. There is an analogy between Von Wright's 'not-P' and Strawson's 'un-P'. For, Von Wright (1959, p. 5) interprets 'not-ill' as 'well'. In other words as a complement of 'ill'. So we see that in Von Wright's interpretation of 'not-P' a complement idea is present. If we interpret the role of 'not' in 'not-P' as a kind of operation for producing the complement of P, or eventually the anti-pole, then it is, of course, clear why Aristotle sees an extreme difference between 'not equal' and 'not-equal'. For, the complement of 'equal' is also an affirmative and it is obvious that an affirmative can never contradict another affirmative. At the utmost, it can eventually be a contrary of another affirmative. It is, in this respect interesting to recall how Aristotle in his argumentation about the differences between 'not equal' and 'not-equal' *reduced* the last one to 'un-equal'. In this term the complement operation is more obvious.

Another confirmation of the necessity to introduce a complementary-operation as differing from a negation-operation is Zimmer's observation (Zimmer, 1964) (⁷) that when in a certain language the contrary of a certain term P exists, then mostly the 'affixnegation' can not be added to this term (In fact, the adding of a prefix to P gives rise to a possible word

in the language L to which P belongs, which is not realized; not used by the language community which uses L). This is rather easy to explain, if one interprets the 'affixnegation' as a polarization operation, viz. as an operation which in a certain dimension generates the most extreme value, contrary to the value of the term to which the affix was added. In this case, it is clearly a lexical redundancy to simultaneously have the contrary of a word and the affixnegation added to this word ([8]).

So, if the role of 'not' in 'not-P' or of an affixnegation is considered as a polarization operation (in the sense indicated previously), then P and 'not-P' are in a contrary opposition to each other. Pretending 'not-P' implies 'not P' would be the same as saying 'black' or 'grey' implies "not white".

Now it would be of importance to question which role of 'not' is represented by the negation in modern logic.

I.C.4. Russell

The same aspects can be found in Russell's writings as well. This results in the ambiguity of the term 'negation', because it can indicate the one as well as the other role of 'not'. About this, he writes: "La négation exprime un état d'esprit dans lequel certaines impulsions existent mais sont inhibées ... La négation est le rejet d'une proposition, c'est l'inhibition des impulsions qui pourraient engendrer la croyance en la proposition ... (Russell 1969, 233 en 278)".

So we see that on the one hand he interprets the negation as an exclusion, a rejection operation, while on the other hand in the same book we regularly find the assertion that 'non-P' equals 'P est faux' (P is here a proposition).

One can wonder if this last point of view is a third proposal of an entirely new type for interpreting the role of 'not', or on the contrary a special case of the negation seen as a polarization operation. The last interpretation seems to be confirmed by the following statement: "If one utters P, then factly one intends 'It is true that P'. If one utters '-P', then one means

'It is not-true that P', or in other words 'It is false that P'. This for the reason that 'false' is the contrary of 'true'."

This interpretation seems to have rather strongly influenced the modern logical approaches, where the negation only seems to be present as a polarization operation, but then only limited to the term 'true'. Then, however, the problem arises how to interpret 'true' and 'false'. Here a definition of 'false' which does not use the negation in its definiens is certainly important.

The use Russell makes of the negation as a polarization operation, however, illustrates, clearly that, as opposed to what Von Wright thought, the negation in modern logic (Von Wright thought here also of Russell) may not be identified with a general polarization operation (⁹) but rather with a polarization operation limited to the term 'true'.

I.D. *When are the elements of the several types of oppositions true or false ?*

Up to now we have found three different interpretations of the role of negation. Before discussing them, it may be useful to first of all go into some of the proposals for ascribing the predicate false or true to the elements of the several types of opposition more fully.

I.D.1. Aristotle's point of view.

Aristotle distinguishes the opposition 'privative-positive' from the contrary opposition on the basis of the characteristic, that in the latter the subjects, in which one of the elements of the contrary opposition is naturally present (or eventually an intermediate), necessarily must contain either the one or the other of them (or eventually an intermediate) (I.B.I.2.). In the former, however this is only true when a certain condition is fulfilled, so e.g. that the subject has reached a certain point in the development, etc... (I.B.I.3.).

But Aristotle's differentiation of the contrary opposition from the affirmation-negation opposition proves that the con-

trary opposition too must fulfil a condition for the above mentioned necessity to be true, (see I.B.I.4.) ([10]).

So we see that the necessity character only is unconditionally true in case of the affirmation-negation opposition.

I.D.2. Russell.

When treating this problem, Russell introduces a new element. So, the trichotomy 'true-false-meaningless' appears. His point of view is very well characterized by Von Wright (Von Wright 19, p. 6): "Those things of which, on Aristotle's view, it is true to deny that they are, say, white, but false to affirm that they are not-white, may, not unnaturally, be regarded as things to which the predicate "white" has no application whatever -not even for propose of denial. Subject and predicate simply "do not match". To say of a sound that it is white and to say that it is not equally make no sense. On this view, the distinction between things which are not white and things which are not-white vanishes, and its place is taken by a distinction between things which are not white and things of which whiteness cannot be significantly affirmed or denied at all."

I.D.3. Von Wright's arguments against Russell's solution are briefly summarized in the following quotation: "A disavantage of the Russellian trichotomy seems to me to be that it removes from meaningful discourse forms of expression which have a use and which furthermore may be said to have logical relations to admittedly meaningful propositions. The meaningful-meaningless distinction has been badly misused in modern philosophy, and should, whenever possible, be abandoned in favour of more discriminating logical tools. One such tool, I suggest, is a new theory of negation". (Von Wright, 1959, p. 7).

I.D.4. Strawson.

Strawson's solution seems to be an intermediate between the Aristotelian and the Russelian approach. In a discussion

about 'All John's children are asleep' he argues: "The more realistic view seems to be that the existence of children of John is a necessary precondition not merely of the truth of what is said, but of its being either true or false" (Strawson 1964, p. 174). This does not, however, mean that a sequence which does not fulfil this precondition — or as he will call it later on the presupposition — is meaningless. For he says: "It is important to understand why people have hesitated to adopt such a view of at least some general statements. It is probably the operation of the trichotomy 'either true or false or meaningless', as applied to statements, which is to blame. For this trichotomy contains a confusion: the confusion between sentence and statement. Of course, the sentence 'All John's children are asleep' is not meaningless. It is perfectly significant. But it is senseless to ask, of the sentence, whether it is true or false. One must distinguish between what can be said about the sentence, and what can be said about the statements made, on different occasions, by the use of the sentence. It is about statements only that the question of truth and falsity can arise; and about these it can sometimes fail to arise. But to say that the man who uses the sentence in our imagined case fails to say anything either true or false, is not to say that the sentence he pronounces is meaningless. Nor is it to deny that he makes a mistake" (Strawson 1964, pp. 174-175).

So, on the one hand Strawson comes very closely to Russell in the sense that he refuses to call such sentences 'true or false', but at the other hand he departs from the Russellian position with his disagreement for calling them 'meaningless'.

The Aristotelian conditionality on the privative-positive opposition and on the contrary opposition is enlarged by Strawson as also being true for the affirmation-negation opposition. It is also worthwhile drawing attention to the fact that nowadays the conditionality introduced by Strawson obtains more attention in modern linguistics and psycholinguistics. More about this later.

Strawson's motive for introducing the presupposition is

found in his attempts to escape the orthodoxical criticism on the Aristotelian logic.

He summarizes the orthodoxical criticism on the Aristotelian logic as follows: "It has become orthodoxy ... to maintain that the constants of the system cannot be given any interpretation such that (a) they have roughly the same meaning as in ordinary speech and (b) all the rules of the system hold good together for the interpretation. It is in other words, maintained that no consistent interpretation can be found for the system as a whole which approximates to the naive interpretation" (Strawson 1964, p. 164).

Strawson extensively discusses this criticism which centers essentially on the dilemma whether the A and E the forms of the propositions (A = all x is y; E = No x is y) have existential implications or not. For: "If they do, one set of laws has to be sacrified as invalid; if they do not, another set has to go. Therefore no consistent interpretation of the system as a whole, within the prescribed limits, is possible".

I.E. Attempts for solving the problem about the interpretation of the constants of the Aristotelian logic in such a way that the Aristotelian solution I.D.1. can be conserved.

I.E.1. Introduction.

We could ask ourselves if there is no other type of solution than the Strawsonian one to avoid the orthodoxical criticism on the Aristotelian logic. A solution which keeps more closely to the Aristotelian.

First of all we need to be aware of the fact that the dilemna, which forms the orthodoxical criticism, is based on an interpretation of the Aristotelian system in terms of modern class or predicative calculus.

The basic forms of the Aristotelian system are:

A : All x is y
E : No x is y
I : Some x is y
O : Some x is not y.

Strawson (1964, p. 167) illustrates that if one interprets these forms as in table I, one gets into trouble if one does not introduce the notion 'precondition'.

Table I

	or	
$\sim(\exists x)(fx . \sim gx)$	or	$(x)(fx \supset gx)$
$\sim(\exists x)(fx . gx)$	or	$(x)(fx \supset \sim gx)$
$(\exists x)(fx . gx)$	or	$\sim(x)(fx \supset \sim gx)$
$(\exists x)(fx . \sim gx)$	or	$\sim(x)(fx \supset gx)$

The same is true, if one adds to the interpretation of A and E as a conjunctive member, the specification $(\exists x)(fx)$. We can, however, wonder if this translation of these Aristotelian forms is the only possible translation.

What is very peculiar to these modern formulations of these forms, is that the quantification is strongly connected with the existential specification: a negative or a positive specification.

I.E.2. An existence-free quantification ?

One could ask oneself if in principle it is impossible to disconnect the quantification from the existence specification. If this would be possible, then one could specify interrelations between quantifiers in independence on considerations of the existence of the elements which are in the domain of the quantifiers.

A question which immediately arises, is why introduce an existence-free quantification ? It is clear that I can imagine one unicorn, although I know that no unicorn exist. In can even fancy two unicorns. I am also able to make assertion about all unicorns. Of course, characteristics which are valid for each of the elements which are unicorns, are also valid for one such element or for some of them.

Nevertheless, we mostly quantify existing elements and not imagined elements (such a difference in a conceptual frame-work is treated in II.D.). As a result of this, it is easy to under-

stand that there is a strong connotation of existence connected with the notion 'quantification'. But, this does not imply that quantification and existence are unseparately connected to each other in all their meanings.

I.E.3. The interpretation of 'A is B'.

Another important problem here is why the translating in modern logic of sequences as (1) 'A is B' the implication form 'fx → gy' is used, combined with a quantification ?

The explanation can perhaps be found in the translation (2) of (1).

(2) the element x that is f, is g.

In other words, one isolates the typical characteristics of x and y. This happens by the intermediary of a relative sentence. Here, however, the problem arises about the ambiguity of a sentence such as (2). A relative sentence (see Vandamme 1971) can be interpreted — as far as the interpretation of its relation to the main sentence is concerned — in at least three ways: (a) as an implication (if ... then), (b) as a conjunction (eventually an ordered conjunction), and (c) as a restrictive meaning, illustrated in (3).

(3) The man standing near the door, is happy.

One can distinguish two kinds of operations on concepts, viz. those giving propositions as result, and those which do not. Predication to a constant — otherwise we get a propositional function — belongs to the first kind; quantification (without existential implication and not accompanied by a predicate and particularization (or restriction operation) belongs to the second kind.

As a result, if we translate the particularization or restriction operation on an object as (xf), then (1) will be formed as (4).

(4) g(xf)

In (4), 'xf' is not a propositional function, but it isolates an object. g(x) and g(xf) are propositional functions. So if a constant is substituted for x, then g(x) and g(xf) will be true or false. This is not the case for (xf). (xf) will result in a determinated constant. It is also important to note that in g(xf), 'g' is not a predicate modifying (xf). It only modifies the x which is particularized by means of f.

The restriction operation can clearly be linked to the well-known ɿ-operator, used for descriptions and interpreted as 'the one and only object x such that ...'. This ɿ-operator is also an operator which results in indicating more specified objects — here in fact intended for indicating 'one' object.

Besides the classical implication translation, one could also translate the basic Aristotelian forms by making use of the restriction interpretation.

The third possible interpretation is the interpretation of (1) as $(x) (fx . gx)$.

The restrictive interpretation seems to us rather interesting in principle, because it avoids to translate A as a proposition f(a). It is on the contrary an operator which results in a more determined object as said already. In this sense it is more near the speaker's intuition.

I.E.4. The laws of Aristotelian Logic in the light of the newly introduced interpretations.

With the interpretations of quantifications proposed above as existence-free and the restrictive interpretation of the relative sentence, is the orthodoxical criticism on the Aristotelian logic still valid ?

Let us translate the Aristotelian basic forms in the new interpretation.

A	All x is y	(Ax) g (xf)
I	Some x is y	(Sx) g (xf)
E	No x is y	— (Ex) g (xf)
O	Some x is not y	(Sx) — g (xf)

I.E.4.a. Conversion.

Simple conversion of E and I obviously holds for this interpretation, but not for A and O. Simple conversion (Strawson 1952, p. 156) consists in transposing the subject and the predicate, the quality and the quantity remaining unchanged.

In our formulation:

1. — (Ex) g (xf)   →   — (Ex) f (xg)
2.   (Sx) g (xf)   →   (Sx) f (xg)
3.   (Ax) g (xf)   →   (Ax) f (xg)
4.   (Sx) — g (xf)   →   (Sx) — f (xg)

1 and 2 are valid, while 3 and 4 are not. We want to note that in the existential interpretation (EI) of the quantors and also for the implication interpretation (II) in the predication relation, a good result is obtained, (thus in the classical logical formulation). This is not true for the conversion per accidens.

I.E.4.b. Conversion per accidens.

This consists of transposing the subject and the predicate of a statement and changing its quantity from universal to particular, the quality remains unchanged. In other words:

$$x \, A \, y \supset y \, I \, x$$
$$x \, E \, y \supset y \, O \, x$$

This is in our interpretation:

5.   (Ax) g (xf)   →   S(x) f (xg)
6. — (Ex) g (xf)   →   (Sx) — f (xg)

It is clear that 5 and 6 are true from the moment one introduces the existence-free quantification (EFI). The introduction of the restrictive interpretation (RI) has no special function here.

I.E.4.c. Obversion.

This consists in negating the predicate and in changing the quality of the statement, subject and quantity remain the same.

Thus we have
$$x \, A \, y \supset x \, E \, y'$$
$$x \, E \, y \supset x \, A \, y'$$
$$x \, I \, y \supset x \, O \, y'$$
$$x \, O \, y \supset x \, I \, y'$$

In our formulation we respectively get:

7.	(Ax) g (xf)	→	— (Ex) — g (xf)
8.	— (Ex) g (xf)	→	(Ax) — g (xf)
9.	(Sx) g (xf)	→	(Sx) — (— g (xf))
10.	(Sx) — g (xf)	→	(Sx) — g (xf)

The obversion does not imply any difficulty neither for our interpretation, nor for the classical logical interpretation, viz. with EI and II.

I.E.4.d. The square of opposition.

This consists if the doctrine that A is the contrary of O, and E of I. A and E are contraries, I and O are subcontraries. A entails I, and E does O. These laws are:

1. $x \, A \, y \equiv - x \, O \, y$
2. $x \, E \, y \equiv - x \, I \, y$
3. $- (x \, A \, y \, . \, x \, E \, y)$
4. $x \, I \, y \lor x \, O \, y$
5. $x \, A \, y \supset x \, I \, y$
6. $x \, E \, y \supset x \, O \, y$

In the existence interpretation of the quantifiers, the laws 3 to 6 are not true. When using the existence-free quantifier this is avoided. For in this case, if for each of all n objects of a certain kind, a certain characteristic is valid, then it is also valid for one or some of them.

I.E.5. Conclusion.

We see that the orthodoxical criticism on the Aristotelian system disappears, if we interpret the quantifier existence-free. This means that the Strawsonian motivation for introducing the notion 'presupposition' certainly is weakened. For it is only in case of using the existential interpretation of the quantifiers that any needs exist for this type of approach.

It became also clear that the restrictive interpretation of the Aristotelian basic forms did not play any role in avoiding the orthodoxical criticism.

I.F. *The problem about the relation between logic and natural language.*

To what extent are natural language, logic and sciences in general related with each other? As a matter of fact, it is clear that in the natural language many differentiations and characteristics are communicated. The modern logic and the sciences in general illustrate how by making abstraction of some of these differentiations in the natural language (e.g. by considering them as not permitted elements in their system) or by scientific reconstruction (Carnap, 1950) of them, it is sometimes possible to construct very interesting and eventually useful systems.

In a first rough approach, one could call the specific sciences, constructions inside the field of possible constructions of the differentiations, characteristics and relations communicated by the natural language. They are constructions in which some of the important characteristics communicated by the natural language may be neglected. And what is of greater importance, this neglection can be very useful. Another point is that combinations of the characteristics communicated through the natural language, are not restricted by the natural language. Natural language permits contradictions, etc. It never restricts the possible combinations of differentiations, characteristics or relations which it communicates. What it does, is putting

restrictions on the combinations of word categories, etc...;
in other words, on communication categories, but never on
the content of what it communicates ....

On the other hand, of course, the natural language is not at
all normative about the possible combinations of characte-
ristics, etc... which it communicates.

An illustration of this is found in section I.E.. In modern
logic, the restrictive interpretation of the relative clause is
neglected or rather considered as non-permitted. One prefers
the implication interpretation of the relative clause, although
both interpretations are rather infrequently used in language.
Most common is the conjunction interpretation (Vandamme
1971 a).

But nevertheless, we see that a fairly interesting and power-
ful system is obtained. Of course, it is clear that other
approaches can be made, e.g. a system with the restrictive
interpretation. "Which system is most powerful, useful, con-
sistent, etc... ?" is an empirical question.

The analogous problem arises about the question which
interpretation must be chosen for the logical constants (e.g.
negation); the quantors, etc... In natural language, several
interpretations are communicated. Aristotle prefers the ex-
clusion negation. Russell seems to use a special form of anti-
nomy negation (a subspecies of the polarization negation).

Here, the same empirical question can be asked, which inter-
pretation will be most useful in the context, now under discus-
sion. Al kind of conceptual differentiations are gathered in
natural languages, etc... which ever come in the human mind,
and which were important enough to be communicated. These
were so many communicated, that they belong to the domain
of the intersubjective language of the group who is using the
natural language.

From this, it becomes clear that a natural language is
dynamic, as far as the content is concerned, it can commu-
nicate. New characteristics, etc. can be taken up and old ones
will disappear, when they come or stop to belong to the general
knowledge of the speakers of the natural language in question.

So, it seems to us that we can see natural language in its

communication of characteristics, relations, differentiations, etc... as a heuristical and very interesting basis for constructing systems ! *However, a open basis of possibilities from which one has to choose. The choice does not need to be limited by this basis.*

II. *THE EXCLUSIVE NEGATION*

As we have repeatedly illustrated in part I (Aristotle, Von Wright, Russell, Strawson), one role of the negation which is mostly distinguished, is the exclusion. This role is sometimes called "denial, inhibition, exclusion, etc...". Here we will make a proposal for an interpretation of this role of negation.

II.A. *The interpretation of the exclusive negation.*

One could ask the question, what could be the role of the negation in a semantic model (see Vandamme 1970 b). There we discussed the notion semantic model and made an outline of some principles for simulating it. The notion 'register' seems very fruitful. We will not go into more detail about this here). In a first approach, it can look attractive to consider it as a meta-modelelement, viz. as stating the absence of a relation or an element in the model. So the negation in "John is not ill" would indicate, that in the model there is no modification relation between 'John' and 'ill'. Such an expression of an absence of a relation in the model is a meta-expression about this model and therefore it is not itself in the model.

This interpretation of the negation is perhaps useful for explaining some uses of the negation in the natural language. But it is certainly insufficient for many uses of the negation. For, if somebody says "John is not ill", then he does not intend to communicate that in his model, there is no modification relation between 'John' and 'ill'. In other words, he does not want to inform us that in his model nothing is said about the relations between 'John' an 'ill'. On the contrary, he intends to communicate that in his model, the modification relation between 'John' and 'ill' is excluded, inhibited, etc...

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This interpretation of the negation is therefore too weak. It would only indicate the indeterminateness in the model. How to describe the alternative interpretation of the negation as an exclusion or inhibiting operation in the model ?

II.A.1. Before making a proposal, let us first discuss the activating strategy in the model in general. Any element is activated, when the activation value surpasses a certain threshold. The threshold must not necessarily be identical everywhere in the model. We have already discussed (Vandamme 1970 b) a rather complex activation strategy, taking into account the role of the extra- and intralinguistic context.

As a result of this approach, a relation R between two activated element A and B will also be activated, if its threshold a_1 is surpassed.

II.A.2. Taking this into account, one could interpret the exclusive negation as an operation which raises the threshold of what is negated. So in a negation of R, the threshold a_1 will be increased. Here the problem arises how much must the increase of the threshold be ? An infinite or a finite value M ? In the former case, an activation of R could only be got by a new operation on the threshold-value. In the latter, however, with a sufficient increase of activation -values of relation R (e.g. by repeating the affirmative), finally the R could also be activated when the new threshold $(a_1 + M)$ is surpassed. This could perhaps be avoided by hypothesizing negation operations by the listener, parallel to each affirmative statement (adding activation values). A result of this could be that a doubter, after receiving enough affirmations (activations) of R, would have R activated. This because he reactivates his

negation claim (viz. adding M to get the threshold values of R increased) weakly or not at all. This is not a strange result, except after an operation on the threshold itself.

For simplicity sake, let us hypothesize that a threshold by the negation-operation becomes infinite. This means that an element with such a threshold cannot become activated.

II.A.3. We can represent the negation in the model as threshold raising operation by connecting the threshold a_1 to an element c (by which a switch is opened or closed).

```
A _____ B
    R       |
            |
            |
            |
      a_i          C
        S_i   |
              |
              |
              |
            a_j
```
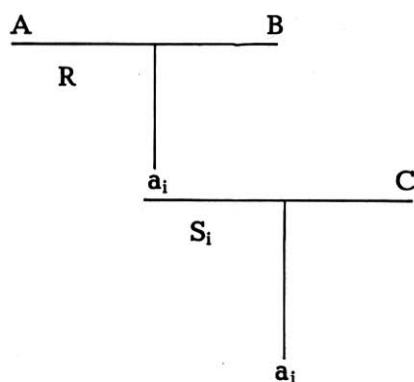
In this approach a negation is itself relational, viz. a relation S_i connects the threshold of the negated element to a certain constant S. So it is clear that (A) the negation itself has an activation threshold a_j, and that (B) the negation itself can be negated.

II.A.4. The negation of a negation.

a_3 is here the threshold of S_2. We have here the negation of a negation as an exclusion or an inhibition of a negation. So we get a representation of — — R(AB). It is, of course, clear that it is even possible in this framework to introduce the double negation as a strengthening of the first negation. This phenomenon is found e.g. in Greek and West-Flemish. The double negation in this function can be seen as an extra raise of the threshold.

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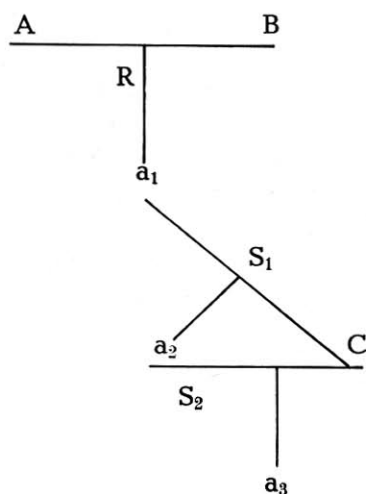
II.A.5. The activation of S_2 too can, of course, be excluded. In this case, we have $———R(AB)$, which can be represented as follows:
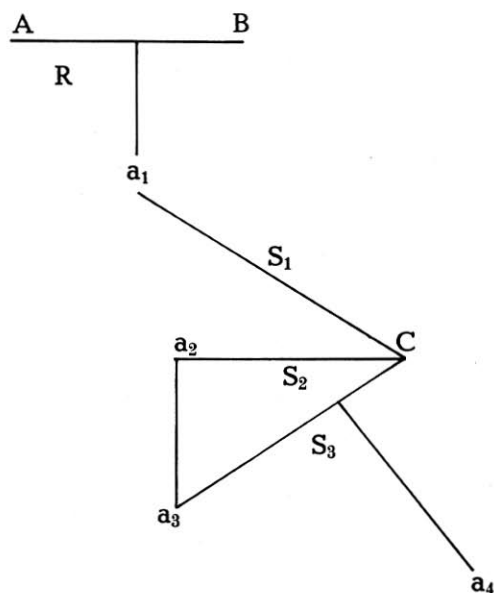
This process can, of course, continue.

II.A.6. What are the consequences of a sequence of negations as far as the activation of the R is concerned ?

II.A.6.a. It is clear that in — R(AB) the activation of R becomes impossible by reason of the activation of S_1. So, we can say:

$$- R(AB) \rightarrow - MR(AB).$$

II.A.6.b. In case of double negation '— — R(AB)', it is the negation or the exclusion operation which becomes impossible through the activation of S_2. This implies the possibility of the activation of R, but certainly not its necessity of being activated. This means that (1) is valid, but not (2).

$$(1) \ - - R(AB) \rightarrow MR(AB)$$
$$(2) \ - - R(AB) \rightarrow \quad R(AB)$$

In other words, the double negation of R implies only the possibility of the activation of R. We know that the activation of R is only possible, when the activation threshold a_1 has been surpassed. When we accept — — R(AB), then the sole happening is that the exclusion of — R(AB) is activated and accepted. It does not mean that the threshold a_1 is surpassed. The operation of inhibiting the surpassing of the threshold a_2 is in principle neutral as far as the activation a_1 is concerned. Of course, in many circumstances on the basis of association relations, it is possible that in fact the inhibiting of a_2 results in the surpassing of a_1. But, in principle, this is not necessary.

In general, we can say that given — — R, the activation of R will be dependent on the activation value which it normally would have in the model, taking into account the context, if — — R has not been given. So, if it would have a value which surpassed the normal threshold, R will be activated. If it is slightly or much below the normal threshold, then R is not activated, etc...

II.A.6.c. In case of — — — R(AB), the exclusion of the exclusion of R is impossible. In other words, the exclusion of activa-

ting — R(AB) is possible, but also not necessary. Therefore (3) is valid, but not (4).

$$(3) \quad \text{———} R(AB) \rightarrow M - R(AB)$$
$$(4) \quad \text{———} R(AB) \rightarrow \quad - R(AB)$$

II.A.7. Conclusion.

The exclusion negation is seen here as *an operation on a relation between two or more elements.* (Another question is: May the exclusion negation only operate on relations ?)

The consequence of this operation is that *certain kinds of other operations cannot be executed on the negated relation.* In the example mentioned, it was the activation operation which became impossible. Eventually, other interpretations could be made, e.g. realization instead of activation, etc... In this sense, negation seems to be *a special type of inhibiting operation.*

With this structure in mind, let us look at other phenomena, which generally are described when using the terms 'negative, negation, etc...', and see if the same structure can be found in them. We shall combine this investigation with some thoughts on the problem about the genesis of negation and the place of the concept-negation in this evolution.

II.B. *Some thoughts about the genesis of negation.*

II.B.1. External negation.

Russell (1969) in his 'Signification et Vérité' explicitly differentiates the 'mots-objects' (word-objects) from the 'logical constants'. To him, both belong to another level. Among the 'mots-object, he states also verbs 'run, cry, etc.' and prepositions (relations) 'in, on, etc...'.

He describes the 'mots-óbjet' as follows: "L'essentiel pour un mot-objet, c'est qu'il ait une certaine ressemblance au sein d'un ensemble de phénomènes suffisamment frappante pour établir une association entre les illustrations de l'ensemble et

celles du mot affecté à l'ensemble, la méthode d'instauration de l'association étant que pour un certain temps, le mot est fréquemment entendu quand un membre de l'ensemble est vu."

As far as the conjunction, the negation etc... are concerned, Russell (1969) argues that there is a big difference: "Notre énoncé est a propos d'énoncés, et ce n'est qu'indirectement qu'il a trait à des objets".

More concretely: "Pour juger 'ceci n'est pas du fromage' vous devez préalablement avoir dans l'esprit le mot 'fromage' ou quelque équivalent. Il y a rupture entre ce que vous voyer et les associations du mot 'fromage' et ainsi vous portez le jugement: 'ce n'est pas du fromage'."

One could wonder, if there is no kind of exclusion negation (with the previously proposed structure (see II.A.)), which is also a 'mot-objet' in the sense of Russell. Take the following situation: 'A child reaches for a piece of pie'. The realization of his action can be prevented by the exclamation 'No !' by his mother, accompanied by a slap on his hand or another operation which makes it impossible for the child to get the pie. The realization of a specific kind of relation between the pie and the hand of the child is prevented. This kind of exclusion operation we could consider as a first type of negation.

Of course, such negation is an observable operation that can be experienced. Therefore, it seems also to belong to the Russellian 'mots-objet'. For all the operations covered by this kind of negation interpretations form a set of phenomena with a functional resemblance to one another.

It will also be evident, that in this type of pragmatic negation (a specific realization of the general exclusion negation structure; and probably also the first negation the child experiences and learns) the denial of the negation does not imply the realization of the action.

This can very easily be demonstrated. If the child reaches for the pie, and if a person A takes action to prevent the child from taking it, but person B interacts and stops A from preventing the child to take the pie, then it is clear that the child

will not necessarily succeed in the realization of his action. He can stumble, etc...

Another interesting topic is the relation between this kind of negation and punishment (this came to our attention through a discussion with M. De Mey). Of course, a negation in this sense does not necessarily imply punishment. However, it can be part of the punishment operation. If it does or if it does not will be dependent on context. This is another example of the importance of the introduction of a general context theory.

II.B.2. Interiorized negation.

II.B.2.a. Wants — negation.

A second form of negation seems to be an interiorization of the first one (II.B.1.), viz. of an externally observable pre-venting or inhibiting operation. It seems to us that the Freudian negation can be interpreted as such an interiorization.

We have based our interpretation of Freud essentially on Beulens (1970). Endogenous stimuli are led away. They can only be neutralized by appropriate action.

II.B.2.a.1. The endogenous stimuli could be represented by x and the appropriate actions by y. R is the relation between both.

$$ x \xrightarrow{\quad R \quad} y $$

In this way, a want, a wish, etc. can be represented as an 'x' directed towards a certain 'y'. y can eventually be a set of appropriate actions or a more or less unique element. This depending on the kind of x. It is also trivial to mention that, whereas a human being possesses several x's, the possibility exists that he can be directed to several y's, which can not be realized simultaneously, or even of which the realization of

one y_i makes the realization of another y_j impossible, etc.
II.B.2.a.2. The notion 'activation threshold' can be introduced here rather usefull. When R is activated, then x is actively directed towards y.

II.B.2.a.3. The operation of increasing this threshold could be seen, here again, as the function of the negation operations. This kind of operation results in preventing x being actively directed towards the realization of y. In this sense the negation is clearly an operation of repression. In this approach, it is very easy to understand the essential relation which as Freud asserts (Beulens, p. 12) exists between negation and repression.

II.B.2.a.4. It is clear that the possibility exists for directing x towards another (or other) y('s) (displacement and sublimation), when R is negated (repressed). As a result of this, the relation between sublimation and displacement is easy to understand in this framework.

II.B.2.a.5. Of course, one can eventually be conscious of the negation of a certain R. Which means conscious of the negation operation S_1. This must clearly be distinguished from the activation of R. It is obvious that Freud is right that this consciousness in principle does not need to result in the disappearance of the displacement. For, the displacement is a result of the negation, and by being conscious of it, this negation does not need to disappear, even if one should want this cognitively. It is also true that an R only can be negated, if x is directed to y. This is not paradoxal.

For, the negation does not destroy the tendency (and even the tendency must exist before it could be destroyed), it only prevents its realization.

The displacement can eventually be broken up (a) if the activation of R becomes so high that the threshold is nevertheless surpassed, or (b) of the threshold is decreased, e.g. the relation S (see II.A.) is weakened.

II.B.2.b. The concept-negation.

This negation is introduced into the conceptual model of a human being. Here too — as it was the case with the wants-negation — the negation originates in the pragmatical, social context. It is introduced and used in a more abstract way in the congnitive model as an operation on its own action patterns. The cognitive model permits us to try internally some actions and to foresee its results. So one attains more control and is already able to make evaluations before the actions effectively occur. The use and importance of the negation operation in this model will be discussed in part III. The framework of the negation given in II.A. suits the concept-negation operation very well.

In the light of this interpretation, we will later on look at the problems: (a) if other forms of concept-negation operations must still be introduced II.C., (b) the problem of the non-existence paradox, and (c) how to differentiate the concepts 'true, false, not true, not false...'. In this discussion, of course, we will always take into account the ideas proposed in I.F., about the interrelations between natural language, logic and sciences.

II.B.3. Negation in society.

Proceeding from the pragmatic negation, treated in II.B.1., it is easy to arrive at a social group's negation. A certain group X intends the realization of a certain Y, so we have R(XY). In this framework, the negation exists in preventing X for realizing Y by another group Z. The destruction of X is certainly not the negation, although this can eventually be the result of preventing the realization of Y. In general, the consequences of the negation for X and for its actions in general will be influenced by context.

Taking the earlier approach of thresholds into account, one can again introduce degrees of negations. The complete negation then becomes the complete prevention of the realization. In the system with thresholds, this could be guaranteed (a) by increasing the threshold practically up to infinite; or

(b) by increasing the threshold every time the activation value of R is increased, etc. In the latter case, any new attempt for realizing Y is accompanied by a new adequate action for preventing the realization of Y. In the former, an adequate action is made — if this is possible — which makes the realization Y by X completely impossible.

Weaker forms of negations could be actions which hamper the realization of Y for X, or make the realization less probable, etc...

So, it seems to us that the use of the terms 'negative, negation, etc...', as they occur in descriptions of and in theories about social actions and interactions must be understood in the framework of the exclusion negation as proposed in II.A.7.

We want to illustrate this by referring to a study on 'Utopia' (Plattel, 1970). Plattel (1970, p. 47) argues that every concrete utopia renounces (this he calls negation) the existing world (for him the affirmation) and aims at a better future (that is for him the negation of the negation).

Do we here again find the discussed framework of the negation ? Negating is here an action of renouncing something. What is renounced ? The validity of the actual situation, the intentions or actions to perpetuate the actual world, etc. This negation is itself negated in an utopia, not for restoring the negated situations (which in principle is possible), but rather for making a new situation to construct a new world possible. It is obvious, that the denial of a negated situation X does not imply the situation X. In fact, in order to have an utopia, it is necessary that the denial of the negation does not result in an affirmation of the existing world.

II.C. *Only a relation-negation ?*

In part I, we mentioned how Aristotle (I.B.II.) and Strawson (I.C.3.) have made a differentiation between the affix negation and the true negation. The affix negation seemed rather to be a polarization operation and therefore not a true negation.

We do not speak about this kind of phenomena here. We are only asking if it is necessary and justified to introduce several kinds of true negations.

As an argument for several kinds of negations, one could propose the following one. Take a sentence such as (1) of the type — R(AB):

(1) John does not eat bread.

and ask someone to draw a picture, representing the happening, one can get many different types of dessigns. (¹¹) One can have e.g. structures such as R(AC), S(AB) or R(CB), which respectively refer to (2), (3) and (4).

(2) John eats meat.
(3) John makes bread.
(4) Harry eats bread.

So three possibilities are already obtained, taking only one substitution into account. One can, of course, substitute two or even three elements. All these possibilities are open when a structure R(AB) is negated.

A possible explanation could be that, in its dependence on the alternatives which are made to the negated sentence, the negation operates on the subject (the first argument of the relation), on the relation, or on the object (second argument of the sentence), etc... This means that a sentence such as (1) is n-ambiguous, because the negation can operate on many different elements.

Another and a more economical approach could be made, in which the negation always operates on the relation (a predicate being a special case of a relation). Of course, when the relation is negated on basis of the context, alternative relations can be chosen. However, the negated relation itself does not, strictly speaking, imply another one. 'John does not eat bread' does not imply 'John eats nothing' or 'he eats meat', etc... All these alternative relations are doubtlessly possible or probable, even if or eventually because 'John does not eat

meat'. If an alternative must be chosen — which is the case, when a drawing is asked of a negative sentence — then the choice will be dependent on the context.

Some examples of the important role, context plays, are discussed in "Een algemene Wetenschap van de Communicatie" (Vandamme, 1971).

Which benefit does the last hypothesis carry, by which the domain of the negation operations is limited to relations ? In the alternative approach, one obtains a rather clumsy complexity. Many different kinds of negations need to be introduced. Negations which are contra-intuitive and impossible to be linguistically justified. What is still worse, is that the context is necessary to determine which kind of negation one has to do with: the negation of the first or the second argument or of both, etc... But, if context seems to be sufficient and necessary for introducing the needed differentiations, why introduce extra-complexity by distinguishing several negations, although one does not gain explanatory power.

One could try to refute this argumentation by arguing that there is in fact linguistic evidence for several negations. Negations which must be differentiated from one another in dependence on the elements they are operating on.

As evidence, one could in Dutch refer to the difference between 'niet' (not) and 'geen' (no + noun). So one could try to state that 'niet' (not) is a relation negation, while 'geen' (no + noun) is an argument negation.

This point of view is easy to refute. Sentence (5) poses the same problem as (1), viz. the same possibilities of alternatives arise.

   (5) Jan eet geen brood. John eats no bread.
   (6) Geen brood eet Jan. No bread eats John.
   (7) a. Geen man slaat een kind. No man slaps a child.
        c. Geen cent heeft Jan gegeven. John gave no cent.
        b. Geen man heeft Jan gedood. John killed no man.
        d. Geen man draagt een hoed. No man wears a hat.

If (5) is true, then John can eat meat and Harry can eat

bread, etc. That a specific alternative is not presupposed (the notion 'presupposition' was discussed previously) is illustrated by (6) and (7a, b, c, d).

When in normal context, (6) is heard with a stress on 'brood' (bread), then one is perhaps inclined to think that John eats something in any case. Therefore the alternative is rather of the type of sentence (2).

Sentences (7 a to d) illustrates clearly that this alternative is only one among several possible ones. In fact, the alternative 'Somebody slaps a child' or 'John has killed in any case something, but no man' is certainly not very strong as far as sentences (7 a to c) are concerned. This proves that the explanation of the type of inclination in (6) must be found in performance, viz. that the use of a sentence such as (6) mostly occurs in a situation, where the alternative of type (2) is present. We must, however, — in view of 7 a to d — take care not to consider the inclination to an alternative of a certain type as a result of the structure of sentence (6), nor as a presupposition for the interpretation of such a structure.

This illustrates, how on this account 'geen' (no + noun) cannot be distinguished from 'niet' (not). Therefore, it is not an argument for introducing several negations, which the relation negation is a special type of.

It must, however, be stressed that a difference in value exists between 'geen' (no + noun) and 'niet' (not).

(8) Ik zie twee mooie paarden niet.
    I do not see two beautiful horses.

(9) Ik zie geen twee mooie paarden.
    I see no two beautiful horses.

(10) Ik zie de groene doos niet
    I do not see the green box.

(11) Ik zie geen groene doos.
    I see no green box.

(12) Ik zie de paarden niet.
    I do not see the horses.

(13) Ik zie geen paarden.
    I see no horses.

In Dutch, sentence (8) means that of two specific beautiful horses it is true that I do not see them, while eventually I can see two other beautiful horses. Sentence (9), however, signifies that I do not see any two horses which are beautiful, or still in other terms, for any two horses, I see, it is not true that they are beautiful. The same difference can be repeated for (10) and (11), and for (12) and (13).

How can this difference be formally represented ? (12) could be represented as (14) or (15). (15) is a representation in the restrictive formulation.

(14) — R(AB)
(15) — R(xf, yg).

How to represent (13) ? One could do it by means of (16)

(16) — (Ex) (f(x) → R(Ax))

or in the restrictive interpretation by (17).

(17) — (Ex) (R(A(xf))).

However, it is important to note that, if one does *not* consider Ex and (x) as judgements which can be true or false, as they stand there alone, even if a constant is substituted for x, then one could argue that the quantors are, strictly speaking, rather elements of a restriction relation. (^{12}) What kind of existential restriction relation it is, will be discussed in the treatment of the non-existence paradox. So, in consistency with the symbolization convention introduced earlier, one would strictly speaking have to represent them as 'xE' (one x) and 'xA' (all x's).

Taking this into account, what is then the difference between (12) and (13) ? In (12) a propositional relation is negated. In (13) rather a restrictive relation, introduced in the existential quantifier seems to be negated. This accounts for the differences noted in sentences (5) to (13). This characteristic of negating a restrictive relation in an existential quantor instead

of a propositional relation seems to be generally true in Dutch, in English, etc. for all cases, where 'niet' (not) is contracted with some other morpheme. Examples of this kind are 'nobody, nothing, never, ...' (niemand, niets, nooit, ...). So (18 and (19) can respectively be represented as (20) and (21).

(18) Iemand eet het brood niet.
(19) Niemand eet het brood.
(20) (Ex) — R(Ax)   (A = iemand)
(21) — (Ex) R(xB)   (B = brood)

We have something analogous in (22), which can be represented as (23).

(22) Niemand eet nooit niets.
(23) — (Ex) — (Ey) — (Et) R(x y t).

It is necessary to take into account the importance of the ordering of the quantors. In this sense (24) is surely different from (25).

(24) Ooit, ziet niemand Jan.
(25) Niemand ziet ooit Jan.

They can be represented as (26) and (27).

(26) (Et) (— Ex) R (A x t)
(27) (— Ex) (Et) R (A x t).

The importance of the order of the quantifiers has also been neglected in many linguistic analysis. So Chomsky and Lakoff take as an argument that the passive transformation changes meaning, the fact that sentences of the type of (28) differ from those of (29).

(28) Weinig mensen lezen veel boeken.
     Few people read many books.
(29) Veel boeken worden door weinigen gelezen.
     Many books are read by few people.

It is true that (28) differs from (29). But the same difference is found between (30) and (31) in Dutch.

(30) Veel boeken lezen weinig mensen.
(31) Weinig mensen lezen veel boeken.

In (30), the object precedes the subject. In (31) the subject precedes the object. This means that the quantifier 'veel' (many) precedes in (30) the quantifier 'weinig' (few). In (31) the opposite is true. The differentiation between (28), (29) and (30) and (31) is easily explained, if one takes the order of the quantifiers into account. This means that the semantic structure of (28) and (30) must be different from (29) and (31). The difference in meaning is not dependent on the passive operation, etc. Rather one could argue that in order to generate a sentence on the basis of the meaning structure of (29), such a structure needs to be chosen by which the quantifier 'veel' (many) precedes the quantifier 'weinig' (few). The reason is that in interpretation the first met quantifier receives priority to the following one (Vandamme, 1971 a, for such an analysis). This is another argument for the hypothesis of two independent linguistic competences: a generative and an interpretative one (in feedback relation with each other).

Following this approach the structure of (28) and (29) is respectively (32) and (33) (for justification see Vandamme 1971 a).

(32) (Mensen ⊓ weinig) ⊗ (boeken ⊓ veel)
       ⊗ mensen ⊓ lezen/boeken
(33) (boeken ⊓ veel) ⊗ (mensen ⊓ weinig)
       ⊗ mensen ⊓ lezen/boeken

Of course, the interesting topic is also the combination of the restrictive negation with the propositional relation-negation (PRN), e.g. 'Niemand eet niets niet' (nobody does not eat nothing) (— Ex) (— Ey) — R (x y).

More difficult is the problem, of course, of determining equivalents. To answer this, it will be important to ask in which context. For, of course, we shall need to consider the nature of the relation between natural language and logic (see I.F.).

We have already seen that the restrictive negation permits the same possibilities of alternatives as the propositional relation-negation. In other words, it does not as such direct the choice of the alternatives, as is often wrongly believed.

The reason why is easy to understand. From certain restricted objects (e.g. objects which are horses) it is said that they cannot be restricted (by the quantifier) in such a way that a certain relation R is true for this object. But, of course, this needs not to be true for other relations or eventually for other objects restricted by other predicates (e.g. cats) as far as the same R or other relations are concerned.

The believe that a special kind of positive alternatives is the result of a negation, is also the basis of many wrong approaches to the negation phenomena. Sometimes one uses as criteria for determining whether the negated sentence is understood, the generation of the right alternative to the negated sentences. But one can certainly have understood a certain negation without being able to make the choice of an alternative. In such a choice, the context needs to play a rather important role. This is also discussed by Wason (1971).

If the context is not specified, as is the case in many experiments, then it becomes much more difficult to find an alternative. Then, the subjects may try to produce more or less equivalent, although eventually not-synonymous sentences.

Sequences as (34) are also no argument against the hypothesis of introducing only relation negations.

> (34) Ik zie een niet groene doos.
> I see a not green box.

If, in (34) we have to do with the polarization negation, viz. 'niet groen' (not green), then no problems arises (see earlier). If this is not the case, then the meaning structure of (34) is roughly an ordered conjunction of p and q (Vandamme, 1971 a).

> p = een doos is niet groen (a box is not green).
> q = ik zie deze doos (I see this box).

In this last alternative, the negation is also a relation negation.

II.D. *A conceptualistic solution of the non-existence paradox.*

The non-existence paradox is wellknown from Plato's dialogue called "Parmenides". It consists of the problem that for being able to say about something that it does not exist, this thing must exist. Otherwise we could not predicate it, as we do by saying that a thing x does not exist (existence is treated as a predicate). If x would not exist, we could not ascribe the property of non-existence to x, but to something other than x. If x does exist, then it is contradictory with the property of non-existence. This is the paradox. In other terms, the negation is made by an operation on the positive. That is, the negative involves the positive as a constituent, and therefore it presupposes its existence.

(Toms (1962, 1971) treats the non-existence paradox rather extensively.)

A small fragment from the Parmenides (Plato, Parmenides 161 e) illustrates this:

Parménide

Ainsi l'un qui n'est pas, participe, semble-t-il, et de l'égalité et de la grandeur et de la petitesse.

Aristote

Il semble.

Parménide

Ce n'est pas tout: il faut encore qu'il participe de l'être en quelque manière.

Aristote

Comment ?

Parménide

Il faut qu'il en soit de l'un comme nous disons; sinon, nous ne dirons pas vrai, quand nous disons que l'un n'est pas. Mais, si nous disons vrai, il est évident que nous disons ce qui est. N'en va-t-il pas ainsi ?"

The solution Plato proposes for the paradox in his later work "The Sofist" is highly insufficient. It consists — as many later tentatives — in the trial to define negation in positive terms. Plato (The Sofist, 256 d, 2) interprets negation in terms of 'otherness':

L'étranger

Nous affirmons donc sans crainte et nous maintenons énergiquement que le mouvement est autre que l'être.

Théétète

Oui, sans le moindre crainte.

L'étranger

Ainsi donc il est clair que le mouvement est réellement non-être et qu'il est être, puisqu'il participe de l'être ?

Théétète

On ne peut plus clair.

He summarizes his point of view rather clearly in the following expression (The Sofist, 258 d):
"Ainsi, à ce qu'il semble, l'opposition de la nature d'une partie de l'autre et de la nature de l'être, quand ils sont opposés l'un à l'autre, n'a pas, s'il est permis de le dire moins d'existence que l'être lui-même; car ce n'est pas le contraire de l'être qu'il exprime, c'est seulement autre chose que lui.

Théétète

C'est clair comme le jour."

We (Vandamme, 1970 a) have already shown how as a result of this, the principle of non-contradiction is no longer valid. For instance, 'green' is different from 'big'. This means 'green' is not 'big'. As a result, a table can be green and not green as well as it can be green and big. Apart from us, Plato's solution has been critizised by Toms (1971).

Toms (1962, 83) argued that the non-existence paradox even exists in a conceptualism, if one interprets 'negation' of a relation between, or an operation on, the elements A and B, as an indication that R(AB) does not suit the facts. By this, a new relation between the description and the facts is introduced and at the same time it is negated. So one is again confronted by the non-existence paradox. Must the relation exist so that one can deny its existence ?

Our negation interpretation as an operation in a model gives us the possibility of avoiding the paradox (this is a conceptualistic approach).

In the model, we want to distinguish the true from the false descriptions, the descriptions of existing objects from those of non-existing elements.

The latter distinction could be got by connecting some descriptions in the model to the concept 'existence'. The descriptions we are uncertain about, we would leave unrelated. Of some descriptions we would negate the relation with existence, viz. by increasing the activation threshold. In this approach, it is clear that each negation does not imply non-existence.

However, how to interpret the concept 'existence' ? A proposal one could make, is to interpret 'existence' as a concept indicating that in principle some operations can be executed on this element to which the existence is ascribed. If this is the case, then one could argue that it is possible to differentiate several types of existence. This in dependence on the characteristics of the operations. So one may propose to differentiate (1) existence in reality (being this a realistic, idealistic, or solipsistic construction) and (2) conceptual existence.

The kinds of operations for determining the first type of existence, could be all possible operations and their derivations by which an individual changes his environment or assimilates

information from his environment. All kinds of conceptual oper-
ations would refer to the second type. Existence in its most
general form (existence 3) would then mean the possibility
of being an argument of any operation whatsoever.

It is clear that this last definition of existence is self-con-
firming, in this sense that it can on its turn be an argument of
some operations (in fact conceptual operations) and therefore
it itself exists. This self-confirming interpretation is also
true for the interpretation (2) of existence, but not for the
interpretation (1). Existence (1) only exists according to the
second and third type of existence. This is evident. Existence
(1) is a concept indicating the possibility of executing (roughly
speaking) manual operations. It is certainly impossible to
execute manual operations on a concept.

What does it mean that the relation between a concept A
and the concept existence (1) is negated ? It means that there
is no *denotatum* for A, on which operations denoted by the
concept existence (1), can operate (e.g. manual operation,
etc...).

The denotatum of A exists according to the interpretation of
existence (2) or (3), but not according to the notion existence
(1).

As a result, negating a concept, which is related to the
existence (1) concept, does not imply that it is existent in
the existence (1) interpretation, viz. that in fact, it is an object
on which is operated manually, etc. Here, the non-existence
paradox disappears.

This cannot be said about the negation of non-existences (2)
and (3). It is clear that the execution of the negation operation
itself (viz. to increase the threshold) is a concept operation;
and therefore the exclusion of being an argument of a con-
ceptual operation always implies a conceptual operator. So
it seems contradictory to negate existence (2). The same
happens to existence (3).

This does not imply that we cannot say that for a certain
individual A a certain concept T does not exist. But as far as
he who negates is concerned, T must exist as a concept.

This solution is discussed in a broader framework (Van-

damme, 1970). Apart from us, Toms (1971) introduces a solu-
tion along the same lines (his B-theory) in a platonistic frame-
work, ours was a conceptualistic approach. Shortly it is as
follows: "According to the B-theory affirmative and negative
propositions result from performing different operations upon
a third entity which is neither affirmative nor negative. The
third entity cannot be an intentional object, because it has
to exist whichever operation is performed upon it. The only
possibility seems to be that it is a universal. Affirmation would
then be due to the claiming instantiation, negation to the
operation of claiming non-instantiation."

The analogy between both approaches is that the evitation
of the paradox results from the introduction of an affirma-
tion and a negation operation on a neutral term, as far as
these operations are concerned.

So, in our solution, the terms whose existence (1) is negated
or affirmed are in any case existent in the second interpreta-
tion. This means that they are neutral relative to the notion
existence (1). When affirming the existence, we connect them
with the concept existence (1), while by negating it, we negate
their relation. In this way, the affirmation is not implied and
so we avoid the paradox. Something analogous is the approach
of Toms. However, we propose another interpretation of nega-
tion, affirmation and the neutral term. It is also important, that
if we want to negate the existence (2), a paradox, or in our
view rather a contradiction, will be obtained. The reason is
then the absence of a neutral term.

Proceeding from our interpretations of existence, what
could be the significance of the existential quantifier? An
interpretation could be that Ex means the indication of an
object, which is in a restrictive relation to the existence (1)
concept (more about the restrictive relation in I.E.3.). If one
would use the concept existence (2) or (3) then one would get
a trivial specification, because every concept can be restricted
in this way.

If one uses the existential quantor in an existence-free inter-
pretation, then x is restricted by a pure quantificational ele-
ment, viz. the unity. In the case of 'some' and 'all', we then

can say that x is restricted to a domain of some elements or
of all elements.

II.E. *The differentiation between 'true, false, not true, not
   false'.*

Proceeding from our approach to negation up to now, one
can conclude that 'true and false' are contraries, just as 'illness
and healthy'. Negating one of these elements does not mean
the same as confirming its contrary.
   In the several experiments on the negation phenomena, an
analogous difference appears. In general, one finds it neces-
sary to make a differentiation between sentences (1, 2, 3 and
4) (see e.g. Wason, 1971).

   (1) There is red.
   (2) There is green.
   (3) There is not green.
   (4) There is not red.

(1) and (2) are alternatives in one dimension; (3) and (4) are
negations of alternatives.
   In the truth dimension, the alternatives are true, false and
senseless. Their negations are not true, not false and not
senseless.
   In the dimension of health, the negation of the alternatives
'ill and healthy' are 'not ill and not healthy'.
   How to define the several alternative elements ? Having
defined e.g. 'red' and 'green', it is not difficult to define 'not
red' and 'not green', once the negation operation is defined.
   The negation of an element x can, however, not be used to
define the alternatives inside the dimension of this element.
This is even the case for 'negation' in the interpretation of a
polarization operation.
   'Unhealthy' will result in indicating the other pole of
'healthy' in the dimension 'health'. It is, however, clear that
this pole in principle needs to be determined by other means

too, without using the polarization operation. Maybe, by describing a structure which represents the opposite pole of the dimension 'health'. The exigence of such a possible determ- ination is obviously not the same as the exigence, that in the natural language a morpheme exists to denote the alterna- tive pole. In fact, we have already mentioned that very proba- bly such a morpheme does not exist.

So the problem also arises: how can the structure be des- cribed which is signified by the term 'true' or 'false'. The approach, Russell made, by defining 'false' as 'not true' is correct, if 'not' is used as a polarization operation, although it is insufficient. If 'not' is used as a negation operation, then the definition is surely incorrect for natural language.

But how then to define the terms of the truth-dimension? A suggestion, Apostel once made, which is only useful for a certain limited case of arithmetics, was the following: 'n = m' is true, if 'n' is the R-th successor of 0 and 'm' also the R-th successor of 0. 'n = m' is false if 'n' is the R-th successor, 'm' the S-th successor of 0, and if R is a successor of S or vice versa.

Is it possible to give a more general definition of false?

Proceeding from sentences (1 to 4), we can have four evalua- tions for each sentence, as far as the truth is concerned. Let us exemplify the possibility for sentence (1).

(5) It is true that the ball is red.
(6) It is false that the ball is red.
(7) It is not true that the ball is red.
(8) It is not false that the ball is red.

An important feature of the differentiations 'true/false' and 'not true/not false' has already been found in Aristotle, and was also stressed by Franz Brentano. It consists of the fact that affirmation e.g. always implies existence. ([13]) This is not true for the negatives, as was illustrated in Aristotle's approach and in Brentano's writings. ([14])

So we can say that sentence (1) is true in our conceptualistic framework (in our model), if there is such an input (in princi-

ple) that it can be projected on the conceptual structure represented by the sentence (1).

The second condition is (taking into account the implication of true) that the objects mentioned in sentence (1) are in a restrictive relation to the existence (1) concept.

Sentence (1) is false, when the second condition is fulfilled and when all the inputs are projected as a whole or partly on structures other than the ones mentioned in (1). The example of partial definition of 'truth' and 'false' is along these lines.

Sentence (1) is 'not true', if one of both conditions for being true is not fulfilled. Analogously, sentence (1) is not false, if one of both these conditions are not fulfilled. ([15])

It will be clear that with the approach, the notion of analytical truth is not captured. We do not want to discuss this here. The important problem of differentiation between analytical and synthetical truth is at stake here. The discussions of Quine, Tarski, Carnap and others are here rather crucial. It would lead us too far to discuss this here.

Does this treatment of true and false with its existence implication weaken the existence-free quantifier hypothesis ? Not at all. To be existent is not sufficient to be true to begin with, and secondly the requirement that the elements in question must exist in order for a statement to be true or false (but not in order to be not true) does not imply that the quantifiers must be treated as extential. An example will make this clear. The statement that five chairs ('all' is also treated as a kind of undeterminded cardinal number) in the house are dirty implies that one chair in the house is dirty. But both statements are not true if there are no chairs in the house. It is along these lines, that one must look at the existence free quantifier.

Another remark that must be made is the following one. The fact that in natural language 'true, false, not true, not false' are differentiated from each other, does not mean that logic, or science in general, may not neglect these differentiations. In fact, as we already argued (see I.F.), in principle nothing prohibits the identification of e.g. 'false' and 'not true' in view of some constructive purposes.

III. *NEGATION AND INTELLIGENCE.*

III.A. *General role.*

What could be the relevance of negation for intelligence, viz. for a general strategy to solve problems ? If one considers the cognitive model (a world model representing data of several degrees of abstraction) as providing the necessary data for the construction and the choice of alternative strategies, then the important role an exclusion negation can play for the action control must be clear.

Negation can play an important role as a registration of failures of certain actions, presumptions, prohibitions, etc. This can happen by exclusion of certain relations or of complexes of relations. It is certain that these exclusions will frequently be dependent on certain contexts. This is also true for the use of negations in communication.

The exclusion negation operation here is certainly preferable to an erasement operation (destruction operation). For, in this last case, one risks starting the same actions again, which proved to be unsuccessful, and to form the same ideas, which proved to be wrong, etc... So one stands to loose much time, and to run dangerous risks ...

Why this is so follows from the fact, that if one erases something x, then nothing excludes the possibility of starting immediately with the rewriting of x.

With the exclusion negation, what is excluded, is preserved. It only has *another functional role* in the system !

Of course, the exclusion of some elements can eventually lead in certain contexts to the destruction or erasement of this element.

An example, how the elimination or exclusion of some elements can be very useful in a decision problem is easy to give. If in principle one knows several alternatives for a certain problem, and if one can eliminate some of them, then the choice is certainly facilitated.

This is repeatedly illustrated, although in a simplified

manner, in the rather popular TV program "Rien que la vérité" (a TV play realized by the O.R.T.F.).

One gives the players two problems A and B with the respective solutions A' and B'. They know that one of the proposed solutions is wrong. Now they have to choose the couple 'problem — solution' which they think to be the correct one. It often happens that the players explain their choice of a certain couple on the basis of the fact that they knew or thought it more probable that the other couple was not correct.

Stating this role of negation in a general form: if a disjunction e.g. (p ∨ q) is true, and if one of the alternatives is not true, then the other one is true. In fact, the so important logical principle 'modus ponens' is based on this principle, viz.

$$\frac{-p \lor q}{q} \quad .$$
$$p$$

III.B. *Limitations on negative information.*

It is, however, important to be aware of the necessity of limiting one's negative knowledge: the number of exclusions in the model. And this chiefly for economical reasons: (A) the limitations of the memory, (B) the waste of time by regarding all the negative information when decisions must be taken.

It is interesting to mention Campbell's (pp. 168-169) arguments on the problem of the destruction of negative information.

(a) He argues: "... it (destruction of negative information) is perhaps most strikingly characteristic of mutation-selection. The present generation contains in its genes a record of the variations that were adaptive (or were not maladaptive). The record of the totally unsuccessful variations is lost with the organisms which embodied them."

This is also the characteristic of the 'homeostat' of Ashby (1952). It records only its successful steppings-switch combinations, and these are preserved by being left intact.

(b) He makes the following remarks on the conclusion of Bruner et al. (1956): "Bruner emphasizes the overwhelming memory load introduced by problem-solving strategies which require memory for hypotheses ruled out as well as hypotheses that have withstood a given probing and they have called attention to the availability of adequate strategies which do not require memory for the disproven."

(c) In laboratory studies of concept information persons have long been known to profit mainly from positive instances.

Taking this into account, one could suggest that intelligence uses a strategy where the negative information is temporarily conserved. Once, one have made an adaptation of the general theory or decision strategy, which takes the negative information into account, then this information can safely be destroyed.

This does not imply the prohibition of introducing negations in the general dicision strategy or theory.

For, if one introduced only positive features, as far as they could map the negative information, one would always run the risk of once again getting not adapted results, identical or analogous to earlier mistakes. As this is the case in the 'mutation-selection'. *This means that eventually one cannot avoid taken into account the negative information without introducing exclusions in the theory.*

For certain types of strategies, it would be rather disastrous, if the used rules would generate the negative information. Certainly, if the rules are induced from a set of material (including negative and positive information), and if they are intended for generating only the positive information, avoiding the negative ones. Look for instance at a competence grammar. Such a grammar must characterize grammatical sentences and differentiate them from the ungrammatical ones. In the construction of such a grammar (e.g. Chomsky stresses this very

much) the negative information as well as the positive information play an important role.

But how to understand such a competence grammar, if (a) it does not reflect the data of grammaticalness (A) and ungrammaticalness (B), on which the grammar was constructed; and (b) in principle sequences (S) noted as ungrammatical $(S \in B)$ can be correctly generated by this grammar. This problem poses itself for all skills.

Another strategy for reducing the negative information is suggested by Campbell. It is based on a fact established by Hovland: "about an ecological feature in the tasks typically employed which makes negative instances intrinsically less informative than positive instances. Of all of the possible concepts that the discriminable features of the stimuli make possible, only a very small proportion are called correct by the experimenter, leading to an imbalance in which a single positive instance reduces equivocality much more than a single negative instance."

Campbell suggests that the strategy of neglecting wrong trials should be limited to such settings. He also argues that the ecology of organisms during the course of evolution was such, viz. there were many more ways of constructing an organism that won't work than one that will ...

L. Apostel ([16]) made an alternative suggestion for limiting the role of negative information. He proposed to make it dependent on the history of the process in intelligence. In a certain sense this is analogous to Campbell's suggestion. If negative information will be more informative than positive information, then it will be preserved; in the other case, it will be neglected. In other words, it will be dependent on the context. The difference between Apostel and Campbell is that the former makes a relativation to the intelligence process, which he considers to be the relevant context of the negative and the positive information; while the latter rather stresses the relativation to the alternative possible information in relation to the task intended. But both approaches are complementary.

An example of Apostel's point of view is the following. If

a certain error has been made rather frequently, it may be worthwhile nothing it in order to avoid the repitition. The loss of memory can then be compensated by the utility of avoiding the stubborn error.

In the relativation suggested by Apostel, it is also important to remark that the role and utility of the negation concept can be strongly dependent on the problem solving strategy used, or on some part of it. So it may seem that in a max-max decision strategy, where one takes any risks, in order to get a gain however small or big, the use of negation information will be minimal while the positive information predominant. In a mini-max, viz. minimum of risks for a maximum gain, the negative information can be of great importance.

To handle these problems better, a formalization of intelligence (of the problem solving strategy) is needed. Much work has already been done in this respect.

Apostel suggested that it could be rather useful to treat the formalization of such a strategy in an interrogative logic. Problems being questions, solutions being answers to questions.

It is clear that in such a treatment, the interrogative logic may not be restricted to no or yes questions, viz. to the questions which can be answered by yes or no.

Also it seems to us that such an interrogative logic only can be a subpart of the frame needed for treating problem solving strategies. We believe that a general axiology needs in any case to complement an interrogative logic and this for the following reason.

In an interrogative logic, a question could be interpreted as follows: $W(I(x))$, viz. a wish to get information of a certain type about something. The answer is $I(x)$, viz. information about x. The notion of complete answer can easily be introduced in this framework (Vandamme, 1971).

Nevertheless, it seems to us preferable to represent a problem not as $(W(i(x)))$, but rather as $W(I \cdot A(x))$ and a solution as $I \cdot A(x)$. A or I can eventually be empty. In other words, a solution to a problem exists in a certain genera-

tion of information and/or in a certain (accompanying) action about a certain object x.

In this framework, we need to reformulate the question: What could be the role of negation (?) ... But this falls outside the present scope of the article.

III.C. *The importance of both the negative and the positive information.*

All the quoted arguments for reducing the negative information seem very much to weaken the basic hypothesis of Alexander (1964) about the central role of negation. He argues that negative information is much more important and easier to handle. So he states that (a) it is easier to communicate negative adaptation requirements, and (b) that the perception of inadaptation is much more direct.

Before discussing these arguments, it seems to us worthwhile to sketch the framework of Alexander's approach. He wants to describe the design-process with the intention of bringing this process from the intuitive level to the more rational one. Or if this is not completely possible, to bring it at least also in the rational level. This is possible — he argues — from the moment that, what is intuitively done, can be described and so be compared with actions, which do not happen in the intuitive level (Alexander 1964, p. 8). In this approach a method for describing the design process is crucial. The negation plays a crucial role in this method.

To understand this, we must look at the design problem as Alexander sees it. The problem is to achieve fitness between two entities: the form in question and its context. The form is the solution to the problem; the context defines the problem (Alexander, 1964, p. 15). As a result, when speaking of design, the problem is concerned with both form and context and good fit is a described property of this ensemble. Alexander illustrates this abundantly and he also stresses how the division line between form and context is relatively arbitrary. In other words, that the border line between both can be drawn

rather easily on different places. Any good designer needs to be conscious of this relativity.

This phenomenon in fact can be compared with the necessity of scientific reconstruction of certain terms in sciences (Carnap 1950, 190). This is also a method for taking into account the relativity of the problem and the answer. By reconstruction the problem (the context for Alexander) can be changed, so that a solution is easier to achieve ...

An example which Alexander gives, can be useful for understanding his point of view better: "Let us consider an ensemble consisting of the kettle plus everything about the world outside the kettle which is relevant to the use and manufacture of household utensils. Here again there seems to be a clear boundary between the teakettle and the rest of the ensemble, if we want one, because the kettle itself is a clearly defined kind of object. I can easily make changes in the boundary. If I say that the kettle is the wrong way to heat domestic drinking water anyway, I can quickly be involved in the redesign of the entire house, and thereby push the context back to those things outside the house which influence the house's form. Alternatively I may claim that it is not the kettle which needs to be redesigned, but the method of heating kettles. In this case the kettle becomes part of the context, while the stove perhaps is form." (Alexander 1964, p. 17).

Rightly, Allexander stresses also the danger of making to much use of this relativity. It can be a means for flying the design problem: "There are two sides to this tendency designers have to change the definition of the problem. On the one hand, the impractical idealism of designers who want to redesign entire cities and whole processes of manufactures when they are asked to design simple objects is often only an attempt to losen difficult constraints by stretching the form-context boundary." (Alexander 1964, p. 17).

An analogous point can be made about the use of scientific reconstruction in science. In fact, the continuous change of terms in some prescientific discussions can perhaps be explained along these lines.

Now the problem arises of describing the condition which must determine a good fit between the form and the context. Alexander says:"It is the easiest thing in the world to name the specific kinds of misfit which prevent good fit. These misfits are the forces which must shape it, and there is no mistaking them. Because they are expressed in negative form, they are specific, and tangible enough to talk about" (Alexander 1964, p. 13).

But the conditions which prevent good fit are not only easiest to express, but they are — in his view — also easiest to perceive: "The same thing happens in perception. Suppose we are given a button to match, from among a box of assorted buttons. How do we proceed ? We examine the buttons in the box, one at a time; but we do not look directly for a button which fits the first. What we do, actually, is to scan the buttons, rejecting each one in which we notice some discrepancy (this one is large, this one is dark, this one has too many holes, and so on), until we come to one where we can see no differences. Then we say that we have found a matching one. Notice that here again it is much easier to explain the misfit of a wrong button than to justify the congruity of one which fits" (Alexander 1964, p. 23).

But would one not equally describe this task in a positive way ?

When one is asked to look for the buttons which match the given button, one could start by selecting the ones with the same colour, then the ones with the same size, and so on. When one has decided that the isolated ones have enough characteristics in common with the given one, then one may stop the process. Of course, the order of the controlled attributes will also be dependent on the importance which has been given to each attribute.

In any case this strategy looks easier, because it is no longer necessary to use a new criterion for every new button one compares.

It is also rather trivial to stress that even in Alexander's approach, the perception of positive characteristics of the first button is very crucial. For, the misfits of every other

button must be determined by the positive characteristics of the first button.

Perhaps one can remark that in the process of selecting — with exception of the trivial situation, where one selects all the elements — the selection will be accompanied by rejection. But the opposite is also true, when rejecting elements — with exception of the trivial situation, where one rejects all the elements — one we also always have a selection.

In connection with this point of view of Alexander that conditions of misfits are most easily perceived and expressed, we did a little experiment. We asked a few students to formulate personal requirements about some design problems. We asked them to give five requirements about an ideal car, a house, wine, and so on. We got the following results: 75 % of the requirements were positive, 18 % negative and 7 % were a mixture, viz. a conjunction of a positive requirement with a negative paraphrase. In any case, the positive was always the first member of the conjunction. The mixture of positive and negative requirements generally seemed to be used when one felt the positive characterization inadequate. But even with the negative paraphrase, one was in these cases unhappy about the phrasing of the requirements.

These results seem clearly to contradict the Alexander's argument.

Taking this into account as well as the earlier discussion on negative information, the following synthesis could be proposed.

Alexander is right in stressing the importance of negation. For in the knowledge processes, even in sciences, the negative information is too greatly neglected. In linguistics, it is only rather recently with Chomsky, that one has learnt to make systematical use in a constructive way of the negative information on the grammaticalness of a certain sequence, etc. In the architectural problem this neglect seems also to be a fact.

But, although the negative information is under certain conditions very informative and useful, it is nevertheless dangerous to overestimate its importance, and eventually as

Alexander does to underestimate and neglect the importance of positive information.

In the process of determining requirements for good fits for instance, we do not only have bad experiences with old solutions and designs; and so not only negative requirements for each new design. The bad experiences will certainly lead to requirements of exclusion (negation) of certain characteristics of the old design. But, we do also have at least partial positive (eventually conservative) feelings about the old designs. So it will be clear that there certainly will be positive requirements, and whereas conservatism is generally important, then the importance of positive requirements will also be rather extensive. This conservatism could be linked to the specification of continuity, as this is found in Ashby's 'Homeostat'. The positive requirements can also be seen in a framework of a positive feedback, viz. that one wants to accentuate in the new design the characteristics which were positively experienced in the old design. But the positive feedback — as we all know — needs to be limited. Therefore, at a certain point, the negation will be important for stopping such an increase of accentuation. It also seems wrong to us to see only (as Alexander does with his stress on the generation of negative requirements) the unhappy experiences with the old designs as the sole motivation for the search towards new designs. (¹⁷) The bad experiences will certainly lead to requirements of exclusion (negation) of certain characteristics of the old designs.

But, the human explanative tendencies and the trend for renovation (Kruithof, 1968) are also important parametres. Of course, the feelings of unhappiness, etc... can strengthen these tendencies.

But, one must be aware that these feelings of unhappiness cannot be avoided by some negative requirements as such. This simplifies the matter too much.

The negative requirement must be seen as a statement of conditions which, when avoided, would have made the old solution better. But eventually in another context, the same negative requirements are pointless.

This means that in a general explicit theory for determining solutions (the construction which is the intention of Alexander), the relativity of the negative and positive requirements must be kept in mind. This relativity strongly minimizes the use which can be made of negative information for eliminating new solutions. For the reason of its great dependence on the specific context.

Once one is aware of this relativity, Alexander's next principle is not of much use: "We take just those (negative) relations between forms and context which obtrude most strongly, which denote attention most clearly, which seem most likely to go wrong".

For, the so called negative relations are only clearly negative in the specific old contexts and forms. The same characteristics, once taken up into a new solution, do not need to be still negatively validated. What is clearly negative in the old setting, is not necessarily so in the new setting (new, on the basis of another internal structure of the form or of the context). This relativity is still stronger, if one also takes into account the arbitrariness of the differentiation between the context and the form (the determination of the boundary), which Alexander also stresses.

ADDENDUM

A common structure to the several types of negation?

In our exposition we have tried to prove that the exclusion negation was the mother structure of several types of negation. Taking into account the contributions in this issue of the other articles on negation, some extra-comment in this account seems to be wishful.

Apostel f.i. regards the opposition 'positive — negative' as based on the opposition 'aggression — flight', which he formally interprets as follows: "An operation is maximally positive, if it maps from a very large domain on a very small codomain, maximally negative if it maps from a very small

domain on a very large codomain". This formalization becomes clear, when taking into account the following topological characteristics of flight and attack: "One can fly in all directions (negation), and only attack in one direction of the object attacked (affirmation)".

An analogous approach can be found in Peeters (1972). He tries to relate cognitive positivity bias (as this is illustrated in the findings of De Sota and Kuethe and recently McNeel and Messich. So were subjects f.i. more inclined to assume a priori to any information a positive rather than a negative inter-personal relation between two fictive stimulus persons) to a behavioral approach, which can be related to the dynamics of 'mere survival' and 'self actualization'. In this, he regards as the fundamental opposition 'the avoidance' (mere survival) and 'the approach' (self-actualisation).

As Apostel does, he also relates 'positive' with 'approach' and 'negation' with 'avoidance'. They both come to some very interesting conclusions and explanations of facts.

A question, however, arises, viz.: If negation can be related to avoidance — there is even agreed on the fact that negation always is a type of avoidance — does this then mean that all types of avoidance are types of negations ? In other words, if it is true, that general features of avoidance are valid for negative phenomena, is then everything about negation said when these features are described ? Or is negation rather one of the many ways for realizing avoidance ? In this last case, negation may not be identified with 'avoidance'. Nevertheless it can be very important for understanding negation, to relate it to its more general category and to its opposition: 'approach'.

Negation is then an operation which intends to realize 'avoidance'. This happens by blocking, empeding the actua-lization of certain relations; what results in an exclusion of certain actions, etc. Stated more concretely: A person A f.i. is driving his car from x to y. He remarks his foe B on this way. To avoid B, he stops his car (he negates the driving in the direction of B), then he may choose another direction (when he is afraid that B will approach him). In this case, the avoidance of B by A implies only as a subpart the true negation (stopping

the car; what means empeding the approach to B). The moving into the other direction is the complementary part.

In his article, Beulens made — as we already have illustrated — an analogous approach to negation as an exclusion operation. De Mey takes a view along the same lines. He states: "Negation introduces a shift in attention: what is attended to becomes suppressed (A) and an alternative becomes activated (B)". Here, we want to make the following comment: if cognitive negation is considered to be an operation on attention, we believe that it is redundant to include the part (B) 'activating an alternative' in the function of negation. In a certain context, it is clear that attention is always directed on something. As a consequence, it is evident that by blocking the attention on a certain object, the attention will be directed on something else. Eventually on something in opposition with what has been blocked, but this is not necessary. This depends on context. Therefore part B does not seem — in our view — to be essential for negation.

In this light, it might be interesting to mention Frijda's views on negation.

Frijda N. H. (in a discussion with the subgroup Negation) does also accept the excluding function of the negation, but as many people does, he defends the point that the production of alternatives essentially belongs to negation and that this is not the result of the whole contextual system. Nevertheless he does agree that the choice of the alternatives certainly is dependent on the context. Interesting to mention is also his view on the generation of alternatives as connected with negations. He argues that in the search for alternatives, one tries to make a maximal change along one feature (a dimension) of the element to be changed (this is equivalent with 'to be negated') keeping all the other ones intact.

Here again it is not clear if this phenomenon is inherent to negation. It seems that —as we have said in the discussion about De Mey's point of view — one could argue that the exclusion rather naturally (but not necessarily; this depends on the type of negation) leads to the search of alternatives; to the search of change. A very important strategy in the search

for alternatives is certainly the method mentioned by Frijda. (He has very interesting illustrations of this method.) This method is surely not always used — Frijda does agree with this — when changes or alternatives must be produced, nor in all cases where negations are present. Also the eventual identification of the changed object with the negated object seems to us unclear. For all these reasons the search for alternatives seems to us not to be a real subpart of the true negation, although it is much used in negation contexts, but not only there. (I also believe that in the context of negation the eventual search for alternatives has no special features, which differentiate this searching from the searching in other contexts.)

The conclusions of Wason seem to throw an important light on the cognitive function of the phenomenon 'negation'. Negation plays an important role not so much for the description of the world, but rather for informing us about the world, taken into account our expectations of the world. If we are expecting money in the drawer, we can exclude the idea by negating the fact that there is money in the drawer and we will communicate such a negative sentence to whom it may concern, viz. to the person who may expect money in the drawer.

In this light it is clear that Wason is certainly right that the Clark and Trabaso model, which relate negation to the description of the world, end up with a real distortion of the negation phenomena. Negation in natural languages is an operation of exclusion of expectations. There where no expectations are, the use of negation is abnormal. And right there where expectations are present, one can find the important utility of negation in human communication and interaction: in other words in pragmatics. This is the reason of our thinking that negationless logic — as important as it is — cannot be a real substitute for a logic with negation as far as pragmatics is concerned.

NOTES

(1) If this is not the case, then also in a negationless logic, false propositions could eventually be possible.

(2) An extensive discussion can be found in Vandamme 1970 a.

(3) It is worthwhile to mention that in the Chomskyan transformational grammar, one find an analogical approach, viz. in the deepstructure one gets an analysis in NP and VP, where the VP itself is analyzed as a relation V and NP as an argument of V.

(4) As it happens with the correlation opposition.

(5) Another argument of Aristotle (51 b 10-25) runs as follows: "The relation between (1) and (2) is similar to the relation between (3) and (4).

    (1) It is white.

    (2) It is not-white.

    (3) He can walk.

    (4) He can not-walk.

When two pairs correspond if the one pairs are different from one another, the other pair also must be different. But, if then 'he is not able to walk' means the same as 'he is able not to walk', capacity to walk and incapacity to walk will belong at the same time to the same person (for the same man can both walk and not-walk and is possessed of knowledge of what is good and what is not-good) but an affirmation and a denial which are opposed to one another do not belong at the same time to the same thing, etc.

(6) The italics are ours.

(7) In a discussion with the subgroup Negation of Communication & Cognition, L. Derijck-Tasmowski attracted our attention to the observation of Zimmer.

(8) Zimmers observation is in general valid. In this sens that generally he is right. But there are, of course, exceptions.

waar	vals	onwaar	
true	false	untrue	
nuttig	schadelijk	onnuttig	onschadelijk
etc.			

This is also to be expected, for, if the only reason is that there is too much redundancy when we have both of them, then it is clear that in some cases the necessity of redundancy can make exceptions. In fact we find them. This is an affirmation of the explanation.

Zimmer's observation with the explanation, we give here, seems also to be an important argument and illustration how the language uses can influence the development of language. For, it shows that there is a strong tendency to adapt the language system, so that the redundancy is optionalized. This seems to be confirmed by the fact that in almost all languages, it seems that Zimmer's observation holds true.

(9) Von Wright (1958, p. 6): "The denial of 'x is P' is treated as equivalent

to the affirmation of 'x is not-P'. Non-membership in a class is identified with membership on the complementary class.".

([10]) Aristotle, however, argumentates further (Categoriae 20-25) the justification of the differentiation of the contrary opposition from the privative-affirmative opposition by attracting attention to the fact, that in the latter the same condition of existence is needed (which is also a condition in the case of the contrary opposition) besides some other conditions. However, it seems that for some contrary opposition, other conditions exist as well. So, for instance, for a proposition to be true or false, it must have sense, etc... So the differentiation seems dimmed (more about this document 2 workgroup 'Negation').

([11]) In this respect, the work of Bregson is important.

([12]) Even if one does not agree here, this does not change our argument. If one considers the quantifiers as being in a prediction relation with x, then it is this relation which is negated.

([13]) Stegmuller (1960, p. 5-6) states these thoughts of F. Brentano as follows: "Eine Vorstellung kann noch so absurd sein; solange ich nicht behaupte, dieses Vorgestellte existiere in Wirklichkeit, ist es sinnlos, diese Vorstellung 'falsch' zu mennen."

([14]) Stegmuller (1960, 12-13): "Die allgemein bejahende Urteile sind daher in Wahrheit apodiktisch verneinende ... dass ein Dreieck nicht eine Winkelsumme von 2 R besitze. Ob es tatsächlich irgendwo in der Wirklichkeit ein Dreieck gibt oder nicht, bleibt dabei völlig dahingestellt."

([15]) Of course, the sentence "The ball is red is not true" is true implies the existence of the sentence between quotation marks and not of the the elements mentioned inside the quotation marks. If this would not be the case, we would need to introduce a new 'truth' interpretation if a negative sentence is affirmed.

([16]) A personal discussion.

([17]) But even then, the negation only limits the set of possible solutions to the problem. For the bad experiences (not a negation) explain (a) the search for explanations of the bad experiences (here the negative conclusions can be important) and (b) the search for new solutions.

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