SEMANTICS FOR A UTILITARIAN DEONTIC LOGIC

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I. *Background.*

This development of an act-utilatarian deontic logic was stimulated by Castañeda [1]. In [1], as well as in [2] and [3], Castañeda observed that a naive, but very natural, act-utilitarian statement of necessary and sufficient conditions for "X ought to do act A in circumstances C," such as U below, is logically unacceptable.

U: X is morally obligated to do A in circumstances C if and only if X's doing A in C will bring about a greater balance of good than his doing any alternative act open to him in these circumstances.

Castañeda showed that such a naive formulation of act-utilitarianism cannot consistently give necessary conditions for "being obligatory" if $\mathbf{O}(p\,.\,q) \supset \mathbf{O}(p)\,.\,\mathbf{O}(q)$ is the form of a logical truth, where $\mathbf{O}(\ )$ is the deontic ought-operator and p,q are propositional variables. (In this essay, I shall always treat deontic operators as propositional operators.) If p and q describe different but jointly performable acts and if we have $\mathbf{O}(p\,.\,q)$, we have $\mathbf{O}(p)$. If we admit now that (p . q) and p describe alternative acts, the "only if" part of U says that (p . q) describes the best way of acting in some circumstances while the "only if" part of U says that plain p also describes the best way of acting in these circumstances. In a reply to Castañeda's [1], Lars Bergström in [4] noted that he had worried about a similar issue in his book: [5]. Bergström claimed that he realized that a fundamental step in reformulating act-utilitarianism so that it is consistent with the principles of deontic logic is defining "alternative to an act," or some complication of the notion, so that only incompatible acts are compared in regard to which is productive of more good. Åqvist in [6] and Castañeda in [3], also realized that such a definition of "alternative to an act" was required to make act-utilitarianism consistent with the principles of deontic logic. If someone holds that act-utilitarianism is worthy of consideration and also holds that there are principles of deontic logic, amongst which is the distributivity of "ought" over logical "and," it is only natural that problems such as that of Castañeda and Bergström would stimulate him to search for a reformulation of act-utilitarianism so that it is at least consistent with these principles of deontic logic.

However, in this essay, I shall not pursue the details of atempts to reformulate act-utilitarianism so that, while remaining faithful to utilitarian moral insights, it is consistent with the principles of deontic logic. Here, it suffices to note that Åqvist [6], but especially Castañeda [3], are extremely thorough searches for such a reformulation. I do not want to pursue the details of such reformulations since Castañeda's observations stimulated me to a radically different investigation of the relation between act-utilitarianism and deontic logic. It suggested to me the following example. And the example shows that an act-utilitarian may not be bothered by the fact that "ought," as he uses it, does not distribute over logical "and."

Consider a young man called Tom, of whom the following is true. If Tom gets into college K and does well in college K, he will be an asset to humanity. However, Tom must study hard while in high school if he is to do well once he gets into K. Unfortunately, though, if Tom is to get into K he must cheat on the entrance exam, which will be given sometime while he is in high school. Let p be "Tom cheats on the entrance exam" and q be "Tom studies hard while in high school." In these circumstances **O**(p . q) could hold while neither **O**(p) nor **O**(q) held in an act-utilitarian sense of "ought." What is needed to produce the most good in these circumstances is that Tom cheat to get into college K where his hard study in high school can lead him on to a glorious career. By itself the hard study is vain effort while the cheating alone is mere ignoble behavior. Clearly, many more such examples could be generated. It is easy to pick circumstances C such that A and B together produce the most good in C while neither A alone nor B alone produce the most good.

To be sure, those for whom $\mathbf{O}(p . q) \supset \mathbf{O}(p) . \mathbf{O}(q)$ is analytic for all senses of "ought" would not deny that such a situation as that about Tom could arise. They would deny only that an act-utilitarian criterion for "ought," when properly understood, applies to all of (p . q), p, and q of our example about Tom since, being consistent with one another, they do not describe genuine alternatives. But to me the example suggests that $\mathbf{O}(p . q) \supset \mathbf{O}(p) . \mathbf{O}(q)$ is not analytic for the act-utilitarian sense of "ought."

The doubt that $\mathbf{O}(p . q) \supset \mathbf{O}(p) . \mathbf{O}(q)$ is analytic for all senses of "ought" leads to a doubt about the legitimacy of holding that there are any principles of deontic logic. By holding that there are principles of deontic logic I mean holding, implicitly or explicitly that certain formulae containing ought-operators and permitted-operators express analytic truths for all senses of "ought" and "permitted," and that certain other formulae with these operators cannot express analytic truths for any sense of "ought" and "permitted." Examples of explicitly holding that there are principles of deontic logic are A. R. Anderson's requirement of deontic normality on pp. 168-170 of [7] and Castañeda's stricter requirements on pp. 259-260 of [3]. In this essay, I shall not defend my fundamental assumption that there are no principles of deontic logic. In effect, I am assuming that the job of a deontic logician is to discover which formulae express analytic truths for specific senses of "ought" and "permitted" instead of trying to discover formulae which do so for all senses of "ought" and "permitted." So in this study of act-utilitarian deontic logic, I shall simply reformulate a naive act-utilitarian position in such a way that the reformulation readily lends itself to the development of rules for telling which deontic formulae are analytic for this sense of "ought" and "permitted." For this reason, I say that my study is radically different from Åqvist's and Castañeda's.

Before I start my study, I shall cite some references where one can find a basis for evaluating the assumption that there are no principles of deontic logic and I shall say a little bit about the value of studying an ethical theory as I shall study act-utilitarianism. Sidorski in [8], Stennius in [9], von Wright in [10], [11], and [12] reject some of the most fundamental of the alleged principles of deontic logic, if they do not go so far as to suggest that there are no principles of deontic logic. I would maintain that Sidorski and von Wright on p. 29 and pp. 78-81 of [12] do go so far as to deny that there are any principles of deontic logic. But in this connection it is only fair to note that on p. 159 of [3] Castañeda lists several references where he has argued for his principles of deontic logic. The major advantage of taking deontic logic as the determination of which formulae are, and are not, analytic for specific senses of "ought" and "permitted" is that it makes deontic logic a tool for the evaluation of the specific senses. (Here I shall regard specific senses of "ought" and "permitted" as ethical theories.) Let me illustrate this use of deontic logic with some examples. If investigation of a particular sense of "ought" revealed that $(p \supset \mathbf{O}(p))$ is analytic one could judge that the ethical theory was cynical while, on the other hand, if $(\mathbf{O}(p) \supset p)$ were analytic one could judge the ethical theory to be naively optimistic. Investigation of an ethical theory, such as Boh's investigation of Ockham's in [13], may reveal that for it $(\mathbf{O}(p) \supset \mathbf{P}(p))$ is not analytic, where $\mathbf{P}(\ )$ is the permitted-operator. Investigation of an ethical theory may reveal that it has gaps in so far as for it $\sim\mathbf{O}(\sim p) \supset \mathbf{P}(p)$ is not analytic, i.e. it does not have the *nullem crimen sine lege* principle. See pp. 85-88 of [10] for such an investigation. Of course, if an investigation of an ethical theory revealed that for it a paradoxical formula, such as the Good Samaritan formula: $\sim\mathbf{P}(p) \supset \sim\mathbf{P}(p \,.\, q)$, is not analytic, the investigation would reveal a merit in the ethical theory. However, such merits and demerits cannot be discovered in an ethical theory if the ethical theory is reformulated to that it conforms to certain principles of deontic logic. I should add here that my study also differs from Åqvist's and Castañeda's because I use conditional obligation and permission and I modify Hintikka's model system technique for obtaining semantical techniques for deciding which formulae are analytic.

I shall now develop an act-utilitarian deontic logic which I shall call DU. In Part II, I shall present a language for this system. Part III will be definitions of crucial terms plus a sketch of act-utilitarian reasoning. Part IV will be the giving of precise readings for deontic operators. Part V will be an informal development of the semantics while Part VI will be the formal development of the semantics. Throughout the naive version of act-utilitarianism, sketched in III, will be defended by showing that its deontic logic lacks paradoxical formulae and at least hinting that the formulae which are not analytic for it really should not be.

II. *The Language for the System* DU.

Let there be the standard wffs. for a propositional calculus with denumerably many propositional variables: p, q, r, and with subscripts, plus a propositional contstant t, and operator signs: $\sim$, ., $\vee$, $\supset$. Call these the wffs. of propositional logic. To this language and the two deontic operator signs: **P**(/) and **O**(/), plus the rule that **P**(A/B) and **O**(A/B) are wffs. if and only if both A and B are wffs. of propositional logic. Call such wffs. *deontic atoms* and allow the propositional calculus formation rules to apply to deontic atoms. The formulae so obtained are the wffs. of DU.

These formation rules prohibit formulae with deontic operators lying within the scope of deontic operators, where the pair of parentheses after a deontic operator marks its scope. In this essay, I am avoiding problems about the status of sentences such as "It ought to be that it ought to be that p." I shall often talk of deontic formulae. *Deontic formulae* are formulae which contain at least one deontic operator. In this essay, I shall not find it necessary to introduce a conditional forbidden-operator: **F**(/) and unconditional operators: **P**( ), **O**( ), and **F**( ). If I were to use them, I would define **F**(A/B) as **O**($\sim$A/B), **P**(A) as **P**(A/t), **O**(A) as **O**(A/t), and **F**(A) as **F**(A/t), where the propositional constant t is any tautology which contains no variables in A.

To form a system the formulae of this language need a reading interpretation and a logic. A logic is a technique for selecting a subset, hopefully proper, of the wffs. as theses. The reading of $\sim$ as "not," as "and," $\vee$ as "or and $\supset$ as "if ... then ..." are examples of reading interpretations. Logics may be semantical or syntactical. The truth table technique for identifying tautologies is an example of a semantical logic while an axiomatization of classical propositional logic is an example of a syntactical logic. Wffs should be given a reading interpretation before they are given a logic because it is our reading interpretation which guides the development of the logic, if we want the logic to give useful results.

The propositional logic wffs are to be read as they are usually read and the logic that I shall give for them is classical propositional logic. I intend formulae such as **P**(p/q) to mean: "if the situation describable by q occurs the situation describable by p is permissible," and I intend formulae such as **O**(p/q) to mean "if the situation describable by q occurs the situation describable by p is obligatory." However, I shall read them as: "It is permitted that p given q" and "It is obligatory that p given q" respectively. However, these reading interpretations do not give much guidance in developing a semantics for an act-utilitarian deontic logic. So I shall define some terms in Part III which will enable me to give a more precise readings for the deontic operators. These precise readings should express act-utilitarian senses of "permitted" and "ought" and give clear guidance on how to develop a semantical technique for selecting theses. Here it is appropriate to say a bit on the range of the propositional variables in the scope of deontic operators. I intend that the variables to the left of the solidus range over propositions describing human actions and events within the power of human beings to produce. The range of the propositional variables to the right of the solidus is broader. Still, I do not want the q in **O**(p/q) to be a contradiction; I want it to be relevant to human action. However, I have no way in the language of DU for marking that variables in deontic operators are so restricted. I should say that, in this essay, I am hoping, rather than assuming, that this merely informal restriction of the range of the variables will lead to no serious oddities.

III. *Some definitions and Utilitarian Reasoning.*

"Situation" is the primitive term in these definitions. My writing this essay is a situation. The American Civil War is a situation and so is the history of the world up to the present. More formally: An interpretation of a consistent set of wffs from DU, such that not all are theses of classical propositional logic and not all are deontic formulae, is a description of a situation. I would now like to define what I call alternatives in a situation. But I shall not use the term "alternative" since what I would call alternatives need not be incompatible courses of conduct. But, as previously noted, in the literature on utilitarianism and deontic logic alternatives are now taken to be incompatible courses of conduct. So I shall use the artificial term "exit from a situation." If S is a situation, if p and ~p are consistent sentences which are not entailed by any complete or partial description of S, and if q describes all or part of S then, and only then, do: p, ~p, (p . q), and (~p . q) describe *exits from* S. The next definition is the most important but the most complex.

A context C for p given q is a situation S which meets the following conditions.

i) Part of S is described by q.
ii) Both (p . q) and (~p . q) describe exits from S.
iii) A description of S is a description of what is physically possible.
iv) A description of S does not contain any description of events occurring after the time at which the exits (p . q) and (~p . q) are being contemplated.

There is no requirement that the description of S should be a description of what has actually happened. I admit that I have no formal way of indicating physical possibility and time. Still, since the ideas expressed in clauses (iii) and (iv) above guided my development of utilitarian deontic logic, it is important to list them.

Why have I defined "context" in this complex way? Basically, there are two reasons. First there are some general suggestions in Rescher [14] and von Wright [11] on what it means to say that conditional deontic formulae are satisfied. Secondly, I was guided by a rather naive model of act-utilitarian reasoning. The general suggestions on what it meant to say that **O**(p/q) was satisfied amounted to the following. In all (some, most) worlds in which q is true, p describes what is obligatory. I want to read **O**(p/q) in this way and **P**(p/q) in a similar way. Indeed, I want to say in a precise way that **O**(p/q) means that in all possible worlds in which q is true p describes what is obligatory. And this will mean, since I am talking about utilitarianism, that in all possible worlds in which q is true the situation describable by p produces more good than that describable by $\sim$p. However, if I talk of possible worlds I must try to make sense of such talk. My definition of "context" is my attempt to make sense of "a possible world in which q is true."

A characterization of my model of utilitarian reasoning wil show how it guided my definition of "context." When a utilitarian asks whether or not he ought to do the act described by p, hereafter p, he is aware of events that have already occurred and which are relevant to his contemplated action. This is why I say that when a utilitarian asks whether or not he ought to do p, he is asking whether or not he ought to do p given q, where q is the relevant background information. To answer his question the utilitarian must consider all possible consequences of his doing p and of his doing $\sim$p given q. To do this, he would proceed as follows. First, he would make a list of what else physically could have happened in addition to q. Let the items in the list be: A_1, A_2, ..., A_i, ..., etc., where each A_i is a description of how the world could have been granted that q did occur. Let $(q \,.\, A_i)$ represent the set of sentences consisting of q and those in A_i. Each (q . A_i) describes a possible world that the utilitarian must consider. The utilitarian's consideration of a possible world consists of a factual and an evaluative phase. The factual phase is calculating what happens if he does p in the possible world and what happens if he does ~p. Let us take a closer look at this factual phase. Ideally, the closure of the laws of logic applied to the laws of nature, p, and (q . A_i) gives the consequences of doing p in the world described by (q . A_i). Call these consequences: Cpi. Similarly, the closure of the laws of logic applied to the laws of nature, ~p, and (q . A_i) give the consequences of doing ~p in (q . A_i). Call these consequences: C~pi. The factual phase of a utilitarian's consideration of a possible world consists in the calculation of Cpi and C~pi for that world. The factual phase of a utilitarian's deliberation of whether or not he ought to do p given q is finished when he has calculated Cpi and C~pi for each (q . A_i).

Of course, in practice no utilitarian would have a very long list of A_is and the A_is would not be very long. And he would not carry out the deduction of the consequences very far. I call my model of utilitarian reasoning "naive" because I have not introduced the complexities necessary for picking out the relevant possible worlds and the relevant consequences.

The evaluative phase of a utilitarian's consideration of a possible world consists in inspecting the pair (Cpi, C~pi) to determine which is preferable over the other, i.e. which contains more good. The evaluative phase is finished when he has evaluated the (Cpi, C~pi) pair for each (q A_i). But "finished" here means only that he has done all of the evaluating that he can do with the information at hand. It does not mean that he can reach a moral decision. If in all of the pairs (Cpi, C~pi) Cpi is preferable over C~pi, the utilitarian decides that he ought to do p given q. If there is no pair in which C~pi is preferable over Cpi, the utilitarian decides that he at least has permission to do p given q. However, what if there is a pair (Cpi, C~pi) in which Cpi is preferable over C~pi but that there is another pair (Cpj, C~pj) in which C~pj is preferable over Cpj ? In this case, the utilitarian cannot make a moral decision. The preferability of C∼pj over Cpj tells him that p is not permitted given q while the preferability of Cpi over C∼pi tells him that ∼p is not permitted given q. He cannot make a moral decision as long as he gives himself only q as background information. He should now look at $(q \,.\, A_i)$ and $(q \,.\, A_j)$ to see whether or not there are some falsehoods in either A_i or A_j. If he found that there was a falsehood, call it r, in A_j, he would then ask himself whether he ought to do p given q . ∼r. He now starts his factual and evaluative reasoning all over again. He no longer has the troublesome pair (Cpj, C∼pj).

I hope that the above explains why I defined "context" as I did. Intuitively, a utilitarian wants to calculate the consequences of performing an act under certain circumstances, i.e. in a context. But he has only partial knowledge of these circumstances. The partial knowledge is expressed by the q he gives himself. The partial knowledge leaves open many contexts in which he could be acting. To determine all possible consequences of his act he has to consider what results if he performs the act in all of the contexts in which he could be acting. My definition of "context" is supposed to characterize these possible circumstances.

IV. *Precise Utilitarian Readings Deontic Operators.*

P(p/q) says: In no context C for p given q is the (∼p . q) exit preferable over the (p . q) exit.

For instance, if p is "I shoot my wife" and q is "Tom seduced my wife", **P**(p/q) tells me that, no matter what else could have happened up to the present time, I cannot produce more good by not shooting Tom. Admittedly, if the above is what a utilitarian means by "permission," it is extremely difficult to establish that one has utilitarian permission. But in this development of a utilitarian deontic logic, I shall not worry about epistemological problems in utilitarianism. This definition of "permission" is in the spirit of Bergström's definition of "right" an p. 11 of [4], Åqvist's definition of "right" on p. 300 of [6], and G. E. Moore's definition of "right" or "morally permissible" in section 89 of *Principia Ethica.*

$\sim$**P**(p/q) says: There is a context C for p given q such that in C the ($\sim$p . q) exit is preferable over the (p . q) exit.

In other words, an act is not permitted if there is the physical possibility that there are facts not mentioned in q such that if they occurred (p . q) would not describe a course of conduct which produces as much good as that described by ($\sim$p . q). It is very easy to establish that an act is not permitted. But this will not be too bad because we shall see that saying that p is not permitted is not to say that p is forbidden. It might be best to say that $\sim$**P**(p/q) expresses that a claim to have permission to do p given q has been defeated

O(p/q) says: In all contexts C for p given q the (p . q) exit is preferable over the ($\sim$p . q) exit.

In other words, if **O**(p/q) holds you can produce more good by doing p given that q occurred than by doing $\sim$p regardless of what else could have happened besides q. Just as it is very difficult to establish **P**(p/q) it is difficult to establish **O**(p/q).

$\sim$**O**(p/q) says: There is a context C for p given q such that the (p . q) exit is not preferable over the ($\sim$p . q) exit.

In other words, you are not obligated to do p given q if you can point out a circumstance, in addition to q, such that if it had occurred you would produce as much good, or more, by doing $\sim$p. Again, it might be best to say that $\sim$**O**(p/q) expresses that a claim that we have an obligation to do p given q has been defeated. This leaves open the possibility that we could still have the obligation to do p given q plus some other conditions.

In the preceding readings, neither "obligation" nor "permission" are defined in terms of the other. This is as it should be. We should not try to develop the deontic logic for a utilitarian sense of "obligation" and "permission" with some biases about equivalences in deontic logic. In particular, we should not have the bias that *nullem crimen sine lege holds*, i.e. we should not have the bias that $\sim\mathbf{O}(\sim p/q) \supset \mathbf{P}(p/q)$ is a thesis. As we shall see in the next part, $\sim\mathbf{O}(\sim p/q) \supset \mathbf{P}(p/q)$ is not a thesis although $\mathbf{O}(p/q) \supset \sim\mathbf{P}(\sim p/q)$ is.

V. *Informal Semantics.*

I shall now give informal proofs that $\sim\mathbf{O}(\sim p/q) \supset \mathbf{P}(p/q)$ and $\sim\mathbf{P}(p/q) \supset \sim\mathbf{P}(p \,.\, r/q)$ are not theses but that $\mathbf{O}(p/q) \supset \sim\mathbf{P}(\sim p/q)$, $\mathbf{P}(p/q) \supset \sim\mathbf{O}(\sim p/q)$, and $\mathbf{O}(p/q) \supset \mathbf{P}(p/q)$ are theses. I hope these informal proofs will show how I will use the formal techniques and show why I introduce the formal techniques that I do. In these proofs, a formula will be called a thesis if and only if its negation, given the preceding readings, leads to a contradiction.

To show that $\sim\mathbf{O}(\sim p/q) \supset \mathbf{P}(p/q)$ is not a thesis, we show that we can assume that its negation: $\sim\mathbf{O}(\sim p/q) \,.\, (\sim\mathbf{P}(p/q))$, holds. $\sim\mathbf{O}(\sim p/q)$ tells us that there is a context C_1 for p given q in which the $(\sim p \,.\, q)$ exit is preferable over the $(p \,.\, q)$ $(p \,.\, q)$ exit. $\sim\mathbf{P}(p/q)$ tells us that there is a context C_2 for p given q in which $(\sim p \,.\, q)$ exit is preferable over the $(p \,.\, q)$ exit. But this is no contradiction since C_1 and C_2 may be different. Hence, *nullem crimen sine lege* is not a thesis of utilitarian deontic logic.

An example may make this rejection of *nullem crimen sine lege* more palatable. Let q be "Mary is pregnant out of wedlock" and p be "Mary is forced to marry the father." Assume that you are a utilitarian and her brother you asks whether or not he is forbidden to force Mary to marry. I.e., he asks whether or not $\mathbf{O}(\sim p/q)$ holds. But all that he has given as background information is that she is pregnant out of wedlock. On the basis of such meager information you do not want to say that he is forbidden to do so. You can think of other facts which may have occurred and such that if they had occurred her being forced to marry would not produce less good than her not being forced to marry. Here q plus these other possible facts give a context C_1 in which the (∼p . q) exit is not preferable over the (p . q) exit. So you tell Mary's brother that ∼**O**(∼p/q). Yet you caution him that he does not have utilitarian permission to force Mary to marry. You can think of other facts such that if they had occurred her being forced to marry would produce less good than her not being forced to marry. These other facts together with q constitute a context C_2 in which the (∼p . q) exit is preferable over the (p . q) exit. So you tell Mary's brother ∼**P**(p/q).

O(p/q) ⊃ ∼**P**(∼p/q) is a thesis because its negation:

O(p/q) . (**P**(∼p/q)), leads to a contradiction. Neither **O**(p/q). nor **P**(∼p/q) explicitly say that there are any contexts for p given q. So we must assume that if we have **O**(p/q) or **P**(p/q) we have at least one context, call it C_1, for p given q. Now **O**(p/q) says that in C_1 the (p . q) exit is preferable over the (∼p . q) exit. Let us abbreviate this as: (p . q) PREF. (∼p . q). But **P**(∼p/q) says of C_1 that ∼((p . q) PREF. (∼p . q)). So we have the contradiction requsite for showing that **O**(p/q) ⊃ ∼**P**(∼p/q) is a thesis. The heuristic value of this informal proof is that it brings out that the formal semantics shall have to have a rule for assuming that there are contexts for **O**(p/q) and **P**(p/q) if they are not provided by some other formula.

If we try to assume both **P**(p/q) and **O**(∼p/q) we can show that the converse of *nullem crimen sine lege:* **P**(p/q) ⊃ ∼**O**(∼p/q) is a thesis. Again we have to posit the existence of a context C_1 for p given q. For C_1, **P**(p/q) tells us that ∼((∼p . q) PREF. (p . q)) while **O**(∼(∼p/q) tells us that in C_1 (∼p . q) PREF. (p . q). So we have the contradiction requisite for the *reductio* proof that **P**(p/q) ⊃ ∼**O**(∼p/q) is a thesis.

What must be done to have **O**(p/q) ⊃ **P**(p/q) as a thesis ? Try to assume both **O**(p/q) and ∼**P**(p/q) . ∼**P**(p/q) tells us that there is a context C_1 for p given q in which the $(\sim p \,.\, q)$ exit is preferable over the $(p \,.\, q)$ exit, i.e. that in C_1 $(\sim p \,.\, q)$ PREF. $(p \,.\, q)$. $\mathbf{O}(p/q)$ says of C_1 that $(p \,.\, q)$ PREF. $(\sim p \,.\, q)$. We do not yet have a contradiction. But if we assume that preferability is an asymmetric relation, $(p \,.\, q)$ PREF. $(\sim p \,.\, q)$ gives $\sim((\sim p \,.\, q)$ PREF. $(p \,.\, q))$, and we have the requisite contradiction. And it is good for utilitarianism that $\mathbf{O}(p/q) \supset \mathbf{P}(p/q)$ is a thesis. Regardless of what I want to be a thesis of utilitarian deontic logic, one cannot say that it was unreasonable to assume that preferability is an asymmetric relation. In the formal semantics asymmetry is the only property of preferability that I shall assume.

$\sim \mathbf{P}(p/q) \supset \mathbf{P}(p \,.\, r/q)$ could be called a "Good Samaritan" formula. The argument that it is not a thesis will motivate my "exit enrichment rule" as well as showing a merit in utilitarianism, as I have presented it. $\sim \mathbf{P}(p/q)$ tells us that there is a context C_1 for p given q for which uk PREF. uj, where uk is the $(\sim p \,.\, q)$ exit in C_1 and uj is the $(p \,.\, q)$ exit in C_1. C_1 is relevant to $\mathbf{P}(p \,.\, r/q)$, the negation of the consequent, if r has a truth value in exits from C_1. To guarantee that C_1 be relevant to $\mathbf{P}(p \,.\, r/q)$ we enrich the exits uk and uj by disjoining $(r \,.\, \sim r)$ to each. With this enrichement the uk exit becomes in part a $(\sim(p \,.\, r) \,.\, q)$ exit because enriched uk entails $(\sim(p \,.\, r) \,.\, q)$. If enriched uj entailed a $((p \,.\, r) \,.\, q)$ exit $\mathbf{P}(p \,.\, r/q)$ would give us a contradiction by saying that $\sim$(uk PREF. uj). But enriched uj is: $((p \,.\, q) \vee r \,.\, ((p \,.\, q) \vee \sim r)$, which is compatible with $((p \,.\, r) \,.\, q)$ but does not entail it. The refutation of the Good Samaritan formula can be explained as follows. A context in which unpermitted p is done may be one from which there is no exit for rectifying p, viz., doing r. Such a context cannot be used for evaluating doing unpermitted p with its rectification r.

VI. *Formal Semantics.*

I shall now resent an adaptation of the semantic techniques exploited so successfully in [15]: Jaakko Hintikka's *Knowledge and Belief*. However, the following is an extensive addition to Hintikka's techniques so any errors should be attributed to these additions. Besides [15], see [16] and [17] for a discussion of Hintikka's model systems semantics.

A utilitarian deontic model system, a DUMS, is an ordered triple: <S,D,V>. S is a non-empty set of consistent sets of propositional logic formulae. ("S" signifies states of affairs.) Some members of S are included in other members of S and some members of S provide contexts for applying members of D. Members of S are denoted by: u1, u2, u3, etc... Variables used to talk of members of S are: ui, uj, uk, uil, ujl, ukl, etc... D is a non-empty consistent set of formula of which only finitely many are deontic formulae. The sign ua denotes the members of D with no deontic operators in them: ua∈S. V is a consistent set of sentences stating that members of S do or do not stand in a certain asymmetric relation. Of course, the asymmetric relation is intended to be the preferability relation.

A formula F of DU is a thesis of utilitarian deontic logic if and only if the assumption that ~F belongs to the D set of an arbitrarily selected DUMS would lead, by the following rules, to the assumption that one of the S sets, the D set, or the V set of this DUMS contains a contradiction.

The first set of rules are propositional logic rules. They apply to all formulae and sentences. In them the letters b,c stand for any sentence or formula and u stands for any member of S, or D, or V.

PL 1: If b∈u, then ~b~∈u, where "~∈" means "does not belong to."
PL 2: If (b∨c)∈u, then b∈u or c∈u or both do.
PL 3: If ~(b∨c)∈u, then ~b∈u and ~c∈u.
PL 4: If (b . c)∈u, then b∈u and c∈u.
PL 5: If ~(b . c)∈u, then ~b∈u or ~c∈u or both do.
PL 6: If (b⊃c)∈u, then ~b∈u or c∈u or both do.
PL 7: If ~(b⊃c)∈u, then b∈u and ~c∈u.
PL 8. If ~~b∈u, then b∈u.

The above rules suffice to test whether or not a wff. can be assume to be in a consistent set of wffs.. However, if a formula F belongs to one of the sets I talk of, I want to say that some but not all of F's logical consequences belong to the set. In particular, I do not want to say that all wffs. of the form $(p \vee \sim p)$ belong to every consistent set. So if $F \in u$, then the consequences of F and the other members of u, in accordance with Copi's "19 rule system," belong to u. By Copi's "19 rule system" I mean the natural deduction system Copi shows to be incomplete on pp. 54-62 of [18]. However, I modify Copi's addition rule to read as: If $b \in u$, and c occurs in a formula already in u, then $(b \vee c) \in u$.

It will be helpful to give some definitions before giving my DL rules, which apply to the D set of a DUMS. X(p/q) is a variable ranging over deontic atoms and their negations. A member of S, call it ui, is a context for X(p/q) if and only if $q \in ui$ but $p \sim \in ui$ and $\sim p \sim \in ui$. (This requirement comes from clause (iv) of the definition of context which required that the comtemplated action be done after q.) Members of S, call them uj and uk, are exits in ui for X(p/q) if and only if ui is a context for X(p/q), $ui \subseteq uj$, $ui \subseteq uk$, $p \in uj$, and $\sim p \in uk$. (I could just as well have said $p \in uk$ and $\sim p \in uj$, but I shall usually call the exit with p in it uj and the one with $\sim p$ uk.)

DL1 : If $X(p/q) \in D$ and if either $(p \,.\, q)$ or $(\sim p \,.\, q)$ is inconsistent, no further rules apply.

DL 2: If $P(p/q) \in D$, if uj and uk are exits for X(p/q) in ui, and if $p \in ui$ and $\sim p \in uk$, then $\sim$(uk Pref. uj)$\in V$. (In these DL rules the V belongs to the same DUMS as the D set.)

DL 3: If $\sim P(p/q) \in D$, there are exits uj and uk for X(p/q) in ui such that $p \in uj$ and $\sim p \in uk$ and (uk Pref. uj)$\in V$.

DL 4: If $O(p/q) \in D$, if uj and uk are exits for X(p/q) in ui, and if $p \in uj$ and $\sim p \in uk$, then (uj Pref. uk)$\in V$.

DL 5: If $\sim O(p/q) \in D$, there are exits uj and uk for X(p/q) in ui such that $p \in uj$ and $\sim p \in uk$ and $\sim$(uj Pref. uk)$\in V$.

DL 6: The exit enrichment rule. Let $X_1(A/q)$ and $X_2(B/q)$ belong to D. B and A are wffs. of propositional logic and let r be any well-formed part of A. Let uj and uk be exits for X_2(B/q) in ui and let B∈uj and ∼B∈uk. Under these conditions (B∨(r . ∼r))∈uj and (∼B∨(r . ∼r))∈uk.

DL 7: If DL 3 and DL 5 do not give a context ui for X(p/q) with uj and uk exits, there is such a context and exits if X(p/q)∈D.

DL 8: If F is a wff. of propositional logic and F∈D, then F∈ua and ua∈S.

The last rule is a special rule for the V set of sentences. It tells us that the relation of being preferable over, i.e. — Pref. —, is asymmetric.

VL 1: If (uj Pref. uk)∈V, then ∼(uk Pref. uj)∈V.

I shall first illustrate use of these rules by showing that **O**(p/q)⊃**P**(p/q) is a thesis but that **O**(p/q)⊃(p⊃p) is not. (For conditional operators **O**(p/q)⊃(q⊃p) is the analogue of **O**(p) ⊃p.)

1)		**O**(p/q) . (∼**P**(p/q))∈D.	*Reductio* assumption
2)	a)	**O**(p/q)∈D.	PL 4 on (1).
	b)	∼**P**(p/q)∈D.	
3)	a)	ui∈S, uj∈S, uk∈S	DL 3 on (2b) gives the context
	b)	q∈ui, ui⊆uj, ui⊆uk	ui with uj and uk exits for
	c)	p∈uj, ∼p∈uk	∼**P**(p/q).
	d)	(uk Pref. uj)∈V.	
4)		(uj Pref. uk)∈V.	DL 4 on (2a) and (3a, b, c).
5)		∼(uk Pref. uj)∈V.	VL 1 on (4).

(3d) and (5) give the desired contradiction to show that **O**(p/q)⊃**P**(p/q) is a thesis. The other informal proofs of thesishood could also be formally given. However, we can use these formal techniques to show that **O**(p/q)⊃(q⊃p) is not a thesis.

1)		**O**(p/q) . (∼(q⊃p))∈D.	Assume for attempted *reductio*.
2)	a)	**O**(p/q)∈D	PL 4 on (1).
	b)	∼(q⊃p)∈D.	
3)	a)	q∈D	PL 7 on (2b).
	b)	∼p∈D.	

DL 7 on (2a) will give a context ui for $\mathbf{O}(p/q)$ with uj and uk exits. Then DL 4 will tell us that (uj Pref. uk)$\in$V. But there is no contradiction here. Of course, it is a merit in utilitarianism that this formula is not a thesis. Another merit in utilitarianism is revealed when we see that the following Ross paradox: $\mathbf{O}(p/q) \supset \mathbf{O}(p \vee r/q)$, is not a thesis.

1)	$\mathbf{O}(p/q) . (\sim \mathbf{O}(p \vee r/q)) \in D$.	Assumption for attempted *reductio.*
2)	a) $\mathbf{O}(p/q) \in D$	PL 4 on (1).
	b) $\sim \mathbf{O}(p \vee r/q) \in D$.	
3)	a) $ui \in S$, $uj \in S$, $uk \in S$	DL 5 on (2b).
	b) $q \in ui$, $ui \subseteq uj$, $ui \subseteq uk$	
	c) $(p \vee r) \in uj$, $\sim (p \vee r) \in uk$	
	d) $\sim$ (uj Pref. uk)$\in$V.	
4)	a) $\sim p \in uk$	PL 3 on $\sim (p \vee r) \in uk$ in (3c).
	b) $\sim r \in uk$.	We now know that uk has a (p . q) exit in it.

Now PL 2 on $(p \vee r) \in uj$ in (3c) tells us that we must put either p or r into uj. If we put p into uj we have (p . q) in uj. If we have $(p . q) \in uj$, then DL 4 on (2a) together with the result of (4) tells us that (uj Pref. uk)$\in$V. But that would contradict (3d). However, we can put r into instead of p, and if we do this we do not get a contradiction.

A formal version of the informal proof that a certain Good Samaritan formula is not a thesis would reveal another merit in utilitarian logic. However, the last proof I give will show that a "healthy" Good Samaritan formula is a thesis. It is:

$\sim \mathbf{P}(p/(p \supset r) . q) \supset \sim \mathbf{P}(p . r/(p \supset r) . q)$.

1)	$\sim \mathbf{P}(p/(p \supset r) . q)$ and $\mathbf{P}(p . r/(p \supset r)q)$.	*Reductio* assumption.
2)	a) $ui \in S$, $uj \in S$, $uk \in S$	DL 3 on $\sim \mathbf{P}(p/(p \supset r) . q)$ in (1).
	b) $((p \supset r . q) \in ui$, $ui \subseteq uj$, $ui \subseteq uk$.	
	c) $p \in uj$, $\sim p \in uk$	
	d) (uk Pref. uj)$\in$V.	

Now the kind of propositional calculus reasoning I allow tells us that $(p \,.\, r) \in uj$ and that $(\sim(p \,.\, r)) \in uk$. Then, if we apply DL 2 to $\mathbf{P}(p \,.\, r/(p \supset r) \,.\, q)$ in (1), we get $\sim$(uk Pref. uj)$\in V$. And we have the requisite contradiction with (2d).

The suspicion may arise utilitarian deontic logic is very weak in the sense that it does not have as theses many formulae which one would expect to be theses. The suspicion is justified. But I think such a weakness is really a strenght, both logically and ethically. The more one must revert to their primary way of establishing claims of obligation and permission the better it is. There is more chance to catch mistakes in the moral reasoning. Also I think that it is best that we extend our propositional logic as little as possible beyond classical propositional logic. I shall close by tabulating some results. A sequel to this paper would explain these results primarily by explaining why it is not odd that certain formulae are not theses.

Sixteen results about distribution:

	A thesis ?
1) $\mathbf{P}(p \vee r/q) \supset \mathbf{P}(p/q) \,.\, \mathbf{P}(r/q)$	No
2) $\mathbf{P}(p/q) \,.\, \mathbf{P}(r/q) \supset \mathbf{P}(p \vee r/q)$	Yes
3) $\mathbf{P}(p \vee r/q) \supset (\mathbf{P}(p/q) \vee \mathbf{P}(r/q))$	No
4) $(\mathbf{P}(p/q) \vee \mathbf{P}(r/q)) \supset \mathbf{P}(p \vee r/q)$	No
5) $\mathbf{P}(p \,.\, r/q) \supset \mathbf{P}(p/q) \,.\, \mathbf{P}(r/q)$	No
6) $\mathbf{P}(p/q) \,.\, \mathbf{P}(r/q) \supset \mathbf{P}(p \,.\, r/q)$	Yes
7) $\mathbf{P}(p \,.\, r/q) \supset \mathbf{P}(p/q) \vee \mathbf{P}(r/q)$	No
8) $(\mathbf{P}(p/q) \vee \mathbf{P}(r/q)) \supset \mathbf{P}(p \,.\, r/q)$	No
9) $\mathbf{O}(p \vee r/q) \supset \mathbf{O}(p/q) \,.\, \mathbf{O}(r/q)$	No
10) $\mathbf{O}(p/q) \,.\, \mathbf{O}(r/q) \supset \mathbf{O}(p \vee r/q)$	Yes
11) $\mathbf{O}(p \vee r/q) \supset (\mathbf{O}(p/q) \vee \mathbf{O}(r/q))$	No
12) $(\mathbf{O}(p/q) \vee \mathbf{O}(r/q)) \supset \mathbf{O}(p \vee r/q)$	No
13) $\mathbf{O}(p \,.\, r/q) \supset \mathbf{O}(p/q) \,.\, \mathbf{O}(r/q)$	No
14) $\mathbf{O}(p/q) \,.\, \mathbf{O}(r/q) \supset \mathbf{O}(p \,.\, r/q)$	Yes
15) $\mathbf{O}(p \,.\, r/q) \supset (\mathbf{O}(p/q) \vee \mathbf{O}(r/q))$	No
16) $(\mathbf{O}(p/q) \vee \mathbf{O}(r/q)) \supset \mathbf{O}(p \,.\, r/q)$.	No.

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