# HOW NOT TO DEFINE TRUTH-FUNCTIONALITY

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Virtually all modern logic books, and most philosophers, have as part of their conceptual apparatus the notion of *truth-functional compoundedness,* i.e., of a sentence's being truth-functionally compound. (My use of 'sentence' reflects no philosophical view, or prejudice; the reader is invited to substitute 'statement', 'proposition', 'judgement', or whatever, throughout — with a few clear exceptions.) Logic texts, for example, typically make a distinction between simple and compound sentences; and among those that are compound a distinction is made between those which are truth-functional and those which are not. Nor are examples of philosophical disputes in which the notion of truth-functionality is a focal point hard to find. Any dispute concerning the various putative counter examples to the thesis of extensionality (i.e., the view that every meaningful compound either is, or can be "analyzed into", a truth-functional compound) will do. Such controversies include, for example, those focusing on sentences involving modalities, nomological and general sentences, and sentences about propositional attitudes (e.g., 's believes p', and 's is certain that p'). Unfortunately, discussions of the notion of truth-functionality are often marred by a lack of clarity and, indeed, of accuracy, which scarcely befits its apparent importance. What I should like to do in this paper is to indicate where the lack of clarity (and of accuracy) lies, and to make some progress toward getting things right.

I begin with some brief remarks on compoundedness. Most logic books tell us that a sentence is compound if and only if it contains within it another, different sentence. 'It is false that p' is compound because it contains 'p', which is a different sentence from 'It is false that p'. 'p and q', 'S believes that p', and 'If

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(1) I have been helped a great deal in my thinking on the topics of this paper by discussions with David Sanford.

p had happened, then q would have happened' are other examples which are compound for similar reasons. So far, so good. But consider something of the form 'p or p', e.g., 'Either Susie has freckles or Susie has freckles'. Here is a case that shows that it makes a difference whether we couch our remarks on logical theory in terms of sentences, statements, propositions, or whatever. For clearly 'Susie has freckles' is a different sentence from the longer one of which it is a disjunct. But it is by no means obvious that it is (or makes, or expresses) a different statement or proposition. On the other hand, the sentence 'Susie and Anne were both there' makes or expresses what seems an obviously compound statement or proposition, but since it contains no (grammatical) sentence different from the whole sentence it does not satisfy the requirement (as stated above) for being a compound sentence.

Clarity in the concept of compoundedness is partly a function of the outcome of disputes over the relative merits of the above mentioned "logical units", as well as of the decisions or discoveries we make concerning such matters as the criteria of individuation, identity, and so on, to be employed in connection with the various competing notions (*sentence, proposition*, etc.). We cannot solve the little puzzle I've raised until we have a solution to some big ones.

There are other problems with compoundedness. Consider the notion of "containment" employed in its definition. On a straightforward understanding of this word, the sentence 'I will play you tomorrow afternoon on the field behind the library at two o'clock' contains (not counting itself) at least seven different sentences: 'I will', 'I will play', 'I will play you', and so on. It does not, however, seem to be a *compound*. Nor can we avail ourselves of an easy way out by simply defining 'compound' in terms of the truth-functional connectives, and 'simple' as its complement; this would make it too transparently analytic that all compounds are truth-functional, when in fact, this is a philosophically controversial doctrine.

A plausible suggestion, for which I am indebted to David Sanford, is that a sentence is compound if and only if it contains component sentences different from it which can be replaced

by any intelligible sentence without yielding nonsense. Thus, 'I will' is not a component, in the required sense, of 'I will play you tomorrow' because replacing it with 'Two plus two equals four' yields 'Two plus two equals four play you tomorrow' a bit of nonsense. The intuitive plausibility of this suggestion is enhanced by an interesting parallel with truth-functionality. Just as in a truth-functional compound, a component can be replaced by any other sentence with the same truth value without altering the truth value of the compound, so in any compound a component can be replaced by any intelligible sentence without decreasing the intelligibility of the compound. A truth-functional compound is a more restricted kind of compound, and so here we employ a test in which not only intelligibility but truth value must be preserved. Some difficulties remain: again, how shall we handle cases like 'Susie and Anne were both there' ? But such questions seem to spring from the deeper problems the sentence-statement-proposition issue, and so do not, as far as I can see, constitute a legitimate complaint against the suggestion mentioned.

Assuming that we have an adequate grasp of the notion of compoundedness, what shall we say about being truth-functionally compound ? Again, many logic books say something like this: A sentence is truth-functionally compound just in case its truth or falsity depends on (is a function of, is determined by) the truth or falsity of the component sentences (²). Such a definition is defective, however, in that it can be taken in at least two ways, one of which is quite wrong. First, it may be taken in such a way as to require that the truth value of a component of a truth-functional sentence be a determinant of the truth value of the compound. This, as I shall show, is a common but wrong interpretation. Second, the definition may be read as asserting that only the truth value of a component of a truth-functional compound can be relevant in determining the truth value of the compound. This way is right.

The first version is exceedingly common and is often em-

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(²) See, for example, Copi's *An Introduction to Logic*, p. 238, or his *Symbolic Logic*, 2nd edition, p. 10, or Barker's *The Elements of Logic*, p. 102.

ployed in discussions involving sentences about propositional attitudes, e.g., belief sentences. When we focus on sentences like 'S believes p', we may be tempted to suggest that the truth value of the component must play some role in determining the truth value of the compound; such sentences seem to be something more or less than truth-functional because the truth value of p can be altered without altering the truth value of the whole sentence. For example, Urmson, in his *Philosophical Analysis* says that 'if p is an element in the complex proposition then the truth or falsity of the complex proposition will be in part determined by the truth or falsity of p'. The problem with sentences like 'S believes p', he goes on to say in the same place, is that the truth of the belief sentence '... in no way depends on the truth of p' (p. 72). The same point is urged in both of the above cited Copi books (in the vicinity of the definitions of 'truth-functional') and in numerous other places (³). A standard slogan of this version by which its employment can be recognized, is something like 'beliefs, after all, can be true or false'. The point of this pronouncement seems to be that since the truth value of the belief sentence is independent of the truth value of the sentence believed, belief sentences are not truth-functional.

This way of construing the definition is wrong. If belief sentences were truth-functional this would not entail that the truth values of such sentences are even partly determined by the truth values of the believed sentences. For many truth-functional sentences are independent in this way of their components. I do not mean just that *in some circumstances* the value of the component is irrelevant to the value of the compound. It is true that if q is false, the value of p is irrelevant in determining the value of (p and q), i.e., one may alter the truth value of p without altering the value of the compound. But there is a line in the truth table for (p and q) where the value of p makes

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(³) Some prominent examples: John PASSMORE's *A Hundred Years of British Philosophy*, p. 235; Max BLACK's *A Companion to Wittgenstein's Tractatus*, p. 298; Albert E. BLUMBERG's article entitled «Modern Logic» in vol. 5 of *The Encyclopedia of Philosophy*, Paul Edwards, (ed.), p. 15; and George PITCHER's *The Philosophy of Wittgenstein*, p. 149.

all the difference. Obviously the interpretation taken by Copi, Pitcher, Urmson, and others, ought to be understood in this way. Let us suppose that in saying that a sentence is not truth-functional if the value of a component is irrelevant to the value of the compound, what is meant is that the compound is not truth-functional if on no assignment of truth values to the components does the altering of the value of some component alter the value of the compound. But this is still wrong.

Consider (p or not-p). On no assignment of values to the components of this sentence does the altering of the value of p alter the value of the compound. If we make our criterion a matter of the alteration of the value of the compound by altering the value of the component, the consequences will include the absurd result that there are no truth-functional tautologies (or self-contradictions). There does seem to be a sense in which the value of p determines the value of (p or not-p), in which we want to say that the truth of this compound is somehow partly constituted by, or a function of, the value of p. I won't bother here to explicate that sense; the need can be skirted with a different example. Consider the compound ((p or not-p) or q). Here, the value of q is completely irrelevant to the value of the compound. Altering it makes no difference; nor does the truth of the whole depend, as far as I can see, in any way upon it: it is not "constructed" from it.

It is possible, of course, that one might be able to extricate the problematic interpretation from this predicament by constructing more complicated definitions of 'truth-functional' in which tautologies and self-contradictions are treated differently from other sorts of sentences. Since the spoiling examples I have thus far employed are all logically necessary propositions, it might seem that a definition in which such propositions merit a special clause could succeed at least in the minimum task of putting the distinction in question were we want it. But this appearance is misleading; such an approach would be rather more complicated than my examples thus far would suggest, for it is easy to find examples of truth-functional, logically contingent propositions which contain "irrelevant" components, e.g., ((p or not-p) and q), and ((p and not-p) or q). Neither com-

pound is logically true, or false; yet in neither does the truth value of p play a role. Thus, the relevance of a component's truth value to the truth value of the compound is not a necessary condition of truth-functionality.

It is worth noting in view of the common definitions of truth-functionality, that it is not a sufficient condition either. To see this, consider such sentences as

(A) 'S knows that p'
(B) 'S is under the mistaken impression that r'.

These differ from the standard belief examples in precisely the allegedly crucial respect: (if they are true) we cannot change the truth values of p and r without altering the values of (A) and (B). If we substitute *any* false sentence for (true) p, (A) is thereby falsified; the same remarks hold *mutatis mutandis* for (B). Nevertheless neither sentence is truth-functional.

Again, it is possible that some very complicated principle will work. But a far easier way of dealing with the problem is available. We must abandon the first interpretation and rely on the second, *viz.*, that if a component does play a role in determining the truth value of a compound, it is only in virtue of its truth value, i.e., it is in virtue of *only* its truth value. Some standard remarks about truth-functional components reflect just this point. Some examples: In such compounds a component sentence can be replaced by any other sentence with the same truth value without altering the value of the compound, or: We can think of the components as truth values. The important point which is captured in such remarks is that nothing other than the truth value of a component is relevant to the truth or falsity of a truth-functional compound.

Let us put it in terms of sentences like 'S believes p'. It isn't the irrelevance of p's truth value to the value of the compound that makes the belief sentence non-truth-functional, it is the relevance of something besides truth value. It is not that beliefs can be false as well as true, but rather that we cannot substitute for the believed sentence another sentence with identical truth value, and be certain that the truth value of the compound will

remain as it was. Similarly, the crucial point about (A) and (B), the fact in virtue of which neither is truth-functional, is that the truth value of neither compound is necessarily preserved through substitution of like truth values. If 'S knows p' is true, p is true; and although, as noted earlier, we can falsify the compound by substituting for p any false sentence, we cannot substitute for p just any true sentence, nor for r in the (true) sentence 'S is under the mistaken impression that r' just any false one. A person who knows one thing doesn't necessarily know everything, and the man with one mistaken belief need not have them all.

The reason why (A) and (B), and more ordinary belief sentences, are non-truth-functional is that something about the components other than their truth value is relevant to the truth value of the compound; what they say is relevant. (Indeed, in some cases one might want to say that *how* what is said gains expression can be important. If 'w$_1$' and 'w$_2$' are synonymous, but S doesn't know this, it would be misleading, or even false, to say that S knows that x is w$_1$ when what he would say is that it is w$_2$, and when he would deny, or refuse to assent to, 'x is w$_1$').

To sum up: It is commonly thought that what I have called the 'relevance' of a component's truth value to the value of a compound is both a necessary and a sufficient condition of the compound's being truth-functional. On this view, a component occurs in a compound truth-functionally just in case its truth value somehow partly constitutes, or partly determines the truth value of the compound. I have shown that this provides neither a necessary nor a sufficient condition of truth-functionality. What is necessary is that nothing other than the truth value of a component can be relevant to the truth or falsity of the compound. Perhaps the best way to put it is this: If a compound contains a component *truth-functionally* then, if anything about that component is relevant to the truth value of the compound, it is its truth value.

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