INTENSIONAL ISOMORPHISM

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Several philosophers have suggested intensional isomorphism as a necessary and sufficient condition of synonymy. (1) *One* motive behind this suggestion has been the desire to provide a condition strong enough to guarantee interchangability *salva veritate* in opaque contexts, e.g. belief-sentences. The concept has been discussed almost entirely from this angle. Does it, or does it not, guarantee interchangability in opaque contexts ? Lewis, Carnap, and Putnam have said it can. Mates, Scheffler, and Church have said that it cannot. (2) It is unfortunate, I feel, that intensional isomorphism has been discussed only from this angle. For it blinds us to the facts (a) that there are other reasons for insisting upon such a condition for synonymy, and (b) that there may be objections to it other than that it fails to guarantee interchangability in opaque contexts. At any rate, I am not going to discuss it from this angle, both for the reasons I have mentioned and others. For one thing, I doubt if much more can be said about intensional isomorphism vis-a-vis opaque contexts. More important, I believe that interchangability in opaque contexts is irrelevant to the question of synonymy. That is, I think it possible to provide a criterion of synonymy which distinguishes it from both extensional equivalence and L-equivalence without appealing to interchangability in opaque contexts.

(1) see C. I. Lewis, "The Modes of Meaning". *Phil. and Phen. Research.* 1943-44., and R. Carnap, *Meaning and Necessity.* (Chicago. 1967. 5th impression).

(2) H. Putnam, "Synonymity and the analysis of belief-sentences". *Analysis* 1954: B. Mates, "Synonymity" in *Semantics and the Philosophy of Language. ed. Linsky* (Illinois 1952). I. Scheffler, "On Synonymy and Indirect Discourse". *Phil. of Science.* 1955: A. Church, "Intensional Isomorphism and Identity of Belief". *Phil. Studies.* 1954.

I do not here defend this contentious claim. My aims here are purely negative.

Apart from the intrinsic interest of discovering a criterion of synonymy, there is a further reason why discussion of intensional isomorphism is valuable. Philosophers have claimed that logical truths are true in virtue of synonymy relations holding between constants. On this view "S" is a logical truth if it can be reduced to an instance of the law of non-contradiction once synonyms replace synonyms. (1) For example, $(p \vee q) \rightarrow \sim(\sim p \& \sim q)$ will be a logical truth because antecedent and consequent are synonymous. This view, however, must be false if intensional isomorphism is made a condition for synonymy, since the formulae $(p \vee q)$ and $\sim(\sim p \& \sim q)$ are not isomorphic. To leave open the view that logical truths are such in virtue of synonyms, then, it is necessary to repudiate the suggestion that intensional isomorphism be made a condition for synonymy.

I am going to argue that intensional isomorphism should not be made a condition for synonymy. The results of doing so would be too counter-intuitive to swallow. What, quite, is intentional isomorphism ? I shall consider Carnap's account, and, although other accounts differ from his, I think my objections to his account would also be objections to these others. Carnap's formal definition of intensional isomorphism runs as follows:

> "a. Let two designator matrices be given, either in the same or in two different semantical systems, such that neither of them contains another designator matrix as proper part. They are intensionally isomorphic $=_{Df}$ they are L-equivalent.
> b. Let two compound designator matrices be given, each of them consisting of one main submatrix (of the type of

(1) It should not be thought a knockdown objection that, on this view, the law of non-contradiction cannot itself be true in virtue of synonymy relations. It might be argued that this logical truth is quite special on various other grounds. See, for example, P. STRAWSON, *Intro. to Logical Theory*.

a predicator, functor, or connective) and n argument expressions (and possibly auxiliary signs like parentheses commas, etc.). The two matrices are intensionally isomorphic $=_{Df}$ (1) the two main submatrices are intensionally isomorphic, and (2) for any m from 1 to n, the mth argument expression within the first matrix is intensionally isomorphic to the mth in the second matrix ('the mth' refers to the order in which the argument expressions occur in the matrix.)

c. Let two compound designator matrices be given, each of them consisting of an operator (universal or existential quantifier, abstraction operator, or description operator) and its scope, which is a designator matrix. The two matrices are intensionally isomorphic $=_{Df}$ (1) the two scopes and intensionally isomorphic with respect to a certain correlation of variables occuring in them, (2) the two operators are L-equivalent and contain correlated variables." [1]

In less complicated terms, and ignoring the question of quantification, which need not concern us, Carnap is saying this: For two expressions — words, phrases, or whole sentences — to be intensionally isomorphic, it is necessary (a) that the two be L-equivalent, and (b) that they be structurally similar, both grammatically, and in that for every component expression in the first there exists a corresponding L--equivalent component expression in the second. The conditions are jointly sufficient. He says, for example, that "2 + 5" and "II sum V" are intensionally isomorphic "because they not only are L-equivalent as a whole, both being L-equivalent to "7", but consist of three parts in such a way that corresponding parts are L-equivalent to one another, and hence have the same intension". [2] On the other hand, "7 > 3" and "Gr(sum (II, V) III)" are not isomorphic since, although they are L-equivalent as a whole, the second contains expressions for

(1) *Meaning and Necessity*. op. cit p. 59.
(2) ibid. p. 56.

which there are no L-equivalent corresponding components in the first.

According to Carnap, two expressions which are isomorphic in the above way are synonymous, and there are no synonyms which are not isomorphic. He says

> "It has often been noticed by logicians that for the explication of certain customary concepts a stronger meaning relation than identity of intension seems to be required. But usually this stronger relation is not defined. It seems that in many cases the relation of intensional isomorphism could be used." (1)
> "Synonymity ... is explicated by intensional isomorphism". (2)

One reason for this claim, as we have seen, is that Carnap is searching for a condition which will guarantee interchangability in opaque contexts — for, according to him, synonyms must be so interchangable. With this aspect I am not concerned. However, following Lewis, Carnap employs a quite separate argument. The expressions "round excision" and "circular hole" are L-equivalent. So are the expressions "equilateral triangle" and "equiangular triangle". Intuitively, however, the latter pair are not synonymous in the manner of the first pair. Why ? Because although "equilateral triangle" and "equiangular triangle" are L-equivalent as a whole, the component corresponding expressions, "equilateral" and "equiangular" are not. That is, the two compound expressions are not intensionally isomorphic. In general, it is claimed, we may often come across L-equivalent compound expressions which, intuitively, are not synonymous. Where this is so, it will be found that the expressions are not intensionally isomorphic. Now no doubt Carnap, and Lewis, are right to say that "circular hole" and "round excision" are synonymous in a way in which "equilateral triangle" and "equiangular

(1) op. cit. p. 59.
(2) ibid. p. 64.

triangle" are not. But they misinterpret the significance of such examples. Such examples do not, despite appearances, point towards a need for intensional isomorphism as a general condition for synonymy. I shall return to this point after levelling some objections against the suggested condition.

The first objection is as follows: the suggested criterion for synonymy cannot cover the favourite paradigms of synonymy. If one looks closely at Carnap's formal definition of intensional isomorphism, it can be seen that no place is provided for the synonymy of an elementary expression with a complex expression. Clause (a.) of that definition deals only with the synonymy of two elementary expressions (i.e. those matrices having no proper part). Clauses (b.) and (c.) deal only with the synonymy of two complex expressions. Yet the paradigms of synonymy are those cases where an elementary expression is said to be synonymous with a complex one. For example, "bachelor" — "unmarried man", "lioness" — "female lion", "brother" — "male sibling". Any account which can have no place for such cases of synonymy is surely unacceptable. C. I. Lewis' account of intensional isomorphism does allow for the synonymy of an elementary expression with a complex one, and so he admits there is an exception to the general condition of isomorphism. The trouble here is: how are we to react to a generalization about synonymy which must be provided with an escape clause for the most common type of synonyms ? First, it makes the theory look ad hoc. Second, it makes one suspect that intensional isomorphism is itself an exceptional condition which synonyms must meet only in special cases. I shall shortly try to show that examples such as those concerning the triangles are indeed special, and should not tempt us into any general demand for isomorphism.

A second objection is this: it is often the case, in most languages, that there exist different grammatical ways of saying the same thing: alternative grammatical devices which do not, intuitively, alter the meaning of what is said. A good example of this occurs, quite commonly, in French. It is very often the practice in that language to form adjectives from place-names, e.g. "alsacien", "marseillaise", "lyonnais". Where this is done, it seems immaterial whether we describe something by using the adjective, or describe it as being "from" or "of" the place in question. For example, we can say either "il est alsacien" or "il est d'Alsace"; either "une mère marseillaise" or "une mère de Marseille". (Sometimes, of course, the grammatical difference marks a difference in meaning. "Pommes de terre lyonnaises" are not simply potatoes from Lyon). Intuitively, I am sure, the above pairs would be marked as synonymous. Yet, clearly, they are not intensionally isomorphic. "Il est d'Alsace" contains a component expression, "d(e)", which has no corresponding L-equivalent component in "il est alsacien". To take another type of example though this time a more contentious one: it seems often the case that passive forms are synonymous with active forms. Yet, clearly, actives and passives are not intensionally isomorphic by Carnap's criterion.

A third objection is this: there are examples of complex expressions which are L-equivalent, synonymous, structurally similar in grammatical terms, but which are not intensionally isomorphic. For example, the pair "illustrative schema" — "schematic illustration" are surely synonymous. Moreover, they are structurally similar in grammatical terms, since each contains an adjective and a noun. However, they are not intensionally isomorphic, for (1) the two adjectives, whilst grammatically similar, are not L-equivalent, and (2) the adjective in the first, and the noun in the second, whilst they are related in meaning, are not grammatically isomorphic. That is, "illustrative" and "schematic" are not even alike in meaning; and "illustrative" and "illustration" are not structurally similar. This is by no means an isolated example. A common device in many languages is to take a complex expression "a b" and to transform this into an expression synonymous with it by transforming the adjective, "a", into its noun form, and the noun, "b", into its adjectival form. For example, "professorial dean" — "decanal professor", "foolish male" — "male fool". (This is not to suggest that every such transformation ratains the meaning of the original. "Athletic fool" and "foolish athlete" do not mean the same. An athlete is not simply one who is athletic. My athletic doctor is a doctor and not an athlete).

No doubt this list of counter-examples, of varying types, could be extended. But we need not do so. We can see that any general demand for intensional isomorphism as a condition for synonymy is unacceptable. What, though, can we say of the example showing the difference between "round excision" — "circular hole", and "equilateral triangle" — "equiangular triangle", which tempted us into demanding the isomorphism criterion ? Well, Carnap and Lewis misconstrue what should follow from the non-synonymy of the latter pair. What does *not* follow is either (1) that complex synonyms must be structurally identical, or (2) that corresponding components in complex synonyms must themselves be synonymous. All that does follow is a much more restricted conclusion, namely: for two complex expressions of identical (grammatical) structure to be synonymous, it is necessary for two corresponding grammatical components to be synonymous, if and only if all *other* corresponding components are synonymous. That is: suppose "a b" and "c d" are synonymous. And suppose, too, that "a" and "c" are synonyms. In that case, but only in that case, "b' and "d" must also be synonymous. "Equilateral triangle" and "equiangular triangle" are structurally (grammatically) similar, and the two nouns are, of course, synonymous. Therefore, if the two complexes are to be synonymous, then the two adjectives must be synonyms — which, in this case, they are not. Clearly, then, Carnap's example was a special one. Where two complexes are not grammatically similar, or where they are grammatically similar but the corresponding components are not L-equivalent, there is no reason to insist upon intensional isomorphism. "Une mère marseillaise" and "une mère de Marseille" are synonymous but not structurally similar. "Professorial dean" and "decanal professor" are structurally similar, but none of the corresponding components are L-equivalent; yet they are synonymous. So, in these cases, the conditions in which intensional isomorphism can be demanded do not exist. Those conditions are special, and should point to no general demand for isomorphism. Let me give an analogy. Two maps, in order to be accurate maps of an area, need not be drawn to the same scale, or employ the same symbols. However, if you take two maps both drawn to the same scale, and employing the same symbols, then, no doubt, a given bit of the one map must be identical with a given bit of the other map if *both are* to be accurate representations of the area. Similarly, two expressions need not be intensionally isomorphic to mean the same. However, if you take two expressions "a...n" and "a'...n'" such that each component in the sequence "a...n — 1" is synonymous with a corresponding component in the sequence "a'...n — 1'", then, no doubt, "n" and "n'" must be synonymous if the two complexes are to be synonymous as a whole.

Let us see how these considerations apply to the synonymy or otherwise of logical constants. (1) The objection to saying that constants may be synonymous with one another is this: logically equivalent formulae containing different constants are structurally dissimilar, that is, they are not intensionally isomorphic. In fact, Carnap does allow for a certain type of synonymy relation to hold between constants — but this is only where we are dealing with a pair of elementary expressions, or a pair of complex ones. This, in effect, means that Carnap only admits of synonymy relations to exist *across* different notations. He says, for example, that we can treat "$\vee$" and "A" as synonymous. Presumably we can also treat $Ap\bar{q}$ and $p \vee \sim q$ as synonymous. Clearly, though, Carnap's criterion cannot allow for the synonymy of "$\vee$", in some construction, with "$\sim$" and "&" in some construction. For $p \vee q$ and $\sim(\sim p \,\&\, \sim q)$ are not isomorphic. The latter formula contains components which are not L-equivalent to corresponding components in the former. His reason for denying synonymy here would be the same as for denying synonymy in the case of "$7 > 3$" and "Gr(sum(II,V)III)".

Surely, however, to deny that two structurally dissimilar

(1) Strictly speaking, it is not constants that can be synonymous, but, rather, formulae containing them. This is a result of it being impossible to give other than contextual definitions of the constants.

formulae can ever be synonymous meets with a glaring exception. The constant "$\leftrightarrow$" was invented to abbreviate certain formulae otherwise requiring the constants "$\rightarrow$" and "&" for their expression. Few logicians, whatever their views on synonymy in general, would deny that $p \leftrightarrow q$ means the same as $(p \rightarrow q) \& (q \rightarrow p)$. I am sure Carnap would not deny it — yet, if we take him literally, his criterion for synonymy implies such a denial. Let us waive this point, and assume that such abbreviations can be dealt with as special cases.

The main point is this: we have seen that, in general, there is no reason to demand intensional isomorphism as a condition for the synonymy of complex expressions. I can see no reason why this conclusion needs amendment because the expressions involved happen to include logical constants. Now, we did admit that there were certain cases in which corresponding components must themselves be synonymous if the expressions as a whole were to be synonymous. However, no such cases can arise *within* a single logical notation. They can only arise across notations. For example, if "A" and "$\vee$" are synonymous, and if $Ap\bar{q}$ and $p \vee \sim q$ are synonymous, then "$\sim$" and "$^{-}$" must be synonymous likewise. This is exactly parallel to the demand we made in the case of "equilateral triangle" and "equiangular triangle". But, as I said, no such cases can arise within a notation, $p \vee \sim p$ and $\sim(\sim p \vee q)$ are L-equivalent, but since they are structurally dissimilar, it is not a case where intensional isomorphism can be demanded as a condition for synonymy. The synonymy of these two formulae, if there be one, would parallel that between "il est alsacien" and "il est d'Alsace".

We can see, then, that the demand for intensional isomorphism in the field of logical constants, if they are to be synonymous, is either mistaken or irrelevant. It is mistaken if the demand is for structural identity between formulae: it is irrelevant if the demand is for L-equivalence between corresponding components of structurally similar formulae — since, within a notation, the L-equivalent formulae never will be structurally similar. (Except in the trivial case where the two formulae are identical). I am not saying that L-equivalent formulae in logic are synonymous. I claim, only, that the demand for intensional isomorphism cannot show they are not, for that is a mistaken demand. It is mistaken both in the field of logic, and, more widely in the field of language at large.

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