APPLICATIONS OF EPISTEMIC LOGIC TO THE PHILOSOPHY OF SCIENCE

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0. *Introduction.*

One of the main insights of Kant's (1) work, in a nutshell, is that an adequate epistemology should deal with the interaction between "knowers" and "objects of knowledge" rather than treat each of them separately. The only claim that the above formulation makes is that such an approach allows for certain theoretical insights which otherwise would be inaccessible.

Following the theory of the "semantic ascent" (2), this insight can be reformulated in more manageable terms, i.e. it can be interpreted as claiming that deeper insights can be gained by discussing "propositional attitudes" (3) rather than propositions or persons. Thus, from the point of view of the philosophy of science, a scientific theory is viewed as forming a "belief system", i.e. a system of relations defined on the set of beliefs that a person entertains at a certain time, rather than a system of statements analyzed exclusively by means of the traditional methods of Logical Syntax and Semantics.

An obvious corollary to this approach is that problems which have exhibited an admirable resistance to the syntactic-semantic treatment, might yield after all to the non-conventional

(1) I. KANT, *Critique of Pure Reason* (Smith, N.K. tr.) London, 1933; a similar conception of Kant's ideas is developed in J. HINTIKKA, "*Kant's 'new method of thought' and his theory of mathematics*", Ajatus, vol. 27, pp. 21-30.

(2) The term 'semantic ascent' was coined by W.V.O. QUINE, in *Word and Object*, M.I.T. and John Wiley Press, 1960, p. 270.

(3) The term was coined by Bertrand RUSSELL in *An Inquiry into Meaning and Truth*, George Allen & Unwin, London, 1961, p. 65; it refers to binary relations from persons to propositions (or statements).

weapons of Logical Pragmatics (4). In other words, "Pragmatize your problem !" The idea sounds promising, especially since notorious problems such as the problem of the counterfactuals (5) and the analytic - synthetic (6) debate have strongly suggested the need for more powerful tools. Although the 'vision' of Pragmatics can be traced back as early as Peirce (7), no substantial contribution to this field has been made up to very recently. However, the situation has been radically changed, since the publication of Hintikka's *Knowledge and Belief* (8).

Making use of a simple logical apparatus, Hintikka offers in this book an impressive systematization of the various uses of '*a* knows that' and '*a* believes that' and thereby shows the special attraction inherent in Logical Pragmatics. Hintikka further applies his results to prominent epistemological pro-

(4) The term refers to the study of the relations between speakers and linguistic and logical entities, or, more generally, to the study of the actual use of language — in contradistinction to the study of its structure; it was introduced by Charles MORRIS, see, e.g. *Signs, Language and Behavior*, Brazilia, New York, 1955.

(5) The problem was introduced by R.M. CHISHOLM *"The contrary to fact conditional"*, *Mind*, vol. 55 (1958), pp. 289-307, and N. GOODMAN, *The problem of counterfactual conditionals*; *The Journal of Philosophy*, vol. 41 (1942), pp. 113-128 In N. RESCHER, *Hypothetical Reasoning*, North-Holland Publishing Co., Amsterdam, 1964, pp. 89-90, one can find a partial bibliography.

(6) The problem was brought up by W.V.O. QUINE, *"Two dogmas of empiricism"*, *The Philosophical Review*, vol. 60 (1951), pp. 20-43.

(7) *Collected Papers of C. S. Peirce*, vol. 5 (Heartshone and Weiss, eds.), Harvard University Press, Cambridge University Press, 1927.

(8) J. HINTIKKA, *Knowledge and Belief: An Introduction to the Logic of the two Notions*. Cornell University Press, 1962.

Later contributions of HINTIKKA to Epistemic Logic, a subfield of logical Pragmatics are:

'Knowing oneself' and other Problems in Epistemic Logic, *Theoria*, vol. 32 (1966), pp. 1-13,

Existence and identity in epistemic contexts, *Theoria*, vol. 33 (1967), pp. 138-147;

Individuals, possible Worlds and Epistemic Logic, *Nous*, vol. 1 (1967), pp. 33-62;

"Semantics for Propositional Attitudes", *Studies in Philosophical Logic* (J.V. Davis, ed.), Synthese Library, Reidel Publishing Company, Dordrecht, 1969.

blems concerning the meaning of 'knowing that one knows' and the roots of Moore's Paradox and thus exhibits its usefulness as a general conceptual framework for epistemology. This pilot pragmatical study may also contain the solution to the Strawson-Carnap debate on linguistic naturalism versus linguistic constructionism [10,11]. It indicates the artificiality of the whole issue, since constructed systems, according to Hintikka's practice and preaching [12], offer no substitute for the analysis of natural language locutions; rather, they are the means for the analysis. Constructed systems provide the *theories* for natural language analysis; they are, to use well-known recent Chomskyan terminology, the explanatory models while the natural language material serves to test them and is systematized by them.

This study intends to carry further the "campaign for pragmatization". I shall apply the system of Epistemic Logic (henceforth referred to as EL) as developed in *Knowledge and Belief* to a few philosophical problems which have been widely dealt with in recent literature, more specifically, to that of the analytic-synthetic debate and that of the counterfactual connective which were mentioned earlier, and the problem of the theoretical foudations for Inductive Logic. I shall rely heavily on Hintikka's work, both in EL and in other areas. All the relevant results will be reviewed.

1. *A Review of Knowledge and Belief.*

The review to be presented here gives mainly the technical aspects of Hintikka's EL and is restricted to that part of the

(9) J. HINTIKKA, "*The modes of modality*", *Acta Philosophica Fennica*, vol. 16 (1963), pp. 65-81.

(10) P.F. STRAWSON, "*Carnap's view on constructed systems verses natural languages in analytic philosophy*," The Philosophy of Rudolf Carnap (P.A. SCHILPP, ed.), Open Court, La Salle, Ill., 1963.

(11) R. CARNAP, "P.F. Strawson on linguistic naturalism", *Ibid.*, pp. 933-940.

(12) J. HINTIKKA, *Epistemic logic and the methods of philosophical analysis, Australian Journal oi Philosophy*, vol. 46 (1968), pp. 37-51.

system which deals with the combination of epistemic notions and the concepts of the propositional calculus and is therefore rather incomplete. The reader is referred to the original works to get the right feeling of the scope and importance of the system.

Notation

'a' and 'p' are variables ranging over persons and statements, respectively.
The symbolic abbreviations used are the following:

The phrase	Is abbreviated to
a knows that p	$K_a p$,
a believes that p	$B_a p$,
It is possible, for all that a knows, that p	$P_a p$,
It is compatible with all that a believes that p	$C_a p$.

'K_a', 'B_a', 'C_a', 'P_a' are *epistemic operators* while 'B_a' and 'C_a' are, in addition, also *doxastic operators*.

Model Sets and Model Systems

A *model set*, μ, is the formal counterpart of a partial description of a possible world. It is a set of statements which conforms to the following conditions (besides a few others):

(C.$\sim$) If $p \in \mu$ then not "$\sim p$" $\in \mu$.
(C.V) If "$p \vee q$" $\in \mu$ then $p \in \mu$ or $q \in \mu$ (or both).
(C.$\sim\sim$) If "$\sim\sim p$" $\in \mu$ then $p \in \mu$.
(C.$\sim\wedge$) If "$\sim(p \wedge q)$" $\in \mu$ then "$\sim p$" $\in \mu$ or "$\sim p$" $\in \mu$ (or both).
(C.$\sim$V) If "$\sim(p \vee q)$" $\in \mu$ then "$\sim p$" $\in \mu$ and "$\sim q$" $\in \mu$.
(C.$\supset$) "$p \supset q$" $\in \mu$ iff "$\sim p$" $\in \mu$ or $q \in \mu$.

This is a reformulation of propositional logic in model set terminology. (C.$\sim$) imposes consistency on model sets. It should be noted, however, that the following rule (C.$\sim$ or) $p \in \mu$ or "$\sim p$" $\in \mu$, is not imposed, nor is it derivable from the other rules. This allows for the partiality of world descriptions.

A *model system* is a set of model sets on which a relation called 'alternativeness' is defined. Two model sets stand to one another in this relation if they have some common characteristic (e.g. a predefined intersection set). The model system of all model sets is denoted by 'Ω'. 'μ' is a model set will be symbolized by '$\mu \in \Omega$', when needed.

Epistemic Operators and Model Sets

'$K_a p$' partitions Ω into two mutually exclusive and jointly exhaustive sets. the set Φ such that if $\mu \in \Phi$ then $p \in \mu$, and the complementary set $\bar{\Phi}$. If for any p, if "$K_a p$" $\in \mu$ then $p \in \mu^*$, then μ^* is referred to as an *epistemic alternative* to μ for a. The set of all the epistemic alternatives will be referred to as $\Phi_K(a, \mu)$. The subscript 'K' serves to distinguish this set from $\Phi_B(a, \mu)$, which is the set of all *doxastic* alternatives to μ for a. The latter, in its turn, is defined for the notion of belief in complete analogy to the definition of $\Phi_K(a, \mu)$ for the notion of knowledge — $\mu^* \in \Phi_B(a, \mu)$ iff, for any p, if "$B_a p$" $\in \mu$ then $p \in \mu^*$.

The following conditions will be stated in terms of epistemic (and doxastic) alternatives. Conditions which deal with the relations between statements belonging to different model sets are marked with an asterisk. Unmarked conditions deal with relations between statements within the same model set.

The conditions on 'K' are the following:

(C.$\sim$K) If "$\sim K_a p$" $\in \mu$ then "$P_a \sim p$" $\in \mu$.
(C. P) If "$\sim P_a p$" $\in \mu$ then "$K_a \sim p$" $\in \mu$.

Making use of the '$\equiv$' connective, they can be reduced to $K_a p \equiv \sim P_a \sim p$.

The above conditions impose deductive closure on the set of statements one knows. This is necessary for a successful "pragmatization" of one's problems. In order to assure that all relations of entailment represented at the syntactic-semantic level of analysis will also be represented on the pragmatic level, one has to impose that pragmatic implication hold whenever semantic implication holds (i.e., that if p entails q then $B_a p$ should also entail $B_a q$). However, this requirement is counterintuitive.

(C.P*) If "$P_a p$" $\in \mu$ then there is a μ^*, $\mu^* \in \Phi_K(a, \mu)$, such that $p \in \mu^*$.

This means that to be possible (relatively to one's knowledge) is to be satisfied in a world which is possible from the point of view of one's knowledge.

(C.KK*) If "$K_a p$" $\in \mu$ and $\mu^* \in \Phi_K(a, \mu)$ then "$K_a p$" $\in \mu^*$.

This condition is the main contribution of the "pragmatization". In the case $K_a p_1, \ldots, K_a p_i, \ldots, K_a p_n$, it imposes consistency not only on $p_1, \ldots, p_n$ but also on $K_a p_1, \ldots, K_a p_n$, $p_1, \ldots, p_n$.

(C.K) If "$K_a p$" $\in \mu$ then $p \in \mu$.

This condition, trivial as it seems, has some important functions:
a) It serves to distinguish '$K_a p$' from such locutions as 'John knew Humphrey would win, however he was wrong'.
b) It is the only rule which distinguishes (in this selection of axiomatic conditions) 'K_a' from 'B_a'.

The conditions for belief are completely analogous to the conditions for knowledge when 'K' is replaced by 'B' and 'P' is replaced by 'C' throughout. All the resultant conditions are adopted, except

(C.B.) If "$B_a p$" $\in \mu$ then $p \in \mu$.

This is obviously inacceptable, since one can believe false statements.

It should be noted, however, that by replacing 'K' with 'B' in (C.P*) and (C.KK*) one has changed $\Phi_K(a, \mu)$ into $\Phi_B(a, \mu)$. As to the relation holding between the two sets, Hintikka formulates it in

(c.dox) $\Phi_B(a, \mu) C\ _K(a, \mu)$,

which is easily proved to be equivalent to

(C.KB) [13] If "$K_a p$" $\in \mu$ then "$B_a p$" $\in \mu$.

We shall note a few theorems which will be of use in the sequel:

1) If "$K_a K_b p$" $\in \mu$ then "$K_a p$" $\in \mu$.
2) "$K_a K_a p$" $\in \mu$ iff "$K_a p$" $\in \mu$ (K_a is idempotent).
3) If "$B_a p$" $\in \mu$ then "$B_a B_a p$" $\in \mu$.

To complete the review, we shall present a rule governing combinations of epistemic operators with quantifiers.
(C.109) If "(x)p" $\in \mu$ and "$(\exists y)K_b(a = y)$" $\in \mu$, and p does not contain epistemic operators other than 'K_b' and 'P_b', then "p(a/x)" $\in \mu$.
'(x)' and '$(\exists x)$' are the universal and existential quantifiers, respectively. p(a/x) is the result of substituting 'a' for 'x' throughout 'p'. Hintikka argues [14] that '$(\exists y)K_b(a = y)$' should be interpreted as '*b* knows who *a* is'.
This final rule restricts the applicability of universal instantiation to individuals *known* to *b*. We shall want it later on in one of our proofs.

2. *Epistemic Operators and Quantifiers Over Statements*

In this section, I shall suggest a few additional conditions which will be needed in the following sections.
In general, these conditions belong to an area not treated by Hintikka, viz. that of the relation between epistemic operators and quantifiers over statements, and alethic predicates applied to statements, i.e., such locutions as 'a knows that there is a statement *l* which logically implies the statement q'.
The treatment sketched below should not be taken as a

[13] (C.KB) refers in HINTIKKA's *Knowledge and Belief* (see note 8) to a condition which is differently formulated, though equivalent to the given in our text. The formulation of the conditions on the concept of model set has likewise been slightly changed.
[14] *Knowledge and Belief*, pp. 131-132, 141-144.

systematic study of this fascinating field but rather as consisting of a few ad hoc conditions which are intuitive in all their applications to follow. Whether they might not cause trouble in further applications I do not know, nor indeed whether they are the most economical conditions for the purpose intended. The suggested conditions are, where 'Q_i' (i = 1, ..., n) shall be used as dummies for alethic predicates (e.g. it is logically necessary that 'p') and 'l_j' = (j = 1, ..., m) would serve as bindable statement-variables.

1) (C.Q*) Iff "$Q_i(p) \in \mu$ and $\mu^* \in \Omega$ then "$Q_i(p)$" $\in \mu^*$.

This is a formalization of the idea that a statement which consists of applying an alethic predicate to another statement is *logically* true (if true at all).

2) (C.OK) Iff "Q(p)" $\in \mu$ then "$K_a Q(p)$" $\in \mu$ where 'p' and 'q' serve as dummies for statements. This is an extension of Hintikka's suggestion formulated in (C.K) and (C.P) to ascribe to *a* knowledge of all logical truths.

3) (C.EL) $(\exists l)$ $(l \wedge Q(l))$" $\in \mu$ iff at least for one p, p $\in \mu$ and "Q(p)" $\in \mu$.

4) (C.U) "(l) $[l \wedge Q(l)]$" $\in \mu$ iff for any p, if p $\in \mu$ then "Q(p)" $\in \mu$.

The last two conditions express the rules of E.I. and E.G,U.I. and U.G., as needed in our context.

5) (C.EB-BE) If "$(\exists l) B_a[l \wedge Q(l)]$" $\in \mu$ then "$B_a[(\exists l) l \wedge Q(l)]$" $\in \mu$.

This means that if there is a statement of which *a* believes that it is true and that it has Q then *a* believes that there is a true statement which has Q (C.EK-KE) is likewise adopted. (C.EB-BE) is applicable, however, only when one can assert that there is a statement of which *a* beieves that it is true and that it has Q. When can such an assertion be made ? An answer is provided by

6) (C,BQE) If "$B_a[p \wedge Q(p)]$" $\in \mu$ then "$(\exists l) B_a[l \wedge Q(l)]$" $\in \mu$.

That is, it is sufficient to know that *a* believes in the Q-ness of a certain true statement in order to assert that there is a statement of which *a* believes that it is true and that it has Q. (C.KQE) is adopted in an analogous way.

3. *The Counterfactual Connective.*

An explication of the connective governing counterfactual conditionals such as: 'If this were a crow, it would be black' have been widely attempted. Nagel ([15]), however, is skeptical as to the value of these proposals:

> In any event, it is certainly not possible to construct a general formula which will prescribe just what must be included in the assumptions under which a counterfactual can be adequately grounded.
> Attempts to construct such a formula have been uniformly unsuccessful, and those who see the problem as that of constructing such a formula are destined to grapple with an insoluble problem.

Since this section attempts a refutation of Nagel's pessimistic prediction, I shall first try to outline reasons for the previous failures. I suggest that they were due to the imposition of too strong conditions on the explication.

1) The explication was expected to provide a complete interpretation for the counterfactual connective. It seems that the lesson from Carnap's ([16]) work, which showed both the possibility and the inevitability of partial interpretation, was not appreciated.

2) The explication was expected to provide *truth conditions* for statements made by uttering a counterfactual conditional. This was in accord with the almost general adoption of the

([15]) E. Nagel, *The Structure of Science*, Routledge & Kegan Paul, London, 1961, pp. 68-73.

([16]) Especially R. Carnap, *The methodological character of theoretical concepts*, *Minnesota Studies in the Philosophy of Science* (H. Feigl & M. Scriven, eds.), vol. I, Univ. of Minnesota Press, 1956, pp. 38-76.

traditional logical semantics framework. It might be argued that, in many contexts, this framework leads to a vicious circle and that this circle can be broken through only via *the pragmatic turn* which, in our case, amounts to a consideration of the use of counterfactuals rather than that of their truth-conditions. This approach is feasible only in a pragmatic framework.

Before attempting my explication, I shall enrich the notational and conceptual resources of the pragmatic theory I use, i.e. of Epistemic Logic.

This is needed for the description of the "semantic field" of the connective under discussion as well as for the following sections.

Notational Conventions.

The symbol	is to be read as
$N p$	p is logically necessary
$p \prec q$	p logically entails q
$M_a p$	a is doxastically neutral towards p
$I_a p$	a is epistemically indifferent towards p
$\mu_* \in \alpha_B(a, \mu)$	μ_* is a hyperdoxastic alternative to μ for a
$p > q$	If it were the case that p, then it would be the case that q.

Definitions:

D_1: "$N p$" $\in \mu$ iff, for any μ^*, if $\mu^* \in \Omega$ then $p \in \mu_*$.
D_2: "$p \prec q$" $\in \mu$ iff "$N(p \supset q)$"
D_3: "$\mu_a p$" $\in \mu$ iff "$C_a p$", "$C_a \sim p$" $\in \mu$.
D_4: "$I_a p$" $\in \mu$ iff "$P_a p$", "$P_a \sim p$" $\in \mu$.
D_5: $\mu^* \in \alpha_B(a, \mu)$ iff if "$B_a p$" $\in \mu$ then "$B_a p$" $\in \mu_*$

Te new symbols enable us to formulate an additional condition. However, this condition is merely of technical importance.

(C.N) If, for any $\mu \in \Omega$, not "$\sim p$" $\in \mu$, then, for any $\mu \in \Omega$, "Np" $\in \mu$.

Construction of the Explication

In this section, conditions will be given '>' in a pragmatic context, and since a belief-context is intimately connected with an assertion-context, I shall explicate '$B_a(p > q)$'. However, the relation between these two contexts awaits explication. A minimal adequacy requirement on such an explication would be that it provides for the fact that one's assertions are expected to be *believable* for him, as Hintikka has aptly argued ([17]).

We shall start our analysis from statements which are closely related to counterfactual statements, namely, those which are traditionnally dubbed in the literature as "subjunctives", e.g. "If there is life on Venus, there is water there". It seems that such statements have, in many respects, the same logical force as material implications. The main difference is that, in order to believe such a statement to be true, beliefs concerning the truth values of the antecedent or the consequent of the statement are always irrelevant. Thus, it seems that beliving 'p ⊃ q' without having beliefs concerning 'p' or 'q' is sufficient for believing 'p > q'. Formally:

(C.B $>_1$) If "$M_a p$", "$M_a q$", "$B_a(p \supset q)$" $\in \mu$ then "$B_a(p > q)$" $\in \mu$.

It is clear that the sufficient condition given in (C.B$>_1$) can not be strengthened to a necessary and sufficient one, since '$B_a[(\sim p) \wedge (p > q)]$' which naturally serves as our formalization of "a believes in the *counterfactual* 'if p then q'" would be inconsistent. However, I shall later show a different way of strengthening this condition.

(C.B$>_1$) can be paraphrased, using the notion of "doxastic alternative", as:

(C.B$>_2$) If "$M_a p$", "$M_a q$" $\in \mu$ and if for any μ^*, if $\mu^* \in \Phi_B(a, \mu)$ then p ⊃ q" $\in \mu^*$, then "$B_a(p > q)$" $\in \mu$.

This reformulation does not as yet explain how '$B_a(p > q)$' is possible, so to speak. Under what conditions does one be-

([17]) *Knowledge and Belief*, pp. 67, 72.

lieve in 'p ⊃ q' without having beliefs concerning 'p' and 'q', i.e. under what conditions does one believe in a truth-functional implication for non-truth-functional reasons ? The answer could be gathered from the following paragraph of Nagel [18]:

> A counterfactual can be interpreted as an implicit metalinguistic statement... asserting that the indicative form of its consequent clause follows logically from the indicative form of its antecedent clause, when the latter is conjoined with some law and the requisite initial conditions for the law.

In order to utilize this idea for our needs, one has only to pragmatize it. We shall regard as a sufficient condition for believing 'p ⊃ q' for non-truth-functional reasons, believing in the existence of a true statement *l* which, in conjunction with p, entails q. Formally:

(C.B$>_3$) If "$M_a p$", "$M_a q$" $\in \mu$ and
"$B_a[(\exists l)l \wedge ((l \wedge p) \prec q)]$ then "$B_a(p > q)$" $\in \mu$.

(C.B$>_3$) is equivalent to (C.B$>_1$). That is,
'$B_a[(\exists l)(l \wedge p \prec q)]$'
is equivalent to '$B_a(p \supset q)$', relatively to '$M_a p$', '$M_a q$'.

We shall prove the equivalence. Suppose, first,

1. "$B_a(p \supset q)$", "$M_a p$", "$M_a q$" $\in \mu$.
2. "$\sim B_a[(\exists l)(l \wedge (l \wedge p \prec q))]$" $\in \mu$.
3. "$C_a[\sim(\exists l)(l \wedge (l \wedge p \prec q))]$" $\in \mu$. (2, (C$\sim$B))
4. "$\sim(\exists l)(l \wedge (l \wedge p \prec q))$" $\in \mu^*$. (3,(C.C*))
5. "$(l) \sim (l \wedge l\, p \prec q))$" $\in \mu^*$. (4)
6. "$p \supset q$" $\in \mu^*$. (1.(C.B*))
7. "$\sim[(p \supset q \wedge [(p \supset q) \wedge p \prec q]]$" $\in \mu^*$. (5, 6, (C.U))
8. $((p \supset q) \wedge p) \prec q$" $\in \mu^*$. ((C.N))

9a) "$\sim(p \supset q) \in \mu^*$
9b) "$\sim((p \supset q) \wedge p \prec q)$" $\in \mu^*$.
} (7. (C$\sim$ $\wedge$))

But 9a), contradicts 6) and 9b) contradicts 8). So (C.$\sim$) is violated.

[18] NAGEL, *ibid.*, p. 72.

10.	"$B_a[\exists l)\ (l \wedge (l \wedge p -\!\!3\ q))]$" $\in \mu$),	
11.	"$\sim B_a(p \supset q)$" $\in \mu$.	(counter-assumptions)
12.	"$C_a \sim (p \supset q)$" $\in \mu$.	(11. (C.$\sim$B))
13.	"$\sim(p \supset q)$" $\in \mu^*$.	(12. (C.C*))
14.	"$(\exists l)\ (l \wedge (l \wedge p -\!\!3\ q))$" $\in \mu^*$.	(10. (C.B*))
15.	"$r \wedge (r \wedge p -\!\!3\ q)$" $\in \mu^*$.	(14. (C.E*l*))
16.	$r \in \mu^*$.	(15. (C. $\wedge$))
17.	"$r \wedge p -\!\!3\ q$" $\in \mu^*$.	(15. (C. $\wedge$))
18.	"$r \wedge p \supset q$" $\in \mu^*$.	(17. (D_2))
19.	"$\sim(r \wedge p) \vee q$" $\in \mu^*$.	(18. (C. $\supset$))
20a)	or "$\sim r$" $\in \mu^*$. or	(19. (C. $\sim \wedge$), (C.V))
20b)	"$\sim p$" $\in \mu^*$. or	
20c)	$q \in \mu^*$.	
21)	"$\sim(\sim p \vee q)$ $\in \mu^*$.	(13, (C. $\supset$))
22)	"$\sim q$" $\in \mu^*$.	(21. (C$\sim$V))
23)	$p \in \mu^*$.	(21. (C$\sim$V))

16) and 20a), 22) and 20c), 23) and 20b) are pairs of contradictions, so (C.N) is violated and the proof is complete.

Since we quantify over statements, they can be considered "individuals" in Hintikka's sense and since the number of individuals considered in (C.B$>_1$) is two (namely, p and q) while in (C.B$>_3$) there are three of them (p, q and the individual introduced by the existentially bound statement variable *l*), the equivalence proved is synthetic in Hintikka's third sens of the term ([20]). This suggests that (C.B$>_3$) may serve as an explanation of (C.$>_1$), in spite of their being L-equivalent. In general, if 'p' and 'q' are L-equivalent, neither can serve as a 'syn-

([19]) J. HINTIKKA, *"An analysis of analyticity"*, *Deskription,* Analytizität und Existenz, 3-4, Forschungsgespräch des Internationalen Forschungszentrums für Grundfragen der Wissenschaften (P. Weingartner, ed.), Postet, Salzburg and München, 1966, pp. 193-214. The number of individuals considered in a sentence, S, is the sum of the number of free singular terms in S and the maximal number of quantifiers, the scope of which have a common part in S. (*Ibid.*, pp. 208-209).

([20]) "An argument-step is analytic in sense III if it does not introduce new individuals into the discussion" (*ibid.*).

tactic explanation' of the other, in Hempel's sense [21]. 'p' however may serve as a kind of pragmatic explanation of 'q' in Hempel's sense when 'p' has a higher degree ([22]) than 'q'. Or, using more of Hintikka's ([23]) terminology, though 'p' and 'q' might have the same "depth-information", 'p' might constitute a pragmatic explanation for 'q' if it contains more "surface information". But this was a digression.

By now, we possess three sufficient conditions for '$B_a(p > q)$'. Still this is not enough. Suppose one believes "If the bird over there were a raven, it would be black". Then if he also believes that all ravens are black, and if he is doxastically neutral towards both the species-identity and the color of the distant bird, his belief is justifiable on (C.B $>_3$). If later on he comes to learn that the bird under discussion is not a raven so that his doxastic neutrality is broken then (C.B $>_3$) becomes inapplicable. Yet, intuitively, there is no reason for him to give up his original belief. And, in any case, this intuition can be accounted for by our symbolic apparatus.

One can derive from D_5:

If "$B_a(p > q)$" $\in \mu$ and $\mu_* \in \mu$ $\alpha_B(a, \mu)$ then "$B_a(p > q)$" $\in \mu_*$ i.e.,

if all one's beliefs are retained, one's beliefs in subjunctives also are retained even when some of the reasons for adopting them, namely, the doxastic neutrality towards the antecedent and the consequent no longer hold. In order to complete our treatment, we have to add one more condition over and above (C.B$>$) and D_5, namely

(C.B$_*>$) If "$B_a(p > q)$" $\in \mu_*$ then there is a μ such that "$M_a q$", "$M_a p$", "$B_a(p \supset q)$" $\in \mu$ and $\mu_* \in \alpha_B(a, \mu)$. Since $\mu \in \alpha_B(a, \mu)$, (C.B$_*>$) is compatible with (C.B$>$), since if

([21]) C. G. Hempel, *Aspects of Scientific Explanation*, Free Press, N.Y., 1965, pp. 425-428, 245-291.

([22]) The degree of a sentence S, d(S), is the sum defined in (19).

([23]) 'Depth information is the total information one can extract from a sentence by means of logic. Surface information is what a sentence gives us before we have done any of the many things we can do to it by means of logic in order to bring out all the information that may be hidden in it', Hintikka, J., *ibid.*, pp. 234-253.

"$B_a(p > q)$" is due to "$M_a p$", "$M_a q$", "$B_a(p \supset q)$" $\in \mu$, and not "transferred" to it from another model set, them ($C.B_* >$) is trivially satisfied.
($C.B >$) and ($C.B_* >$) provide, together with D_5, a sufficient and necessary condition for "$B_a(p > q)$".

Theorem B: "$B_a(p > q)$" $\in \mu^*$, iff there is a $\mu \in \Omega$ such that $\mu^* \in \alpha_B(a, \mu)$ and "$M_a p$", "$M_a q$" "$B_a(p \supset q)$" $\in \mu$.

Proof. The left-to-right implication is identical with ($C.B^* >$); we have only to prove the inverse implication. By our assumptions, there is a μ such that:

1) $\mu_* \in \alpha_B(a, \mu)$.
2) "$B_a(p \supset q)$", "$M_a p$", "$M_a q$", $\in \mu$.
3) "$B_a(p > q)$" $\in \mu$. (2, ($C.B >$,))
4) "$B_a(p > q)$" $\in \mu_*$. (1, 3, D_5).

Our theorem is not only implied by, but also implies ($C.B >$) and ($C.B_* >$); the first, since $\mu \in \alpha_B(a, \mu)$ and if "$B_a(p \supset q)$", "$M_a p$", "$M_a q$" $\in \mu$, then the sufficient condition for "$B_a(p > q)$" is trivially satisfied; the second is a still more immediate consequence. Its importance lies in clearly exhibiting the exhaustiveness of our treatment of subjunctives in doxastic contexts. The counterfactual "$B_a(p > q) \wedge \sim p$" does not require any further conditions. It cannot be introduced directly into a
then obviously not "$B_a\, p$" $\in \mu$. The analogous conditions (C.K
then obviously not "$B_a\, p$" $\in \mu$. The analogous conditions ($C.K >$) and ($C.K_* >$) and therefore the theorem corresponding to theorem B, to be called "Theorem K", are likewise adopted as explicating "$K_a(p > q)$". We did not include a definition of $\alpha_K(a, \mu)$ in analogy with $\alpha_B(a, \mu)$. If such a definition were given, the adoption of (C.K) would be sufficient for $\alpha_K(a, \mu) = \Phi_K(a, \mu)$. Therefore, in formulating ($C.K >$), ($C.K' >$) (or, rather, ($C.K_* >$)) and Theorem K, $\Phi_K(a, \mu)$ is to be used, occupying the same place as $\alpha_B(a, \mu)$ within the doxastic conditions.

We have already given a *partial interpretation* of '>' by it a full interpretation ([24]) in two propositional attitudes contexts, 'K_a' and 'B_a'. Hintikka's (C.K*) and (C.B*) enable us, further, to formulate (C.>) which gives a sufficient condition for 'p > q' outside any context. (C.>) is no more than a combined application of (C.B*) and (C.K*) to '$B_a(p > q)$', and '$K_a(p > q)$', respectively

(C.>) "p > q" $\in \mu^*$ if $\mu^* \in \Phi_K(a, \mu)$ or $\mu^* \in \alpha_B(a, \mu)$, for some a and μ, such that "$K_a(p > q)$" $\in \mu$ or "$B_a(p > q)$" $\in \mu$, respectively.

In the light of the intimate connection between subjunctives and counter-factuals and the logical structure of "law-like sentence" ([25]), I will now suggest the general definitions for the concepts "lawlike-sentence" and "Law of Nature":

Definition

(1) "*l*" is a law-like sentence ($l \in g$) iff "*l*" has the form "$Qx_1\ Qx_2 \ldots\ Qx_n p$" where each "Q" is either "( )" or "$\exists$" and at least one of the "Q"'s is "( )".

(2) "$p[(a_1/x_1), (a_2/x_2) \ldots (a_n/x_n]$" has the form "q > r" for some "q" and "r" such that "x_1" occurs freely in "$q(x_i/a_i)$" and "$r(x_i/a_i)$". ('$j^o(\alpha/\beta)^o$ is the result of replacing 'β' throughout 'j' by 'α'.)

It should be remembered that since '>' has been given only a partial interpretation, our (syntactic) definition, likewise, gives "G" only a partial interpretation. In the same way, the following definition gives the definiendum only a partial interpretation.

Definition: *l* is a Law of Nature in $\mu(N(l, \mu)$ iff $l \in G$ and $l \in \mu$.

The above definitions and the previously stated conditions enable us to apply the concept "Law of Nature" within the doxastic and epistemic alternatives of a person in (our world) once we know enough about his belief-and-knowledge sys-

([24]) Giving full interpretation to a term amounts to defining it — obviously, one has to specify — in which terms the interpretation is given.

([25]) See NAGEL, *ibid.*, pp. 49-52.

tem. But we cannot apply it within our world. This is in full accord with the well founded modern conception of laws of nature occurring in scientific theories as hypotheses, and not as "revelations", about the "true nature of the world".

Remarks About Adequacy

1) (C.B $>_3$) reminds us of a number of previous treatments of "$>$". One might wonder whether it is immune to the weaknesses which have effected their failure.

a) Is it possible that

1. "$M_a p$", "$M_a q$", "$B_a[(\exists l)l \wedge {}_p(l)p \dashv q) \wedge N \sim (l \wedge p)]$" $\in \mu$?

If this happens, "$l \wedge p \dashv q$" is trivial and one might doubt whether "$B_a(p > q)$" $\in \mu$, following from 1) by (C.B $>_3$) is intuitively founded, since if this is so "$l \wedge p \dashv \sim q$" is also believed by *a*, (at least in the Hintikkian sense; namely, that one believes in all deductive consequences of all one's beliefs) and therefore "$B_a(p > \sim q)$" $\in \mu$, also by (C.B $>_3$). We shall later deal with the general problem of the compatibility of "$B_a(p > q)$" and "$B_a(p > \sim q)$". So far they seem incompatible. We shall prove that the 'situation described' by 1) is 'impossible'. That is, 1. is inconsistent.

We have already shown in the previous chapter, that 1. entails:

2. "$B_a(p > q)$" $\in \mu$

Since it entails as we have argued above

3. "$B_a[(\exists l)l \wedge (l \wedge p \dashv \sim q) \wedge N \sim (l \wedge p)]$" $\in \mu$,

then, for the same reasons:

4. "$B_a(p > \sim q)$" $\in \mu$.
5. "$p > q$" $\in \mu^* \in \Phi_B(a, \mu)$. (2, (C.B*))
6. "$p > \sim q$" $\in \mu^* \in \Phi_B(a, \mu)$. (4, (C.B*))
7. "$\sim p$" $\in \mu^* \in \Phi_B(a, \mu)$. (5, 6)————
8. "$B_a \sim p$".(7, Definition of "Φ_B")
9. "$\sim C_a p$". (8, (C. $\sim$ C)), (C. $\sim$ B))
10. "$C_a p$". (1, D_3).

9. and 10. violate (C. $\sim$), demonstrating the inconsistency of 1.

b) A question similar to a) is whether our treatment does not allow belief in true, yet vacuous, universal statements as supporting, counterintuitively, belief in subjunctives. Suppose:

1. "$B_a[(x) \sim Gx]$" $\in \mu$.
2. "$B_a[(x)(Gx \supset Qx)]$" $\in \mu$ (1, since "$(x) \sim Gx$" entails "$(x)(Gx \supset Qx)$")
3. "$(x)(Gx \supset Qx)$" $\in \mu^* \in \Phi_B(a, \mu)$. (2. (C.B*))

Suppose also:

4. "$(\exists x)B_a(x = b)$" $\in \mu$ that is, that *a* has beliefs concerning the identity of at least one individual in μ.
5. "$(\exists x(B_a(x = b)$" $\in \mu^*$. (C.EB = EB =*)[26]
6. "$Gb \supset Qb$" $\in \mu^*$. (5, 3, C. 109)
7. "$B_a(Gb \supset Qb)$" $\in \mu$. (6, Definition of Φ_B).

By a precisely similar argument one derives from 1) and 4)

8. "$B_a(\sim Gb)$" $\in \mu$. (1, 4, (109))
9. "$\sim C_a(Gb)$" $\in \mu$. (8, (C. ~C), (C. ~B))
10. "$\sim M_a(Gb)$" $\in \mu$. (9, D_3)

In view of 10) one cannot derive, in our treatment, either "$B_a(Gb > Qb)$" or "$B_a(Gb > \sim Qb)$" on *the strength of 1) alone.*

c) Goodman ([27]) would like to avoid that our '*l*', for which $(l \wedge p) \dashv q$ would satisfy '$p > \sim l$'. One cannot explicitly forbid this without using '>' and thus enter into a *vicious circle.* In our treatment, however, there is no such danger.

Suppose:

1. "$M_a p$", "$M_a q$" $\in \mu$,
2. $(\exists l)B_a[l \wedge ((l \wedge p) \supset q) \wedge (p \wedge \sim l)] \in \mu$.
3. "$B_a[(\exists l)l \wedge (l \wedge p \dashv q) \wedge (p > \sim l)]$" $\in \mu$. (2. EB — BE))
4. "$B_a(p > q)$" $\in \mu$. (1, 3, (C.B $>_3$))
5. "$(\exists l)[(l \wedge p \dashv q) \wedge (p > \sim l)$" $\in \mu^*$. (3. (C.B*))
6. "$r \wedge (r \wedge p \dashv q) \wedge (p > \sim r)$" $\in \mu^*$. (5(C.E*l*))
7. "$B_a[\wedge (r \wedge p \dashv q) \wedge (p > \sim r)]$" $\in \mu$. (6. (def. of Φ_B)

([26]) *Knowledge and Belief*, pp. 160-162 (C.EB = EB = *):
if "$(\exists x)B_a(b=x)$" $\in \mu$ and $\mu^* \in \Phi_B(a\mu)$ then "$(\exists x)B_a(b=x)$" $\in \mu^*$.

([27]) The reference is given in Note 5.

8. "$B_a[(p > \sim r)]$" $\in \mu$. (7)
9. There is a μ_1, such that $\mu \in \alpha_B(a, \mu_1)$ and "$M_a p$", "$M_a \sim r$", "$B_a(p \supset \sim r)$" $\in \mu_1$. (3, (C.B$*$ >)).
10. "$B_a(p \supset \sim r)$" $\in \mu$. (9, (D_5))
11. "$B_a r$" $\in \mu$. (7)
12. "$B_a \sim p$" $\in \mu$. (10, 11). Through (C.B*))
13. "$\sim C_a p$" $\in \mu$. (12. (C. $\sim$C),
(C. $\sim$B))
14. "$\sim M_a p$" $\in \mu$. (13.D_3)
15. "$M_a p$" $\in \mu$. (1)

14. and 15. violate (C. $\sim$). 1. and 2. are, therefore inconsistent.

2) Is "$B_a(p > q)$", "$B_a(p > \sim q)$" possible? That is, can one consistently believe both that $p > q$ and $p > \sim q$? At first sight, the answer seems definitely negative. Anyway, a deeper investigation will show that the situation here is exceedingly complicated. To show this, let us perform the following 'Gedanken experiment': let us envisage three historians, *A*, *B* and *C*, all studying Hitler's regime. *A* has found a note in Hitler's handwriting, expressing his decision to castrate all males of the conquered states. Believing both the authenticity of the note and that Hitler would carry out his decisions as far as he could, he concludes: If Hitler had conquered England he would have castrated all males.

Now, historian *B*, who does not know anything about the note which was found by *A*, happens to find another note, also in Hitler's handwriting and of the same date as the note found by *A*, expressing Hitler's determination never to harm any Aryan male. *B* has the right to conclude that: if Hitler would conquer England he would not castrate all English males assuming, as *A* and *C* do, that Englishmen are Aryans.

After the death of *A* and *B*, all their scientific findings together with their conclusions, are transferred to historian *C*, who has no reason to doubt the authenticity of the notes nor his predecessors' assumptions with regard to Hitler's serious intention to carry out his decision. He can, of course, draw an additional conclusion, which none of them could, namely, that Hitler did not intend to conquer any Aryan state. But what will be his attitude towards the conclusions that his colleagues drew:

A_1: 'If Hitler had conquered England, he would have castrated all English males'.
B_1: 'If Hitler had conquered England, he would *not* have castrated all English males.' ?

These concusions were derived from two mutually consistent assumptions, both of which C believes to be true, namely:

A_0: 'Hitler intended to castrate all males of the conquered states.'
B_0: 'Hitler intended not to harm any Aryan male.'

The inference from A_0 and B_0 to A_1 and B_1, respectively, is intuitively valid. How can we *describe* *C*'s propositional attitudes in this case ? One feels reluctant to admit that *C* believes both *A* and *B*, yet one cannot too easily avoid this conclusion altogether.

Notice however, that the Gedanken experiment described has a similar structure (though not identical structure) to the following situation:

$$S: \mu_1, \mu_2 \in \Omega. \quad \text{"}M_ap\text{"}, \text{"}M_aq\text{"}, \text{"}B_a(p \supset q)\text{"} \in \mu_1, \in \mu_2,$$
$$\text{"}M_ap\text{"}, \text{"}M_aq\text{"}, \text{"}B_a(p \supset \sim q)\text{"} \in \mu_2, \quad \mu \in \alpha_B(a, \mu_1),$$
$$\mu \in \alpha_B(a, \mu_2). \text{"}B_a \sim p\text{"} \in \mu.$$

If this situation is realized (and it is logically possible) then one will have by theorem 13:

$$\text{"}B_a(p > q)\text{"}, \text{"}B_a(p > \sim q)\text{"} \in \mu \text{ (provided that } \mu \neq \mu_1 \neq \mu_2 \neq \mu).$$

The possibility of such a situation seems extremely unfavorable to our treatment, which allows for it, unless one bears in mind the extreme complexity of our logical intuitions in this case as was clearly shown by our Gedanken experiment. Therefore, in order to answer our previous question concerning the description of *C*'s propositional attitudes and to defend our treatment against a possible objection stemming from the counter intuitiveness of the description of S, I would like to remind the reader of the *theoretical* character of the concept of Belief.

The correspondence rules for this concept are far from being obvious. Therefore,
"$B_a(p > q)$", "$B_a(p > \sim q)$" $\in \mu$ is not so counter intuitive if considered as a *theoretical* statement, whose behavioral manifestation, if any, would not consist in asserting both $(p > q)$ and $(p > \sim q)$, but rather, in a's suspending his judgment with regard to both until he is provided with more information, or in his rejection of the whole debate with regard to both as pointless, or something of the kind. Obviously, complicating the correspondence rules for '$B_a p$' to take into consideration other statements, like '$B_a q$' and the internal structure of both 'p's' and 'q's' is inelegant. But our Gedanken experiment has shown that this is not so entirely ad-hoc [28].

4. *A Programmatic Epistemic Framework for Inductive Logic*

All recent attempts to relate Inductive Logic (IL) to EL ([29]) have in common the conception of IL as an independent sour-

([28]) Mr. Asa KASHER has suggested (private communication) that $(C.B>_{\bullet})$ be replaced by

"$B_a(p > q)$" $\in \mu$ only if for all μ_1 such that $\mu \in \Phi_B(a,\mu_1)$, if "$M_a p$", "$M_a q$" $\in M_1$ then "$B_a(p > q)$" $\in \mu_1$. If his suggestion is adopted, theorem B is not valid, and so is the whole argument in this section. That is, '$B_a(p > q)$' and '$B_a(p > \sim q)$' turn out to be inconsistent, and '$B_a(p \wedge \sim p > q)$ is consistent. These results are advantageous, yet the following is quite counter-intuitive: If "$B_a(p > q)$" $\in \mu$ then in all possible states of affairs in which *a* believed only in statements in which he believed in μ (though not necessarily in all of them) and is neutral towards 'p' and 'q' he has to believe in '$p > q$'; I think, though, that neutrality towards '$p > q$' is quite consistent with believing only in statements which are believed in μ, i.e. that "$M_a p$", "$M_a q$", '$M_a(p > q)$' $\in \mu_1$, $\mu \in \Phi_B(a,\mu_1)$, "$B_a(p > q)$" $\in \mu$ is consistent — but it cannot be so if one accepts Kasher's suggestion.

([29]) C. F. HINTIKKA J. and HILPINEN, R., *Knowledge, Acceptance and Inductive Logic, Aspects of Inductive Logic,* North-Holland Publishing Company, Amsterdam, pp. 1966, pp. 1-20.

ce of results which can be used to enrich EL, e.g., to provide rules for 'conditional belief or acceptance' of *h* (hypothesis) on *e* (evidence) based on confirmation measures. The idea that IL should be connected with some kind of pragmatic theory, is, though, very attractive, since pure IL is intended to provide a formalization of the intuitions underlying the practice of scientists and ordinary persons when "confronting" *e* with *h*. Accordingly, applied IL should suggest correspondence rules between the calculus and the intuitions. Yet, the concept of "acceptance" is dubious ([30]), and the specific way of combining IL and EL sketched above stems from ignoring the essentially *pragmatic* character of the intuitions discussed and treating them as essentially *semantic* intuitions, with pragmatic outcomes. This over-simplified conception has also effected the present state of IL research.

That this is an over-simplification is attested to by the vast distance between IL "rational reconstruction" and scientific practice; though this last state of affairs can also be explained by the fact that IL systems deal exclusively with *e* and *h* sentences formulated within constructed languages. Still, this over-simplification can be pointed out more convincingly, by mentioning obvious examples of "inductive intuitions" — in the extended sense of intuitions — relating *e* to *h* which simply cannot be formulated within the existing system of IL because of their non-pragmatic character, i.e. formulating them involves essentially the use of constants, which, when the calculus is interpreted, become names of persons. For example, the 'subjective probability' of *h* becomes higher for me when I come to believe that a man of authority in the field to which *h* belongs believes that *h*, or when *e*, together with some of my current beliefs, jointly imply *h* (though none of them does so by itself). In the last case I would ascribe to *h* on *e* a subjective probability of 1, thoug p(h, e) in any standard IL might be quite low. (One should also remember that among my other beliefs there might be some universal statements, though *e* apparently,

([30]) Y. BAR-HILLEL: *The syndrome of Acceptance* (Lakatos, I., ed.), *The Problem of Inductive Logic*, North Holland, Amsterdam, 1958, pp. 150-161.

consists of singular ones.) I shall not attempt here a new IL system to overcome these disadvantages, but only outline a framework for such a system.

First, let me point out that the phrase "*h* is well established for *a* on e" (in symbols: $Pr_a(h, e)$) means, in many contexts, that if a would believe *e*, he would also believe *h*. That is:

Δ_1) "$Pr_a(h, e)$" $\in \mu$ iff "$B_a e > B_a h$" $\in \mu$.

Since '$>$' was explicated only in '$B_a(p > q)$', $B_b Pr_a(h, e)$' is more handy than '$Pr_a(h, e)$'. *b* might be conceived of as an investigator of *a*'s inductive intuitions. Possibly, though not necessarily, a = b.

Δ_1 is already useful, enabling us to formalize the second example given above for inductive intuitions which are pragmatic.

E_1) If "$B_b B_a(e > h)$" $\in \mu$ then "$B_b Pr_a(h, e)$" $\in \mu$.

I believe E_1 is derivable from the conditions given in the preceding sections, but I have no proof for this. At any rate, E_1 is intuitively sound.

A second step will be to introduce a subjective probability function $P_a(h, e)$. I shall not construct the function; yet, it seems that it might be profitable, in constructing it, to combine ideas from existing systems of IL and subjective probability theory. However, $P_a(h, e)$ should satisfy the standard axioms of the probability calculus as well as the following requirement:

R_1) "$P_a(h, e) = 1$" $\in \mu$ iff "$Pr_a(h, e)$" $\in \mu$

$P_a(h, e)$ helps us to formalize our first example for inductive intuitions which are pragmatic. We need only introduce a new nonlogical constant. '$A(c, S)$', which is to be read as: *c* is an authority in the field of study *S* which is conceived of as a set of statements. The formalization E_2, might further be taken to provide an intuitive meaning-postulate for $A(c, S)$.

E_2) If "$B_b A(c, S)$" $\in \mu$ and "$B_b(h \in S)$" $\in \mu$ then "$P_a(h, e) < P_a(h, e \wedge B_c h)$" $\in \mu$.

The framework as sketched here is a far cry from a full fledged IL theory. I tend to believe though, that the illustrations point out that it might be fruitful. In the next section I shall attempt an application of it to the very famous Analytic-Synthetic Debate.

6. *A Pragmatic Explication for "Analytic"*

Quine's ([31]) basic argument against semantic explications for the concept 'Analytic' is that they involve a vicious circle. It seems that in Carnap's ([32]) last reply to Quine there is an element of pragmatic explication, though only in a very sketchy way. Carnap argues that if a speaker maintains that 'all ravens are black', in face of a seemingly refuting evidence such as the observation of a bird that resembles ravens in every respect, except that it is white, then the sentence 'all ravens are black' is analytic for him. That is, if for any evidence *e*, *h* is established for *a*, *h* is 'analytic for a' ($An_a h$). In symbols:

Δ_2) "$An_a h$" $\in \mu$ iff "$(e) Pr_a(h, e)$" $\in \mu$.

One should notice that Δ_2 relativizes analyticity with regard to a speaker. Yet Δ_2 in no way opposes Quine's ideas. Suppose that I(h) is a semantic information measure on statements (in systems like Hintikka's ([33]) for example). Let us define 'F(h)' as follows:

Δ_3) $F(h) \overset{Df}{=} \{e \mid Pr_a(h, e) \wedge (g)[Pr_a(h, g) \supset I(e) \leqslant I(g)]\}$

i.e., F(h) is the set of all minimally informative statements establishing *h*. Then, from Δ_2 and Δ_3 we get "$An_a h$" $\in \mu$ iff "$(g)[g \in F(\sim p) \supset I(g) = \infty]$", i.e. one needs an infinite amount of evidence to believe $\sim$h. Quine's network ([34]) can be

([31]) See note 6.

([32]) CARNAP, R., *W.V.O. Quine on Logical Truth,* in: *The Philosophy of Rudolf Carnap,* referred to in Note 10, pp. 915-922.

([33]) HINTIKKA, J. and PIETERINEN, J., Semantic Information and Inductive Logic, Hintikka, J. and Suppes, P. eds., North Holland Pub. Co., Amsterdam, 1966.

([34]) Reference given in Note 6, especially section 6. The main idea is, to quote Quine, that "total science is like a field of force whose boundary conditions are experience".

described by equating the degree of centrality of a statement for some person with the amount of information needed to convince this person to believe its negation; i.e., the network, in the same way as the concept of analyticity, is also relativized to persons. One could distinguish between the amount of information needed to make one believe a statement, and that needed to make him stop believing it and believe its negation.

The second metric was used for explicating 'degree of analyticity'; the first metric is suggested for explicating 'degree of obviousness', which, in the limiting case, when h is quite obvious to *a*, ($Ob_a h$) means that h would be established for *a* by an informationally vacuous evidence.

The picture can still be complicated and rendered more interesting if one would distinguish deep and surface semantic information measures (35). I can see no conflict between the ideas suggested here and Hintikka's (36) illuminating discussion where this distinction is suggested. I would rather propose that Hintikka's 'analytic' which historically represents Kant's usage of this term, be rendered in contemporary discussions of philosophical problems, as 'mathematically non-creative', or the like.

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(35) Reference given in Note 24.

(36) HINTIKKA, J., *"Kant, Vindicated", Kant and the tradition of Analysis,* WEINGARTNER, P., ed., Deskription, Analytizität und Existenz, 3-4, Forschungsgespräch des internationalen Forschungszentrum für Grundfragen der Wissenschaften, Salzburg, ed. — WEINGARTNER, P. Postet, Salzburg und München, 1966, pp. 234-253, 254-272.