TRADITIONAL LOGIC AS A LOGIC OF DISTRIBUTION-VALUES

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Even today, something called *Traditional Logic* is applied to the minds of a great many students in a great many universities. If the truth is admitted, this traditional logic is a poor thing, scarcely more than a piecemeal collection of fragments from a system the greater part of which remains permanently submerged. Students acquire a kind of familiarity with the square of opposition, conversion, obversion, contraposition, moods, figures, and so on, but they are rarely permitted a glimpse of the truly systematic properties of the logic behind these devices.

It may occur to one who suffers traditional logic and later learns something of propositional logic that there is a considerable difference in the methods used in these two branches of what is supposed to be the same discipline. It is not just that the standards of exactitude in the older logic seem slipshod; the real problem is that it does not appear to embody a precise conception of validity. The unfortunate student who is trying to find house-room for the rules of equipollence, quantity, quality and distribution, the four figures, sixty-four moods and 256 syllogisms, has no time to be troubled by anything so out of the way as a simple and straightforward technique for testing validity.

The comparison with propositional logic is instructive. The student of propositional calculus generally has at his disposal two methods for testing validity, one based on constructing truth-tables, the other involving deductive techniques. The axiomatic treatment of traditional logic has been extensively developed in recent times, notably by Łukasiewicz and Bocheński; and it is natural to ask whether it might not be possible also to employ in this field techniques analogous to the truth-table calculations of propositional logic. It is common knowledge that traditional logic does not at present have such a method. The first task, therefore, is to make good this deficiency.

In the case of propositional logic, techniques of this kind provide important insights into the general character and systematic properties of truth-functions. Similarly, my principal objective in discussing the decision procedures of traditional logic is to arrive at a better understanding of that logic as an autonomous and exhaustive system. Failing that, it may still be possible to perform a more mundane service; for the provision of an easy mechanical test will surely simplify the task of those obliged to instruct students in the eccentricities of traditional logic.

My design, then, is to inquire whether, taking the inferences of traditional logic as they are, and principles of validity as they may be made, it is possible to establish some just and certain rules for the administration of logical order.

1. The *doctrine of distribution* probably represents the nearest that traditional logic ever comes to a simple testing technique. The doctrine purports to provide a method for determining validity together with a semantical theory that somehow explains why the method works. But with regard to both of these pretentions it has fallen on hard times. Peter Geach has shown that the semantical notions upon which the doctrine is based are confused, and he has also pointed out that it will not work as a test of validity (¹). As it stands, then, the doctrine does not stand at all. The question is, can anything be salvaged from the ruins ? It is best to begin by trying to understand the defects that brought down the old rules of distribution.

Rules of distribution are of course based on the idea that each term in a categorical proposition is either distributed or undistributed. The reasoning by which logicians have hoped to justify the classification of terms in this way will not be

(¹) Peter GEACH, *Reference and Generality*, Ithaca, N.Y., 1962, and "Distribution: A Last word", *The Philosophical Review*, 1960.

discussed here. For present purposes, it is sufficient to say that a term is distributed or undistributed according to the type of proposition it appears in and the place it occupies in that proposition.

A, *E*, *I* and *O* will be used as term-operators corresponding to the four traditional forms of categorical proposition, so that *A* may be read as "All — are —", *E* as "No — are —", *I* as "Some — are — ", and *O* as "Some — are not —". *a*, *b* and *c* are term-variables standing for plural nouns or expressions plausibly construed as the equivalents of plural nouns. The first term of a proposition is its *subject*, the second its *predicate*. The idea of distribution will be expressed by introducing the notion of a *distribution-value* and saying that any term has a distribution-value of 1 or 0.

One of the mnemonics for distribution is SUPN, "*S*ubjects of *U*niversals, *P*redicates of *N*egatives". Thus, the subject term of a universal proposition (*A* or *E*) and the predicate term of a negative proposition (*E or O*) have a distribution-value of 1, and remaining terms have a distribution-value of 0. That is,

$$Eab\colon\ a=1,\ b=1$$
$$Aab\colon\ a=1,\ b=0$$
$$Oab\colon\ a=0,\ b=1$$
$$Iab\colon\ a=0,\ b=0$$

If p and q are categorical propositions with common terms, then $p \rightarrow q$ (an *immediate inference*) signifies that q may be inferred from p, and $p \equiv q$ that each proposition may be inferred from the other. When $p \equiv q$ is true, p and q are *equipollent*. N signifies propositional-negation; n signifies term-negation.

The standard rule of distribution for immediate inferences is that it is not permissible to pass from a proposition in which a term is undistributed to one in which it is distributed. Thus, to give a usual example, universal affirmatives are said to be incapable of simple conversion because, in $Aab \rightarrow Aba$, b

is distributed in the conclusion and undistributed in the premiss. Let us consider where the rule goes astray.

It is evident that the rule cannot by itself specify necessary and sufficient conditions of validity. To give only the most obvious example, both terms of a universal negative are distributed and both terms of a particular affirmative are undistributed, and the rule therefore permits any inference from the former or to the latter, though many such inferences (e.g. *Eab* → *Iab*) are clearly not valid. So the standard rule can aspire to provide only necessary conditions of validity.

However, even this modest project proves to be too ambitious. Consider the notorious case of *inversion*. The inference *Aab* → *Onab* must be valid because it is the summation of a series of inferences that are separately recognized as valid:

Aab → *Eanb* → *Enba* → *Anbna* → *Inbna* → *Inanb* → *Onab*.

On the other hand, *Aab* → *Onab* ought not to be valid if the rule of distribution is sound, because the predicate of *Onab* is distributed and that of *Aab* is not. This argument, borrowed from Geach, shows that the rule cannot manage to specify even necessary conditions. I want to show that the predicament of the formal theory of distribution is in one sense far worse than Geach's argument indicates, in another sense much better. But things must be allowed to get worse before they can get better.

Let me propose a certain policy concerning the effect of negation on distribution-value. Propositional-negation, I will say, reverses the distribution-value of both the terms of the proposition on which it operates. Thus, since *a* and *b* both have a value of 0 in *Iab*, they both have a value of 1 in *NIab*. And, in general:

$$N(11)=00 \qquad N(10)=01$$
$$N(01)=10 \qquad N(00)=11$$

In order to describe the rôle of term-negation, I will first adopt the convention of saying that in propositions like *Aab*, *Enab*, *Ianb*, *Onanb*, and so on, *a* and *b* are the only constituent terms. In other words, the usual practice of speaking of "negative terms" is set aside in favour of regarding term-negation as part of the term-operator. The terminological, and other, difficulties behind this decision will not be discussed here.

Now, term-negation, I will say, reverses the value of the term on which it operates:

$$n(1) = 0 \qquad n(0) = 1$$

What this means is that, for example, since *a* has a value of 0 in *Iab*, it has a value of 1 in *Inab*.

The idea that negation alters distribution-value is not a familiar one, though the rôle assigned to *N* flows naturally from the recognition that, as Keynes puts it, "any term distributed in a proposition is undistributed in its contradictory". And despite the novelty of regarding term-negation in this way it is easy to see that some such procedure must be adopted. If it is said that term-negation has no effect on distribution, then even a straightforward case like *Ianb* → *Oab* (obversion) is rendered invalid. Now, the problem may be avoided for obversion by confining the rule to subject terms. (This is the method of some authors. Others prefer simply to keep quiet about distribution when they deal with obversion and contraposition.) But restricting the rule to one term is an unhappy solution, because the same difficulties arise in sharper form elsewhere. If *n* fails to alter distribution-value, neither of the following contrapositions is acceptable:

$$Aab \rightarrow Anbna, \qquad Oab \rightarrow Onbna.$$

And it is plain that any attempt to confine the rule to subject or predicate must founder on one or other of these inferences. Once we allow that *n* reverses distribution-values both are acceptable. (It may seem strange, by the way, that the logicians who present rules of distribution and then go on immediately to describe obversion and contraposition appear not to notice that the rules do not apply to obversion and contraposition.)

The real weakness of distribution as a mechanical test, and this is what Geach's criticism does not bring out, is the lack of a policy for coping with negation. Consider again the problem of inversion. Each of the steps in the series *Aab* → *Eanb* → *Enba* → *Anbna* → *Inbna* → *Inanb* → *Onab* is supposed to be permitted by the rule of distribution. But this can only appear to be the case because of certain subterfuges. The obversions are handled by confining the rule to subject terms, and a conversion like *Eanb* → *Enba* is regarded as just an instance of $Ea\beta \rightarrow E\beta a$ (which conforms to the rule) with a "negative term" substituted for β. If we turn a blind eye on these manœuvres, the problem arises only for inversion. But what is really problematic about inversion can also be found in other segments of the chain, for example:

Aab → *Enba*, *Eanb* → *Inbna*, *Anbna* → *Onab*.

The ordinary rule of distribution cannot tell us anything about cases such as these. This is the point: it is not that these cases violate the rule but that there is no way of applying the rule to them. To be sure, we might invent a special trick for coping with inferences like the first one. *Aab* → *Enba*, we might say, is a *cobversion*; and the special feature of cobversions is that the rule of distribution applies to the subject of the first proposition and the predicate of the second. This device, as good as many others in traditional logic, parallels the trick for handling obversions. (It is perhaps surprising that no-one discovered cobversions.) But no such patching-up will help us with the second two inferences: the standard rule just does not apply to them at all. The supposed problem of inversion is in reality only a symptom of the general failure of the rule of distribution to apply to inferences involving "negative terms".

What is needed, then, is a clear policy on term-negation, and this has been provided. But the difficulties do not end there. The predicament of inversion is no better than before: that *n* reverses distribution-value does nothing to improve the standing of *Aab* → *Onab*. Indeed, the immediate effect of the policy is to make matters worse: the number of rejected valid inferences increases despairingly. Thus, to demonstrate that inversion is sound we passed through the inference *Anbna* → *Inbna*; and, applying the convention for term-negation, *b* violates the standard rule of distribution. So far as inversion is concerned, then, the standard rule is at least consistent: it rejects the inversion of *A* propositions directly and it also rejects one of the steps by which inversion is shown to be valid. But this is hardly consoling, since both rejected inferences are (recognized as) valid.

We are now in a position to get to the heart of the difficulties. It has been said that the problem of inversion is located in the lack of a policy on term-negation; but it must now be added that inversion brings into the open an even more serious deficiency in the standard rule of distribution. The textbook rule is really an ill-judged attempt to impose an appearance of uniformity on inferences that are importantly different. The traditional topic of Immediate Inference covers, first, inferences passing from a universal proposition to a universal proposition, or from a particular to a particular, and second, those which pass from a universal to a particular. The first kind I will call *non-reductive*, the second *reductive*. Now, inversion, like superimplication, conversion *per accidens*, and so on, is an instance of reductive inference; and the fact is that the standard rule of distribution has only the most accidental bearing on the whole sphere of reductive inference. When it is allowed that *n* reverses distribution-value (which must be allowed if other inferences are to pass muster), a large number of inferences, all of them reductive, fail to meet the requirements of the standard rule. *Ananb* → *Inanb*, *Anab* → *Inab* and *Enanb* → *Onanb* would all be regarded as valid *superimplications* (a superimplication being of the form $A\alpha\beta \rightarrow I\alpha\beta$ or $E\alpha\beta \rightarrow O\alpha\beta$), but they are rejected by the rule.

Let me summarize the difficulties. A number of valid inferences show that the rule cannot be made to work until a policy on term-negation is introduced. When such a policy is introduced, the rule applies to non-reductive inferences (at least to the extent of providing necessary conditions), but it will not work for reductive inferences. Even for non-reductive inferences, the rule does not provide sufficient conditions; and, as will emerge shortly, it works for these inferences only in a way most misleadingly expressed by the traditional formulation. We have arrived at the stage at which the rule of distribution itself must be reformed.

2. When *p* and *q* are propositions both of which have α and β as their constituent terms, and when both α and β have the same distribution-value in *p* and *q*, then *p* and *q* are *equivalent in distribution-value*. And when two propositions are both universal or both particular, they are *equivalent in quantity*. *E*, *A*, *NO* and *NI* are the universal operators; *O*, *I*, *NE* and *NA* are the particular operators. (Equivalence in quantity is of course the mark of non-reductive inference.)

Consider an example of propositions that are equivalent both in quantity and distribution-value. Granted certain obvious presuppositions that will not be discussed now, the following is an exhaustive list of propositions that are universal and such that both their terms have a distribution-value of 1:

Eab,	*Aanb*,	*NOanb*,	*NIab*,
Eba,	*Abna*,	*NObna*,	*NIba*.

Since any proposition must be universal or particular, and any proposition must have a distribution-value of 11, 10, 01 or 00, it is evident that there are eight such groups of propositions equivalent both in quantity and distribution-value; and each group contains eight propositions.

Now, it is easy to show by obversion, simple conversion and the principle of contradiction depicted in the square of opposition (i.e. $A \equiv NO$, $NA \equiv O$, $E \equiv NI$, $NE \equiv I$) that, in each of the eight groups, the remaining seven propositions may be inferred from any member of the group. In short, these are groups of *equipollents*: any member of a group is equipollent with the rest, and no member of a group is equipollent with a member of another group. Hence, the formula for equipollence is: equivalence in quantity plus equivalence in distribution-value.

We may therefore formulate the following simple *rule of equipollence*: $p \equiv q$ if and only if p and q are equivalent in quantity and distribution-value. Similarly, the following is the correct *rule of distribution for non-reductive immediate inferences* : $p \rightarrow q$ if and only if p and q are equivalent in distribution-value.

This rule provides necessary and sufficient conditions for the validity of all non-reductive immediate inferences. Furthermore, the rule covers the greater part of the traditional theory of immediate inference: most aspects of that theory are in fact concerned with equipollences of one kind or another. The elaborate doctrines of simple conversion, obversion, contraposition, rules of equipollence, and so on, are here reduced to one elementary principle, that of distribution-equivalence. (This principle has been presented as a mechanical test of validity, but, of course, it may also be seen as a rule for the mechanical construction of equipollents.)

How does the new rule stand with regard to the old ? As it happens, the old rule works for all inferences between equipollents, provided our policy on term-negation is accepted; but it works in a way misleadingly expressed by the rule itself. All such inferences are between distribution-equivalents, and that a term cannot be distributed in the conclusion if undistributed in the premiss simply follows from the fact that the distribution-value cannot change. The distinction made in the traditional rule between consequent and antecedent is quite irrelevant. And where the old rule hopes (and fails) to provide only necessary conditions, the new rule provides completely necessary and sufficient conditions. Finally, it is a great advantage of the new rule that, unlike the old, it clearly marks out the distinctive logical properties of equipollence.

The correct *rule of distribution for reductive immediate inferences* is simply this: $p \rightarrow q$ if and only if one of the constituent propositions has an unmixed distribution-value (11 or 00) and the other a mixed value (10 or 01). If we say that a term with the same value throughout an inference is *single-valued*, and a term which changes its value is *double-valued*, the same rule becomes: a reductive inference is valid if and only if one of its terms is single-valued. Similarly: a non-reductive inference is valid if and only if both terms are single-valued.

It may easily be shown that this rule is thoroughly effective. We have seen that the propositions of traditional logic fall into eight groups of equipollents. For the purpose of testing, therefore, a single representative of each of the eight groups will suffice. Of the sixteen inferences produced, eight are valid and eight invalid; and the division conforms exactly to the rule.

It may be noted that, as in the case of non-reductive inference, the distinction made in the traditional rule between consequent and antecedent plays no part in our decision procedure.

In the light of the two correct rules of distribution, it should be easy to see how the single traditional rule may have appeared almost to work as a kind of compromise between the two varieties of immediate inference. As we have seen, none of the inferences between equipollents can violate the old rule; and, of the eight varieties of reductive inference distinguishable according to distribution-value, only half (those from 00 to 10, from 00 to 01, from 10 to 11, and from 01 to 11) violate the rule. But there is no room for compromise in logic.

The sometimes tortuous way in which proper rules of distribution have been arrived at should not be allowed to distract attention from the essential simplicity of the rules themselves. Granted our method for determining distribution-value, the entire topic of validity in immediate inference may be disposed of in two or three sentences. An immediate inference is valid only if it is non-reductive or reductive. Non-reductive inferences are valid if and only if both terms are single-valued; reductive inferences are valid if and only if one and only one term is single-valued.

Whereas the traditional rule could aspire to providing only necessary conditions, and fell short even of that small aim, the revised rules provide conditions sufficient as well as necessary for the validity of all immediate inferences without exception. But it still remains to be shown that procedures of this kind are applicable to syllogistic inferences.

II

A method for determining the distribution-value of terms has been described, and effective rules of distribution for immediate inference have been formulated. But the topic of immediate inference occupies a comparatively small place in traditional logic, and it still needs to be shown that the theory of distribution-values is applicable to the compound inferences called syllogisms.

1. It is best to begin by setting aside a number of inconveniences that stand in the way of an easy understanding of syllogistic theory. The doctrine of moods and figures is of little importance. The magic number 256 (the supposed number of syllogisms) may be forgotten. The eight-or-so rules for syllogism provided that (i) the inference contains three distinct terms, and (ii) *p* and *q*, *p* and *r*, and *q* and *r* each have a term in common. The term common to the premisses (*p* and *q*) is the traditional logic and should be given up in our more sceptical age.

First let us define what is to count as a *syllogism*. If *p*, *q* and *r* are categorical propositions, then $p \,\&\, q \rightarrow r$ (signifying that *r* may be inferred from *p* and *q* taken together) is a syllogism provided that (i) the inference three distinct terms, and (ii) *p* and *q*, *p* and *r*, and *q* and *r* each have a term in common. The term common to the premisses (*p and q*) is the *middle term*; the remaining two are *extreme terms*. As in connection with immediate inference, the practice of speaking of "negative terms" is abandoned: *a* and *na* do not count as separate terms.

It may be objected that the conception of syllogism embodied in the above definition is different to the traditional conception, particularly with regard to the admission of negation (*N*) and term-negation (*n*). It follows that formulæ positions constructed out of terms (*a*, *b*, *c*) and term-operators (*A*, *E*, *I*, *O*) together with any number of signs of propositional-negation (*N*) and term-negation (*n*). It follows that formulae like *Aab* & *Enbc* → *Onanc* and *NIab* & *Abc* → *NEac* (to take two examples almost at random) are counted as perfectly respectable syllogisms, even though many traditional logicians would have thought them blasphemous. The point is that the syllogism as it appears in traditional logic is an artifice based on disregarding most of the forms of proposition recognized in the standard treatments of immediate inference. In connection with, for instance, obversion, contraposition and rules of equipollence, propositions involving term-negation and propositional-negation are admitted into the system: it is therefore inconsistent and artificial to omit them from a discussion of compound inference. A logical system must always be, within its terms of reference, exhaustive. It should be stressed, however, that, although the scope of the syllogism is considerably widened by our definition, everything traditionally counted as a syllogism is incorporated in passing. And anyone hide-bound enough to wish to restrict the actual word "syllogism" to the old forms is at liberty to do so.

There are two rules of distribution for syllogisms. The first, which is similar to the old rule for immediate inferences, is that no term may be distributed in the conclusion if it is undistributed in the premisses. This rule is generally presented in the textbooks as two fallacies, "illicit process of the major" and "illicit process of the minor"; but this way of putting it simply represents a needless distinction between the case where the so-called "major term", and the case where the "minor term", violates the rule. The second rule is found in the famous "undistributed middle" fallacy; and what this amounts to is that the middle term of a valid syllogism must be distributed at least once.

Whereas there are definite violations of the rule of distribution for immediate inferences, no such violations can be found among the twenty-four syllogisms usually said to be valid — the reason being that "negative terms" are excluded from the orthodox syllogism. If "negative terms" are admitted, it soon becomes apparent that the standard rules will not work. Thus,

$$Anab \ \& \ Abc \rightarrow Inac$$

violates the rule for extreme terms (granted the method for calculating distribution-value), since *a* is distributed in the conclusion and undistributed in the premisses. But it can easily be seen that *Anab* & *Abc* → *Inac* is merely *Barbari* with one term negated throughout, and therefore valid. Similarly,

$$Anba \ \& \ Anbc \rightarrow Iac$$

is valid, since it is *Darapti* with a negated middle term; but the middle term has a distribution-value of 0 in both occurrences.

If we confine our attention to the standard 256 syllogisms, it is apparent that the accepted rules fail to specify sufficient conditions of validity:

$$Eab \ \& \ Ebc \rightarrow Aac$$

is invalid, but it violates neither of the rules of distribution. Of cource, a "*rule of quality*" (no valid syllogism has two negative premisses) may be adduced to eliminate the above example. But rules of quality are just an embarrassment when term-negation is employed: *Eab* & *Ebc* → *Onac* is valid, despite its negative premisses. In fact, the idea of "quality" is best dropped entirely.

As may be suggested by the uneasy wording of the rule for middle terms (they must be distributed "at least once"), the rules of distribution for syllogisms, like the traditional rule for immediate inferences, attempt to impose false unity on distinct varieties of inference. We must therefore begin by discovering what distinctions need to be made.

Some traditional logicians distinguish three kinds of syllogisms: *weakened, strengthened* and *fundamental* syllogisms. A weakened syllogism is one where a particular conclusion is drawn from premisses that would have justified a universal conclusion. Thus, *Cesaro* is weakened because, although we may conclude *Oac* from *Ecb* and *Aab*, these same premisses justify the conclusion *Eac* (*Cesare*). A strengthened syllogism is one where a particular conclusion is drawn from universal premisses and the conclusion still follows if one of the premisses is replaced by a particular proposition. Thus, *Celaront* is strengthened: although we may conclude *Oac* from *Ebc* and *Aab*, the same conclusion follows when *Aab* is replaced by *Iab* (*Ferio*). A fundamental syllogism is neither weak nor strong, but just right.

In the textbooks, the notions of strengthened and weakened syllogisms are based on a further assumption which is not always made explicit. To speak of a syllogism being strengthened or weakened is really to indicate a relation between it and some fundamental syllogism; and what is assumed is that this fundamental syllogism must be *in the same figure* as its weakened or strengthened version. This assumption, when it is made explicit, is expressed by speaking of "subaltern moods". Since figures should be given up, this part of the meaning of "strengthened" and "weakened" will also be discarded.

Are the three traditional categories soundly based ? If we think only of their quantity, valid syllogisms are of three kinds: *universal syllogisms* (as they may be called), which contain nothing but universal propositions; *particular syllogisms*, which have a particular conclusion and one particular premiss; and syllogisms with universal premisses and a particular conclusion. Using *U* and *P* to signify quantity, the three kinds are:

$$U \& U \rightarrow U$$
$$U \& P \rightarrow P$$
$$U \& U \rightarrow P$$

The first two kinds, universal and particular, together constitute the fundamental syllogisms of traditional logic. I will call them *non-reductive syllogisms*. The third kind, with universal premisses and a particular conclusion, will be called *reductive syllogisms*.

Traditional logic, it has been explained, distinguishes two varieties of reductive syllogism, strengthened and weakened — though some syllogisms (*Barbari*, for example) are both strengthened and weakened. But the category "strengthened" may be given up, because it is based on the erroneous assumption that some reductive syllogisms are strengthened, others not. To say that a syllogism is strengthened is really to say that it is of the form

$$U^1 \ \& \ U^2 \rightarrow P^3$$

and that there is some particular syllogism with a common premiss and the same conclusion, i.e. a syllogism of the form

$$U^1 \ \& \ P^2 \rightarrow P^3 \quad \text{or} \quad P^1 \ \& \ U^2 \rightarrow P^3$$

In reality, this property is possessed by *all* reductive syllogisms; and the one reductive syllogism (*Camenop*) which appears in the standard accounts not to be strengthened seems this way only because there is no corresponding particular syllogism in the same figure — unless, that is, term-negation is permitted.

Nevertheless, even if the notion of a strengthened syllogism is given up, it must be recognized that there are two varieties of reductive syllogism, namely those that would, and those that would not, justify a universal conclusion. The kind that would justify such a conclusion already have a name: they are *weakened* syllogisms. The other kind may be called simply *non-weakened*. This turns out to be the only distinction that matters. A syllogism of the form

$$U^1 \ \& \ U^2 \rightarrow P^3$$

is weakened only if there is a corresponding syllogism of the form

$$U^1 \& U^2 \rightarrow U^3$$

When there is no such corresponding universal syllogism the original reductive syllogism is non-weakened. ("Corresponding" means here: having the same status with regard to validity.)

From this point of view, then, there are three kinds of syllogism:

(i) *Non-reductive*
(ii) *Reductive Weakened*
(iii) *Reductive Non-weakened*

It only remains to describe the mechanical tests appropriate to each of these forms.

2. The method for calculating the distribution-value of terms has already been explained.

A non-reductive syllogism, whether universal or particular, is valid if and only if its middle term is double-valued (distibuted once and only once) and its extreme terms are single-valued (have the same distribution-value thoughout the syllogism.)

Consider the following examples, the first (*Barbara*) a universal syllogism, the second (*Ferio*) a particular syllogism:

$$Aab \& Abc \rightarrow Aac$$
$$Iab \& Ebc \rightarrow Oac$$

In the first case, the middle term *b* has a value of 0 in the first premiss and 1 in the second premiss; the extreme term *a* has a value of 1 in its premiss and in the conclusion; and the extreme term *c* has a value of 0 in both its occurences. In the second case, the middle term *b* has a value of 0 in the first premiss and 1 in the second; *c* has a value of 1 throughout; and *a* has a value of 0 throughout. Thus, in both cases the middle term is double-valued and the extreme terms single-valued. The same holds true of all possible non-reductive syllogisms.

A reductive weakened syllogism is valid if and only if its middle term and one and only one of its extreme terms are double-valued. Take *Bramantip* as an example:

$$Aba \ \& \ Acb \rightarrow Iac$$

is valid because the middle term *b* has a value of 0 in one premiss and 1 in the other, the extreme term *a* has the same value (0) throughout, and the extreme term *c* is double-valued — it has a value of 1 in its premiss and 0 in the conclusion.

It should be easy, remembering the definition of a weakened syllogism, to see the connection between the rule of distribution for syllogisms of this kind and the rule for reductive immediate inferences. A reductive immediate inference is valid only if one and only one of its terms is double-valued. Hence, in a weakened syllogism, the particular conclusion must contain one term with a value different to the value it would have had in the universal conclusion that might have been drawn. To take a simple example, the premisses *Aab* and *Abc* justify the conclusion *Aac* (*Barbara*); and they also justify the conclusion *Iac* (*Barbari*). But the distribution-value of *Aac* is 10, the distribution-value of *Iac* is 00, and *Iac* may be inferred directly from *Aac*. It will be evident that this rule always holds.

The rule of distribution for non-weakened syllogisms is simply this: such syllogisms are valid if and only if all their terms are single-valued. Thus, *Darapti*,

$$Aba \ \& \ Abc \rightarrow Iac$$

is valid because $a = 0$, $b = 1$ and $c = 0$ throughout the syllogism.

These three rules of distribution apply to any syllogism that can be constructed out of the propositions of traditional logic.

It may be noted that it is in connection with non-weakened syllogisms that the old fallacy of undistributed middle goes by the board. The middle term of non-weakened syllogisms must be single-valued, but the value need not be 1. Thus, *Aab & Acb → Onac*

is valid although it has an undistributed middle; for all that matters is that the middle term should be completely distributed or undistributed.

Similarly, the weakened syllogisms demonstrate the unsoundness of the old illicit process fallacies:

Aba & Acb → Inanc

is valid, despite the fact that *a* is undistributed in the premisses and distributed in the conclusion.

This, then, is the simple procedure which may be used by anyone obliged to test the validity of syllogistic inferences. The first thing to be settled is whether the inference is reductive or non-reductive; that is, whether it proceeds from universal premisses to a universal conclusion, from a universal premiss and a particular premiss to a particular conclusion, or from universal premisses to a particular conclusion. (If it does none of these things, the inference is invalid without reference to the rules of distribution.) If the inference is non-reductive, it is valid if and only if the middle term is distributed once and the extreme terms are single-valued. If it is reductive, it is valid if and only if *either* all terms are single-valued *or* the middle term is distributed once and only one of the extreme terms is double-valued. These rules provide entirely sufficient and necessary conditions of validity in syllogistic reasoning, and therefore replace all the traditional rules, most of which are in any case unreliable. Procedures based on these rules also serve to solve a problem which perplexed some of the classical exponents of traditional logic, the apparent lack of "harmony" in the theory of the syllogism. But the light cast by the logic of distribution-values on the true harmony and completeness of syllogistic theory deserves to be discussed separately.

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