MANY-VALUED LOGIC AND FUTURE CONTINGENCIES

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Those who think that the existence of future contingencies entails a denial of the principle of bivalence must give some account of how a proposition which at a particular time is neither true nor false can, at that time, be meaningful and be combined with other propositions in a system of deductive inference. This paper attempts to examine some of the alternatives to ordinary, two-valued logic that arise in this connection, and to explore the logical and metaphysical consequences that result from them.

The first such alternative is the three-valued system of Łukasiewicz (hereafter L_1). The definition of the third truth value as well as the reasons for accepting it are given by Łukasiewicz himself in the following passage: (1)

> I can assume without contradiction that my presence in Warsaw at a certain moment of next year, e.g. at noon on 21 December, is at present time determined neither positively nor negatively. Hence it is *possible*, but not *necessary*, that I shall be present in Warsaw at the given time. On this assumption the proposition 'I shall be in Warsaw at noon on 21 December of next year', can at the present time be neither true nor false. For if it were true now, my future presence in Warsaw would have to be necessary, which is contradictory to the assumption. If it were false now, on the other hand, my future presence in Warsaw would have to be impossible, which is also contradictory to the assumption. Therefore the proposition considered is at the moment *neither true nor false* and must possess a third value, different from '0' or falsity and

(1) Jan ŁUKASIEWICZ, "Philosophical Remarks on Many-Valued Systems of Propositional Logic," in *Polish Logic 1920-1939*, ed. Storrs McCall (Oxford: Clarendon Press, 1967), p. 53.

> '1' or truth. This value we can designate by '$^1/_2$'. It represents 'the possible', and joins 'the true' and 'the false' as a third value.

This passage has two hidden but crucial assumptions. First, it assumes that "I shall be present in Warsaw at noon on 21 December of next year" is a meaningful proposition now. Few would question this assumption, but it is worth noting because it is directly related to a second and more questionable one: a proposition must have a truth value to be meaningful. This assumption will be considered later; for now, it is sufficient to realize that it is crucial to Łukasiewicz' argument. He is claiming that because future contingent propositions are meaningful but neither true nor false, they "must possess a third value."

L_1 is defined according to the following matrix: (I will not use Łukasiewicz' own notation here.)

$\supset$	1	$^1/_2$	0	$\sim$
1	1	$^1/_2$	0	0
$^1/_2$	1	1	$^1/_2$	$^1/_2$
0	1	1	1	1

(Column on the extreme left gives antecedent, topmost row gives consequent.)

The cases deserving special attention are those in which one of the terms takes the third value. If the antecedent takes $^1/_2$ and the consequent 1 or the antecedent 0 and the consequent $^1/_2$, the resulting implication takes 1 since the outcome of the indeterminate constituent cannot affect the truth value of the whole implication. Yet, if the antecedent takes 1 and the consequent 0, the resulting implication takes $^1/_2$ since it can become either true or false. So far so good. But consider the case where both antecedent and consequent take $^1/_2$. This proposition must take 1 for the simple reason that if it did not, L_1 would have no tautologies. If such propositions took $^1/_2$, no formula (not even $P \supset P$) could ever take 1 in all cases. Unfortunately, assigning such propositions 1 in every case avoids one set of difficulties only to create another. We are left with a situation in which *any* two contingent propositions imply each other. At this point, it is important to understand the deeper problem of which this odd result is symptomatic.

The problem arises from the apparently harmless fact that if it is not yet determined whether an event will occur, it is not yet determined whether it will not occur either. This fact is reflected in Łukasiewicz' matrix for negation: the negation of an indeterminate proposition is also indeterminate. But if an indeterminate proposition and its negation always take the same value, they will be logically interchangeable. Let us then reconsider the case of an implication whose terms both take $^1/_2$. Clearly, there are some instances in which we would say that such implications are true. We would not, for example, expect anyone to wait to see whether

(1) If there will be a sea-fight tomorrow, then there will be a sea-fight tomorrow

turns out to be true. But neither would we expect anyone to say that

(2) If there will be a sea-fight tomorrow, then it is not the case that there will be a sea-fight tomorrow

is to be counted as true merely because the occurrence of the sea-fight is now contingent. The problem is that Łukasiewicz had to decide whether to make all such implications true or whether to make none of them true, while our intuitions tell us that some ought to be true and some not.

Similar problems arise in regard to disjunction and conjunction. The tables for both operations are given below:

$\vee$	1	$1/2$	0
1	1	1	1
$1/2$	1	$1/2$	$1/2$
0	1	$1/2$	0

$\wedge$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$1/2$	$1/2$	0
0	0	0	0

In keeping with the belief that the value of a disjunction should be the maximum value of its disjuncts while that of a conjunction should be the minimum value of its conjuncts, L_1 assigns $1/2$ to disjunctions and conjunctions when both terms take this value. The difficulty is that both $P \vee \sim P$ and $\sim(P \wedge \sim P)$ fail to become theses of the system. This is most unfortunate since Aristotle, whom Łukasiewicz cites as his major authority in rejecting bivalence, most certainly believed in excluded middle. (2) Following Aristotle, there is a temptation to say that the value of a disjunction made up of two indeterminate disjuncts is also indeterminate except when one is the denial of the other ——— its value then being 1. Similarly, we could say that when both conjuncts are indeterminate, the resulting conjunction takes $1/2$ except when one is the denial of the other, in which case it takes 0. Yet, this avenue of escape is closed so long as we keep to the principle of truth functionality.

Looking at L_1 as a whole, it should be clear that all of its tautologies are tautologies of classical logic. On the other hand, the converse is hardly the case, and a fair number of standard argument patterns (including reductio ad absurdum) fail to hold in L_1. Other examples are:

(3) $(\sim P \supset P) \supset P$
(4) $(P \supset \sim P) \supset \sim P$
(5) $(P \supset (Q \wedge \sim Q)) \supset \sim P$
(6) $(P \wedge (P \supset Q)) \supset Q$.

(2) ARISTOTLE, *De Interpretatione*, Ch. 9, 19a30-35.

C.I. Lewis points out that classical tautologies not valid in L_1 can often be made the consequents of implications that are valid in L_1 by prefacing them with either P or ~P. (3) This is easily seen since if a classical tautology takes $1/2$ when, say, P does, then an implication that has P or ~P as the antecedent and that formula as the consequent will be an implication whose terms both take $1/2$. Here there is a strong tendency to posit a link between L_1 and classical logic by saying that any formula valid in classical logic but not in L_1 can be made the consequent of a valid implication by making P or ~P the antecedent if the formula is not satisfied in L_1 when P takes $1/2$. But J. C. C. McKinsey found an exception in:

(7) $\sim(((\sim P \supset P) \supset P) \supset \sim((\sim P \supset P) \supset P))$. (4)

There is also a strong tendency to link the two systems by saying that those formulas that are valid in classical logic but not in L_1 can never take 0 in L_1 but only $1/2$. In other words, L_1 cannot actually falsify a classical tautology, it can only validate it or assign it $1/2$. If this claim were true, the relationship between the two systems would be greatly simplified; and it must be noted that both Prior (5) and Rescher (6) at one time maintained that it is true, although recently Rescher has acknowledged his mistake. (7) What both men had failed to see is that their claim is preceisely what was at issue in the thesis McKinsey refuted. If Prior's and Rescher's original claim were true ——— i.e. if classical tautologies not valid in L_1 could take only $1/2$ not 0 ——— then prefacing them with P or ~P when P takes $1/2$ would have to yield an implication

(3) C. I. Lewis and C. H. Langford, *Symbolic Logic*, 2nd ed. (1932; rpt. New York: Dover Publications, 1959), pp. 220-222.

(4) J. C. C. McKinsey, "A correction to Lewis and Langford's *Symbolic Logic*," *Jour. Symbolic Logic*, V (1940), p. 149.

(5) A. N. Prior, "Logic, Many-Valued," in *The Encyclopedia of Philosophy*, ed. P. Edwards (New York: Macmillan, 1967), V, 3.

(6) N. Rescher, *Topics in Philosophical Logic* (Dordrecht: D. Reidel Publishing Co., 1968), pp. 82-83.

(7) N. Rescher, *Many-Valued Logic* (New York: McGraw-Hill, 1969), p. 27.

that is valid in L_1 since, as we have seen, $P \supset Q$ is always true when P and Q both take $^1/_2$. But since there is, as McKinsey showed, an exception to this rule, there must also be an exception to the thesis of Prior and Rescher. Or, put another way, the formula:

(8) $P \supset \sim(((\sim P \supset P) \supset P) \supset \sim((\sim P \supset P) \supset P))$

is a classical tautology which takes 0 when P is $^1/_2$. Two other exceptions are:

(9) $\sim(\sim(P \wedge \sim P) \supset (P \wedge \sim P))$
(10) $\sim((P \vee \sim P) \supset \sim(P \vee \sim P))$.

The reason I have belabored this point is that if Prior's and Rescher's original claim were true, it would be possible to argue that L_1 could be made to include all the tautologies of classical logic simply by treating both 1 and $^1/_2$ as designated values.

Another point worthy of attention is that L_1 makes it impossible in general to infer that $\Vdash A \supset B$ if $A \Vdash B$; this appears implausible since in classical logic we can infer either one from the other. But consider the formula $P \wedge (P \supset Q)$. From this formula we can deduce Q by using inferences that Łukasiewicz would accept. But as we have seen, $(P \wedge (P \supset Q)) \supset Q$ is not valid in L_1 since it takes $^1/_2$ when P is $^1/_2$ and Q is 0. Once more, we have come across an unfortunate result since it forces Łukasiewicz either to abandon or to seriously modify the semantic analogue of the classical deduction theorem. On the other hand, Łukasiewicz can still maintain that for all A and B, if $\Vdash A \supset B$, then $A \Vdash B$.

L_1 also contains modal operators. Translating MP as "it is possible that P" and taking M as primitive, we get the following table, where

$IP = \sim MP$; $SP = \sim M \sim P$; and $QP = MP \wedge M \sim P$:

P	MP	IP	SP	QP
1	1	0	1	0
$^1/_2$	1	0	0	1
0	0	1	0	0

From this table it can be proved that $S(P \supset Q) \supset (SP \supset SQ)$, $SP \supset P$, $SP \supset SSP$, and $P \supset SMP$ are all theses of the system. As Prior notes, the fact that the modal functions never take $^1/_2$ is consistent with our intuitions and enables Łukasiewicz to hold $MP \vee \sim MP$ as a thesis. (8) Furthermore, Tarski has shown that possibility can be defined in terms of implication and negation, as follows: $MP = \sim P \supset P$. This appears counter-intuitive since in classical logic P is materially equivalent to $\sim P \supset P$, but in L_1, $\sim P \supset P$ is false iff P is false and thus has the same truth value as MP.

From the standpoint of the problem of future contingencies, it is significant that in L_1 we can infer the necessity of P from the truth of P and the impossibility of P from its falsehood. Now this does not mean that if P is true, it must be *logically* true. The necessity that Łukasiewicz has in mind is to be understood along the lines of the definiteness we associate with past and present events. While it is now necessary in the sense of being unalterable that Caesar crossed the Rubicon, that he did is hardly a logical truth nor, as far as we know, was he compelled to do so. Hence it is contingent today whether there will be a sea-fight tomorrow; but the day after tomorrow it will be necessary either that there was a sea-fight the day before or necessary that there was not. Prior, however, warns that the fact that we can infer the necessity of P from the truth of P cannot be formalized as $P \supset SP$. (9) When P takes $^1/_2$, SP will be false and thus

(8) A. N. PRIOR, *Formal Logic*, 2nd ed. (1955; rpt. Oxford: Clarendon Press, 1962), p. 249.

$P \supset SP$ will also take $1/2$ and not be a law of the system. Likewise, $\sim P \supset IP$ will take $1/2$ when $\sim P$ does. Consequently the correct formalizations are $P \supset (P \supset SP)$ and $\sim P \supset (\sim P \supset IP)$.

We must now ask whether anything can be done to L_1 to eliminate its more objectionable features. The first suggestion that comes to mind is the introduction of a fourth truth value. By having four values, one could hold that a proposition is contingent but that it and its denial take different values. It should be noted that Łukasiewicz himself rejected L_1 in favor of a four-valued system which validates all the tautologies of classical logic. [10] The four-valued system that follows (hereafter L_2) is not the one that Łukasiewicz actually proposed, but is, I think, a reasonably close approximation to it. [11]

First, form ordered pairs of the traditional values thus: (1, 1), (1, 0), (0, 1), and (0, 0). Next define truth values for implication and negation in the following manner:

(11) $(P, Q) \supset (R, S) = (P \supset Q, R \supset S)$.
(12) $\sim (P, Q) = (\sim P, \sim Q)$.

Finally, accept the following abbreviations: (1, 1) = 1, (1, 0) = 2, (0, 1) = 3, and (0, 0) = 0. This gives us the following matrix, which is the direct power M^2 of the classical, two-valued matrix for implication and negation:
Here "1" and "0" again stand for truth and falsity, and "2" and "3" stand for alternative species of indeterminacy. From the matrix it is clear that the negation of one kind of indeterminacy is not that same kind but the other; hence one "objectionable" aspect of L_1 has been eliminated. But notice what happens when the antecedent and the consequent of an implication both

(9) *Ibid.*, pp. 248-249.

(10) Jan Łukasiewicz, *Aristotle's Syllogistic*, 2nd ed. (1951; rpt. Oxford: Clarendon Press, 1957), pp. 154-180.

(11) For the sake of clarity and simplicity, I have modified Łukasiewicz' own system by not introducing modal functions capable of taking values other than 1 or 0 and by allowing "2" and "3" to designate indeterminate values.

$\supset$	1	2	3	0	$\sim$
1	1	2	3	0	0
2	1	1	3	3	3
3	1	2	1	2	2
0	1	1	1	1	1

take the same indeterminate value. The resulting implication is true ——— precisely that aspect of L_1 we had hoped to avoid. In defense of the fourth value, though, one could argue that this result is not quite as distressing in L_2 as it was in L_1 since at least cases like (2) above have been eliminated. A contingent proposition and its denial now must have different indeterminate values making cases like (2) either 2-indeterminate or 3-indeterminate but never true. Once more, however, our intuitions tell us that if both the antecedent and consequent are indeterminate in the same way, the resulting implication ought to be true if it is a classical tautology like $P \supset P$ and indeterminate otherwise. In general, given a system of logic with n truth values where n is finite, any assignment of values to $n + 1$ contingent propositions will have to give at least two the same value. In other words, the question of whether to count implications whose terms both take the same indeterminate value as true or as indeterminate cannot be avoided merely by introducing a finite number of additional values.

Turning to disjunction, we get the following matrix, keeping in mind that $P \vee Q = (P \supset Q) \supset Q$:
The most startling aspect of this matrix is that excluded middle is a thesis of the system. Unfortunately, this "luxury" was purchased at a very high price. Consider the case where the disjuncts take different indeterminate values; the resulting disjunction is always true. Indeed, the fact that such cases are counted as true is why excluded middle is now a thesis. But it is still hard to accept the fact that in all such cases the disjunction is true. It will be remembered that in L_1 Łukasiewicz

~P	P	Q	P∨Q	P∨~P
0	1	1	1	1
0	1	2	1	1
0	1	3	1	1
0	1	0	1	1
3	2	1	1	1
3	2	2	2	1
3	2	3	1	1
3	2	0	2	1
2	3	1	1	1
2	3	2	1	1
2	3	3	3	1
2	3	0	3	1
1	0	1	1	1
1	0	2	2	1
1	0	3	3	1
1	0	0	0	1

had to decide between making excluded middle a thesis or making the value of a disjunction the maximum value of either of its disjuncts. He chose the latter and assigned disjunctions composed of two indeterminate disjuncts $1/2$. Now, since the value of a disjunction can be greater than the value of either of its disjuncts, the former has been chosen. By introducing a fourth value, we have not resolved the dilemma but merely opted for a different side of it. The fourth value has gotten us nowhere, and it is clear that a fifth value indeterminate between 2 and 3 would fare no better. In addition, it is easy to see that a quandary exactly parallel to this one would arise with conjunction and noncontradiction.

It is true, of course, that all classical tautologies are tautologies of L_2 and vice versa. Yet, the superficial characteristics of this system are again deceptive. The whole reason for introducing indeterminate values was to avoid fatalism. But just as classical logic cannot assign a proposition a value indeterminate between truth and falsity, L_2 cannot assign one a value indeterminate between its two types of indeterminacy. Consider the following four arguments where P predicts an event coming before the one predicted by Q:

(13) Assume that P is 3 and Q is 3. Then $P \supset Q$ is true. Then if P turns out true, Q must turn out true also.
(14) Assume that P is 3 and Q is 2. Then $P \supset \sim Q$ is true. Then if P turns out true, Q must turn out false.
(15) Assume that P is 2 and Q is 3. Then $\sim P \supset Q$ is true. Then if P turns out false, Q must turn out true.
(16) Assume that P is 2 and Q is 2. Then $\sim P \supset \sim Q$ is true. Then if P turns out false, Q must turn out false as well.

What these arguments show is that we can begin with any contingent proposition and form an implication connecting it with any subsequent, contingent proposition, and regardless how the first one turns out, deduce how the second one must turn out simply by knowing what indeterminate value the second one takes. Nor is this difficulty really avoided by pointing out that all these arguments show is that we may not *know* what value the second proposition takes. If we cannot know what indeterminate value a proposition had until it becomes either true or false, then we cannot reason with contingent propositions when they are contingent and one of the primary advantages of this system would be lost. In addition, even though we may not know what indeterminate value a contingent proposition has, unless we introduce a category of having no truth value at all, it will have to have one indeterminate value or the other, making the event it predicts fated even though we are not aware of how. In short, L_2 commits us to much the same type of universe that classical logic does.

Let us now consider a system that (a) does not rest on the tacit assumption that meaningful propositions must have truth values and (b) is only partially truth functional. No doubt, an adequate treatment of the belief that meaningful propositions must have truth values is far beyond the scope of this paper. On the other hand, it is safe to say that although this assumption is by no means obvious, Łukasiewicz and many of his followers seem to regard it as self-evident. For instance, Steven Cahn, in *Fate, Logic, and Time* sounds very much like Łukasiewicz in saying: ([12])

> But if it is not true that a sea-fight will occur tomorrow, and if it is also not false that a sea-fight will occur tomorrow, what truth-value does the proposition "there will be a sea-fight tomorrow" have ? It must have a truth-value to be meaningful, and it certainly is a meaningful proposition ... If it is not true and it is not false, it must have a third truth-value, and it is such a truth-value which three-valued logic provides.

Since truth and falsity are the values we assign to propositions about the past and present, it is misleading to give contingent propositions a value that is somehow intermediate between the

([12]) Steven M. CAHN, *Fate, Logic, and Time* (New Haven: Yale Univ. Press, 1967), pp. 125-126.

two. That is, to say that 1/2 is intermediate between 1 and 0 would seem to require us to alter our traditional notions of truth and falsity since part of what we mean by "truth" and "falsity" is that there is no intermediate status. If a proposition has a truth value, the traditional view tells us that it must be either true or false. It is, then, less controversial to argue that future contingent propositions take no truth value than to posit a third value on a full semantic par with truth and falsity yet midway between them. According to this approach, future contingent propositions do not occupy a mysterious halfway house but, rather, are not *yet* true or false. (13) When they are contingent, they have no truth value and this fact, I would argue, in no way prevents them from being meaningful.

As a result of confusing not *yet* being true or false with having an intermediate value, Łukasiewicz was led to believe that L_1 could be truth functional. Since "1/2" represented another truth value rather than a temporary denial of truth value altogether, Łukasiewicz, it seems, thought that its introduction into the ordinary, two-valued calculus did not have to affect the way that simple formulas combine to determine the values of complex ones. Accordingly, the truth values of complex formulas in L_1 are a simple function of 1, 0, and 1/2 rather than of just 1 and 0. Yet we have seen that both L_1 and L_2 are plagued by certain dilemmas. The seriousness of these dilemmas, as well as our inability to solve them, suggests that the admission of a third alternative does affect the way that values are assigned to complex formulas — — — that 1/2 is not just a third value to be given the same status as 1 and 0 but a determination of a different kind. Moreover, Aristotle himself appears to have arrived at a similar conclusion in Chapter 9 of *De Interpretatione*. Although there is room for doubt, Aristotle appears to be denying bivalence while affirming excluded middle. (14) Thus $P \vee \sim P$ is true even

(13) For a similar argument, see R. J. BUTLER, "Aristotle's Sea Fight and Three-Valued Logic," *Phil. Review*, LXIV (1955), 264-274.
Free Logic," *Jour. of Phil.*, LXIII (1966), 481-495.

(14) ARISTOTLE, *op. cit.*, 19a30-19b4.

if neither of its disjuncts is true yet. But, as Prior remarks, Aristotle's reason for affirming excluded middle could hardly have been that all disjunctions whose disjuncts are "neuter" are to be counted as true by fiat. ([15]) It is true not because its truth value is a function of its disjuncts, but because one disjunct is the denial of the other. That is, it is true because it has the form of a tautology and this consideration takes precedence over the fact that neither of its disjuncts is yet true.

Allowing "A" and "B" to range over formulas and using "V(A)" to designate the value of A, I propose a system in which a valuation V gives some sentence parameters truth values, and leaves others undefined. ([16])

(17) $V(\sim A) = T$ iff $V(A) = F$ or A is a classical contradiction.
$= F$ iff $V(A) = T$ or A is a classical tautology.
Undefined otherwise.

(18) $V(A \supset B) = T$ iff $V(A) = F$ or $V(B) = T$ or $A \supset B$ is a classical tautology.
$= F$ iff $V(A) = T$ and $V(B) = F$, or $A \supset B$ is a classical contradiction.
Undefined otherwise.

It should be clear from these definitions that this system will validate all those and only those tautologies of classical logic. Following Aristotle, $P \vee \sim P$ is true although both its disjuncts may be undefined. In addition, the system allows us to employ our normal argument schemes to contingent propositions since even though $\sim P$ is undefined if P is, it is not true, as it was in L_1, that every contingent proposition implies its own negation.

If my argument is correct, the logical character of the future is unlike that of the past or present not only because some propositions predicting future events are neither true nor false, but because the logic of the contingent future is not truth functional while (excluding cases like category mistakes

([15]) A. N. PRIOR, *Formal Logic*, p. 244.

([16]) Cf. B. C. VAN FRAASSEN, "Singular Terms, Truth-Value Gaps, and Free Logic," *Jour. of Phil.*, LXIII (1966), 481-495.

and presupposition failures) that of the past and present is. In other words, at every instant in time, a totally new set of propositions acquires truth values and thus can be incorporated into a truth functional logic. It follows that temporal passage is a genuine feature of reality and cannot be analyzed as an illusion or an appearance. It also follows that propositions with explicit temporal references cannot be converted into equivalent "tenseless" propositions in the manner suggested by Quine (17) and Williams. (18)

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(17) W. V. O. QUINE, *Elementary Logic*, rev. ed. (1941; rpt. New York: Harper and Row, 1965), p. 6.

(18) D. C. WILLIAMS, "The Sea Fight Tomorrow," in *Structure, Method, and Meaning*, eds. Paul HENLE, Horace M. KALLEN, and Susanne K. LANGER (New York: The Liberal Arts Press, 1951), pp. 282-306.