THE LOGIC OF HOHFELDIAN PROPOSITIONS

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In this paper I want to discuss a certain collection of *propositions*, or, depending on one's ontology (I suppose it ought to be called), sentences, or assertions, or what not. There is a nest of often discussed questions here, which for present purposes I will ignore. But since we must have some background terminology, I shall assume what has been called a "Platonistic" framework, hoping that something can be salvaged from what follows even by those who find such entities as propositions frightening.

The sort of proposition I have in mind has intimate connections with law, morals, action, responsibility, and a host of other notions connected with normative concepts and discourse. In fact, Wesley N. Hohfeld 1923 (see bibliography), according to my reading of him, took propositions of the form I have in mind as the fundamental building blocks of legal and other normative theories. The striking originality of his analysis of legal concepts provided the first of the two reasons for which I have attached his name to them.

The second reason is that I wanted to call the propositions by some name which would not bring immediately to mind a host of philosophical preconceptions. To be sure I may have here failed in my intent, because Hohfeld's writings are known to many, and I may therefore be accused of holding views of his to which I perhaps do not subscribe. But my purpose here is not exegetical; I only want to make it clear that the ideas to which I shall turn were suggested by reflecting on Hohfeld's insights. He should share credit for whatever worth they have, but should not be blamed for any errors I may make in using them.

So much for the terminology; now to try to say what it means.

I

I suppose the easiest way to approach the matter is by considering a series of questions, though I shall eventually be interested in answers to the questions rather than in the questions themselves. Here are a few samples:

(1) Who sent this letter to Dr. Williams?

(2) Is Hugh going to bring suit against the city for negligence in failing to mark the road properly?

(3) For whom was this umbrella intended?

(4) Who killed Cock Robin?

(5) What did Marshall do on his twenty-third move when playing with Lewitsky in Breslau, in 1912?

(6) Who gave George his typewriter?

(7) What corporation does Henry work for?

(8) How did it happen that Françis and Jim won the Nobel Prize?

(9) What did Sam do that made Shirley so angry?

(10) What is the victim's name?

It is *answers* to questions of this sort that I want to call Hohfeldian propositions, where the answers are to be thought of as having three distinguishable parts.

(a) In each case the answer will involve an *agent* of some sort, typically, though not necessarily, a person, as in cases (2), (5), and (9), and perhaps others. But in case (7) it is presumably a corporation which *acts* in employing Henry, and in (8) some committee made the decision to award the Nobel prize to Crick and Watson. Typically, the agent is a human being, but in some cases it might be a corporation, or a committee, or a legislative body, or a dog. Characteristically, and pleonastically, an *agent* is the kind of entity to which we impute or ascribe agency. It may be a human being, but societies recognize many things as *agents* which last much longer than human beings do.

(b) In each case the answer will involve a *patient* of some sort, some entity which is the beneficiary, or maleficiary, or perhaps just the recipient, of the act said to have been executed by the agent, i.e., of *what the agent did to, or for, him.*

(I digress to make an odd observation. A little reflection seems to uncover a curious lack of symmetry in our usual ways of talking about agents and patients. English abounds in terminology for acting: "I do...," "I bring it about that...," "I make it the case that...," all of which are *value-neutral* in the sense that what one does, or how one acts, can be good or bad, well or ill; the notion is carried in Latin by *ago*. But there is a paucity of value-neutral verbs for being at the other end of an action: one can "bear," "suffer," or "endure," unpleasantnesses, and one can "enjoy," or "receive," benefits. If a good thing falls our way, or a bad thing happens to us, we seem to know how to describe the situation. But how do we describe ourselves when an unnoticed visitor brings his presence to our attention by coughing artificially? He didn't *inflict* anything on us, nor did he give us cause to render thanks. He simply conducted himself in such a way as to change our awareness of the situation in which we found ourselves; in the neutral sense I have in mind, we were *patients* to his action or to the change he effected.)

So we have *agents, patients,* and a third item;

(c) In the case of each question the answer will involve some situation, or state-of-affairs, or proposition, or the like, which the agent is said to bring about relatively to the patient.

Writers of the Hohfeldian school have stressed as central to their analysis of "legal relations," the view that such relations must be construed as *two*-termed relations between persons. "... in each case one plaintiff, one defendant, one issue; one privilege or one right is all that needs examination: the one relation between these two people." (Llewellyn, 1930, p. 86.) But it is clear from the examples they discuss that the relations under consideration are in fact three-termed at least, namely, relations holding (say) among two litigants, and some state-of-affairs in dispute. When discussing examples, Hohfeld (and others) explicitly recognize the three-termed character of the relations in question: "... if X has a right against Y *that he shall stay off the former's land,* [then] Y is under a duty toward X *that he stay off the place.*" "X has the power *to transfer his interest* to Y..." (Hohfeld, 1923, pp. 38, 51;

italics supplied.) The point is that rights, powers, duties, etc., are not only rights (powers, duties) *toward* some person or persons, but also rights (powers, duties) *that something-or-other* be the case.

With that lack of sensitivity to the subtleties of natural languages (and everything else) characteristic of mathematical logicians, I shall forthrightly give away my hand at this point (though I reserve the right to try to deal with subtleties later on). I want to consider a three-termed relation

$$H(x,p,y),$$

where the first argument is an agent, the third is a patient, and the second is a situation, state-of-affairs, or what have you, that the agent x brings about relatively to the patient y. (I would prefer, myself, to say that the second argument is "a proposition *p* which x makes true relatively to (or *at*) y," but this locution involves commitments said to be philosophical, and I'd like to avoid these particular commitments in this essay.)

Before getting down to other details about how Hohfeldian propositions are to be construed, I will comment briefly on the questions mentioned above as samples, and on the truth-conditions for propositions for which I have used notation beginning with the Roman letter aitch.

First: in examples (1), (4), (6), and (7) we are interested in finding out *who did it,* i.e., what agent brought about the state-of-affairs we are investigating. In (7), for example, it is perfectly well understood what situation *p* we are talking about (Henry's employment), and who y is affected as patient (Henry); what we want to identify is an agent x. Our motives for trying to find the answers may differ: presumably in (4) we want to fix blame; in (6) we would, I suppose, want to award praise, if appropriate; and (1) may just be a case of disinterested curiosity. But for whatever reason we are in each case attacking the same question: which agent brought about the situation?

By way of contrast, questions (3) and (10) have to do with the patient to the proposition. In case (3) presumably there was some agent x who had in mind fixing things so that a certain umbrella was in the possession of a patient y. We're clear already about the state-of-affairs *p* ("this umbrella is in the possession of y"), and we don't care who the agent was. We want to know who was intended to be affected by the intentions of the intending agent. (10) has a similar cast; apparently something has gone wrong, and without worrying about who did it, or what happened, we are interested in identifying the patient. Again our interests may have to do with blame (10), praise (6), or curiosity (3).

We may also be interested simply in what was done, as in (5) or (9), where the agents and patients are already known (or assumed to be so). Or, as in (2), we may have the whole thing mapped out logically, and simply want to know whether or not, for given x, *p*, and y,

$$H(x,p,y),$$

is true. This brings us to our second topic: truth-conditions.

We have of course all been led, at one time or another, to spend time on questions about "what *makes* our sentences true" or "what gives our sentences *truth*" (as if truth were some sort of gift). I don't want to complain about this sort of question, either as posed by the ancients, or discussed by our contemporaries. But I would like to report, concerning Hohfeldian propositions, what I take to be a sociological, or historical, or perhaps game-theoretical, fact. To wit: both Hohfeldian propositions and their propositional components *p* are taken to be either true or false. I say "taken to be" because I don't want to argue that question at the moment. When we say "Harriet hit Bill over the head with a hand-saw," and suppose ourselves to be speaking correctly, we are committed to the notion that a certain proposition *p* is true (that a hand-saw collided with a head), and that we are correctly attributing agency to Harriet and patient-hood to Bill.

Just how Hohfeldian propositions come to be true is not the topic of this paper, but truth and falsity surely apply to them, at least in the sense that we make decisions on the basis of their truth or falsity. To be sure, they might come to be true simply because someone said so, as when an appropriately empowered authority says "I now pronounce you man and wife"; before the utterance they were not, afterwards they were. Or we might treat Hohfeldian propositions as true for more obviously causal reasons, as when we answer question (1) above by saying "Herbert sent it." But whether or not the truth of such propositions is decided by our views of causation, or by courts of law, or by the whims of the Deity, Hohfeld seems to me correct in saying that this three-termed relation is one of the most important seams in the social fabric.

Of course the statement represented by

$$H(x,p,y),$$

may well be *false* — and on several counts.

First, maybe p didn't happen, in which case we would say $\sim H(x, p, y)$ on the grounds that $\sim p$. (The accused says: "Not only did I not steal her emeralds; they weren't stolen at all! They're right there in her pocket.")

Second, maybe we have the facts right, but have mis-attributed agency. Richard (as patient) got a soap-box (i.e., a proposition p was made true relatively to him) all right, but he didn't get it from his grandfather; his cousin was the chap who gave it to him. Again

$$H(x,p,y),$$

is false.

Or third, we may have wrongly picked out the recipient of the act. It wasn't the second mate who was murdered on shipboard; it was the cook's assistant whose body was found in the passageway. Or maybe we got the heiress wrong; the rightful heir all along was Arthur.

Or maybe (again) Marshall's wicked uncle, long since deceased, sprang up from the grave and *forced* him to make that extraordinary move against Lewitsky in Breslau.

Upshot: (1) $H(x, p, y)$ might be wrong for any of three reasons: (a) p is false, (b) x was not the agent, (c) y was not the patient. (2) All this is surrounded by normative rules: *what the facts are* (what is admissible as evidence ?), *who did it* (how do we attribute agency ?), *to whom was it done* (how do we identify the *bene-male-neutre-ficiary* ?).

I will hope at this point that I have made it reasonably clear intuitively what H(x, *p*, y) is to mean, and that one of the things we ought to suppose true is that

(A) *if* H(x, *p*, y) *then p*.

(I pause to point out briefly that this statement makes the logic of Hohfeldian propositions begin to look like a branch of modal logic; i.e., we have a condition H(x,*p*,y) on *p*, from which the *truth* of *p* follows in analogy to $Lp \rightarrow p$.)

II

Small bit of historical background: I have spoken in this tone of voice before (see Anderson 1962), but two difficulties came my way, both of which have (I claim) now been surmounted. I will simply state dogmatically the solutions.

The first concerns ways of identifying propositions such as might figure as the second argument for the function H. Quine and his forebears have taught us well that if a theory lacks identity criteria, one doesn't know what one is talking about. So we need to know conditions on *p* and *q* such that H(x, *p*, y) and H(x, *q*, y) say the same thing. The answer I propose is that H(x, *p*, y) and H(x, *q*, y) say the same thing just in case, for propositional calculuses, *p* and *q* provably co-entail each other in the system E of entailment. This system has been discussed by Belnap and myself, and has recently been shown by Robert Meyer and Michael Dunn (*Journal of symbolic logic*, vol. 34, pp. 460-474) to be equivalent to a modified form of Ackermann's system of *strenge Implikation* (see Belnap 1967, and references there cited.)

That's one question answered. The second has to do with the sense of "if... then..." in (A). The answer here is that the conditional should be understood in the sense of the system R of *relevant* implication. The arguments are long and complex, and not to be entered into here. The reader can find lots of entertaining information by consulting the references in Belnap 1967 or Anderson 1967.

Having dealt both of these problems a death-blow by fiat, we now get back to H, its connection with deontic logic, and to Hohfeld's own analysis of fundamental legal relations.

III

Hohfeld discussed the "factual component" of his legal relations only indirectly; his primary interest lay, as is appropriate to a philosopher of law, in the sorts of issues which were likely to cause litigation in the courts. But I think it is fair to say that disputes concerning whether one person has a certain duty to another, can be settled only if we have some sort of criteria for telling whether or not a duty has been fulfilled. To say that a duty has been met, or that an obligation has been discharged, is one of the primary duties laid, as I intend matters, on the propositional function H.

Problems concerning who is *supposed* to do what to (or for) whom presuppose that we have ways of answering questions as to who *has* done what to or for whom, and our anxiety about making decisions in hard cases may easily lead us to overlook the fact that in the *vast* majority of cases (say 99.44 %), we know *exactly* how to answer such questions. It is true of course that the kinds of cases which arise in courts of law are also those cases for which something might be said on each side; otherwise they would be settled out of court. But slight reflection on the ten questions with which we began reveals that we do in fact have reasonably clear criteria for answering them all. Having *established* who killed Cock Robin (or, for another example, the name of the translator who mistook *vair*, in Perrault's *Cendrillon*, for *verre*, and consequently inflicted on generations of English school children the notion that Cinderella's slipper was made of glass rather than fur), we *then* worry about whether the act of an agent toward a patient was obligatory, permitted, forbidden, or the like. I don't mean of course that our worries about deontic notions are temporally posterior to our "factual" questions about who did what to whom, but only that making decisions about «it *ought* to be the case that Herbert repays his debt to Simpson" ($OH(x, p, y)$) presupposes an understanding of the notion "it *is* the case that Herbert repays his debt to Simpson"

$$H(x,p,y).$$

All of this palaver has been designed simply to sharpen the analytical target. Whether this is now sharp enough I do not know, but a methodological point now seems to be in order, and an historical point might help elucidate the problem to which I would like now to attend.

IV

When C.I. Lewis originally noticed that the man in the street does *not* mean "not α or β," or "it's false that (α and not β," when he says, "if α then β," he (Lewis) was moved to reflect that "if α then β" means something more like "it is *impossible* that α is true and β false." Thus was born modern modal logic. And the plethora of modal systems he and others developed might be understood to indicate that it is simply not clear *what* assumptions ought to be made about modal operators. On the other hand progress *does* get made, and with the understanding we have been given by Kripke 1965, Hintikka 1963, and many others, we can now be said to understand the topic of alethic modalities reasonably well.

The parallel I would like to draw is this: I don't know exactly what assumptions ought to be made in an attempt to pull deontic logic, relevant implication, entailment, and Hohfeldian propositions together, but it is perfectly obvious that we *do* use these notions all at once. If someone asks me "Did you remember to call Herbert yesterday about the regulations for the bridge tournament," and I say "yes", it is understood by everyone that I was an agent, Herbert a patient, a situation had been discussed, and that what I said was (I hope) true.

A natural question then suggests itself: are there any reasonable assumptions which connect all these notions from a formal point of view. From the standpoint just considered (Lewis's attempts to guess what nice assumptions we might make about *possibility* and *necessity*) I will in the remainder of this paper make a guess about one plausible assumption which can tie these topics together, and then to draw a philosophical moral relevant to law and ethics.

V

I will first add another question to the list with which I began:

(11) Who (wrongly) left the door open ?

Devotees of quantification theory might immediately point out that if the door was left open, then *everyone* left the door open, on the grounds that no-one closed it. But it ought to be clear that the questioner does not want to hear "everyone" in response to his question.

Just *who* left the door open may depend on lots of moot questions, and certainly it depends on the rules governing the situation.

(a) It may be that one is worried because it is a cold day, heat is escaping through the door, and we would like to know just which of our children came crashing in from school and failed to close the door afterwards (so that we can take measures designed to improve behavior).

(b) Or it may be that we are thinking about a highly specialized scienfific laboratory, where it is necessary for the door to be kept closed in order to avoid danger to the citizens at large. In such a case we might appoint someone to take care of the situation, and *he* was the chap who left the door open.

(c) Or it may be that the rule is that the first person (or the last person) through the door is required to close it.

(d) Or it may be that there is no rule about the matter at all. This is typical of the case where we are perplexed. If leaving the door open causes some harm, and we have no clear conventions about who is supposed to close it, we have difficulties in ascribing responsibility, and endless litigation may ensue.

To return: "Who left the door open ?" The obvious answer is "The chap who was *supposed* to close it." And this example suggests a general point which can be formalized with the help of the logical apparatus mentioned earlier.

(B) $OH(x, p, y) \& \sim p . \rightarrow . H(x, \sim p, y)$.

That is, if it ought be that an agent x brings it about that p is true relatively to y as patient, and p is not brought about, then it is x to whom we ascribe responsibility for $\sim p$, and it is y who may be expected to find redress for his grievance. And I hope that this is part of what H.L.A. Hart 1946 meant by saying that we *ascribe* rights and responsibilities, and that the formula above reflects one of the ways in which we make such ascriptions. Or as it is put in Hart and Honoré 1959 (p. 61), "In this use the expression 'responsible for' does not refer to a factual connection between the person held responsible and the harm, but simply to his liability under the rules to be blamed, punished, or made to pay." Their words *suggest* that we ascribe responsibility only for harm, but Hohfeld is more liberal, and indeed insists that being held responsible for good might make give one a "liability," under the rules, to be praised or rewarded. At any rate, I want H(x, *p*, y) to allow for either possibility.

VI

The "philosophical moral" mentioned at the end of IV is that none of us quite know what to make of the claim that "if x ought to bring it about that *p* happens to (or for) y, and *p* doesn't happen, then x is the chap on whom attention is directed; (i.e., *he* is the fellow who failed to close the door)." Properly explained, and with appropriate attention paid to our usual habits of thought in blaming agents for being derelict in performance of their duties, all this *sounds* analytic enough. But the analyticity of the formula just mentioned above is jeopardized by the consideration that the agent x *may* have been at the crucial moment distracted by the necessity of rescuing a baby from a tiger, or the sight of a pretty girl, and was therefore "incapable" of carrying out his duty to y. So what is evidently required is a clause about «other things being equal." We want something like "in the absence of extenuating circumstances, if x is charged with seeing to it that *p*, on y's behalf, and if *p* fails to be the case, then it is to x that we ascribe responsibility for the situation."

I have claimed that co-entailment (in the sense of the system E, for which see preceding references) gives us a sufficient condition for the intersubstitutability of p and q in contexts H(x, p, y) and H(x, q, y), and that the system R of relevant implication is satisfactory for dealing with the "if... then..." in the formula at the end of the last section. I then made qualifications on the interpretation of that formula designed to reduce it to vacuity. And no doubt there will be those who will claim that it is either false, or vacuously true.

But vacuous truth has been for a long time held to be a virtue in logic, and I see no reason why it should be held a fault in this case. Indeed it is hard to see what other than vacuous truth recommends $p \rightarrow p$ to us. In a similar way it seems to me that, given the virtues of E and R, it is analytically true that

$$\mathrm{OH}(x, p, y)\ \&\ {\sim}p. \rightarrow .\mathrm{H}(x, {\sim}p, y).$$

So we are left with a program (which I should be happy to have someone else work out) involving two parts: (a) what relations of a "satisfactory" sort can be developed for deontic logic, with the help of E and R, and (b) what other explicitly deontic assumptions about Hohfeldian propositions are needed to complete the job ?

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A NOTE CONCERNING PROFESSOR A. R. ANDERSON'S THEORY OF HOHFELDIAN PROPOSITIONS

In his interesting analysis of a type of sentences similar to some sentence-structures used by Hohfeld Professor Anderson distinguishes three parts: an agent, a patient, and something the agent is to bring about relatively to the patient. For such sentences he uses the formula '$H(x, p, y)$' as a scheme. He considers such sentences as three-termed relations.

I would like to ask the following question: What is the semantic status of 'p' in Anderson's theory of Hohfeldian Propositions? As the Hohfeldian proposition is defined as a three-termed relation 'p' has in '$H(x, p, y)$' the role of a name. In

(A) if $H(x, p, y)$ then p

and in

(B) $OH(x, p, y) \mathbin{\&} \sim p \rightarrow H(x, \sim p, y)$

'p' is dealt with like with a proposition.

To avoid these difficulties I would suggest to analyze the case that the Hohfeldian proposition is not true in an other way, not in the form '$H(x, \sim p, y)$', yet in the form

$$\sim (E\,x)\ (E\,y)\ H(x, p, y)$$

verbally: The Hohfeldian proposition $H(x, p, y)$ is not true, iff there is no x and no y which would make the formula $H(x, p, y)$ true (sc. for a given meaning of 'p').

Then (B) can be written in the form

(B) $OH(x, p, y) \mathbin{\&} \sim (E\,x)\ (E\,y)\ H(x, p, y) \rightarrow (x)\ (y)\ \sim H(x, p, y)$

or perhaps — if we consider X, Y to be constants

(B'') $OH(X, p, Y) \mathbin{\&} \sim (E\,x\ (E\,y)\ H(x, p, y) \rightarrow (x)\ (y)\ \sim H(x, p, y)$

In this form (B') or (B'') the sentence is analytical, and this is — in my opinion — quite in accordance with intuition.

I believe that Professor Anderson's presupposition of the condition 'other things being equal' (section VI of his paper) in solving questions of fulfilment is not correct. Only if such a condition is given expressis verbis or if it can be presupposed by interpretation we can consider such circumstances as an exculpation for not fulfilling a duty. But in such a case the clause 'other things being equal' is a part of the antecedent of the hypothetical norm-sentence whose fulfilment is analysed.

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DISCUSSION

A. R. ANDERSON

The paper I had planned to discuss with you at this meeting has, as I understand matters, been distributed to you already, and in the time available for discussion I would be happy to try to answer questions or objections concerning what I have called "Hohfeldian propositions," and their role in legal reasoning: But the papers and discussions preceding this one lead me to believe that the time at my disposal might better be spent in trying to characterize, and comment on, some fundamental differences of approach taken by those of us whose interest is primarily in legal and jurisprudential issues, in contrast to those of us who lean more explicitly on mathematical logic and analytic philosophy. I hasten to add that my remarks will not be buttressed with arguments; my aim is rather to tell you how the world looks to *one logician* (myself) who has tried to understand a little about legal reasoning.

Human beings behave in a large variety of ways: they sleep, they sing songs, they come to Brussels for conferences, they write poetry, they slip on the ice, they sneeze, and so on. For reasons to emerge shortly, I will add that they also count money, and braid hair.

Now all of these activities, and countless others, can be analyzed and discussed from a variety of points of view, or, as I shall say, in a variety of tones of voice. Take slipping on the ice, for example. We might discuss a bad fall in the tone of voice of a psychologist studying traumatic experiences, or the tone of voice appropriate to the description of comic pratfalls, or that of a concerned friend of the victim, or perhaps in the tone of voice adopted by critics of the ballet — or we might consider the matter simply as a physical, or physiological, or psychological phenomenon. We might in fact try to formulate a mathematical theory of slips on the ice, thought I haven't the faintest idea of what such a theory might look like, or who on earth would be interested in it.

Certain activities are however, particularly amenable to discussion in a mathematical tone of voice, among them counting, as I mentioned above. In this connection two points need to be made. First, it is clear that human beings could count things effectively (children, sheep, pots, etc.) for *thousands* of years before anything like a satisfactory mathematical treatment of the topic existed. Careful and thorough treatment of elementary number theory may be said to be less than one hundred years old. The topic did not of course begin in a vacuum; important contributions had been made piecemeal for a long time. But elementary number theory as we know it had its inception with the work of Peano and his colleagues toward the end of the last century.

Second, the fact that subjects involving numbers and measurements were among the first to yield to mathematical treatment — were, so to speak, most congenial to the mathematical tone of voice — seems important to the numbers, but quite inessential to the tone of voice. By which I mean that *many* other topics are amenable to mathematical treatment: abstract sets (aggregates, *mengen, ensembles*), algebra, games of strategy, knots, networks, etc., — and braids.

As in the case of counting, human beings were able to braid hair (for example) for thousands of years before a mathematical treatment of the topic was available, or at least this conclusion seems forced on us by archaeological studies. But it was not until this century that Emil Artin formulated a mathematical theory of braids, and cleaned up the principal problems in the theory.

I hardly need add that the application of mathematical techniques to the study of logic has proved to be a gold mine; the literature has grown in the past sixty years to the point where very few investigators are in a position to work in more than a handful of the various branches of the topic. One of those branches is of course the one which brings us together for this conference — deontic logic, a topic which seems to me, at any rate, to involve discussing in a mathematical tone of voice the kind of reasoning we use when considering sets of rules (for games, or morality, or the law, or the like). And this brings us to consider a question which has been much discussed here for the past day or so, namely, what kind of applications can a lawyer expect to find for the mathematical treatment.

I can answer this for myself, but of course others with similar interests may give different answers.

My own interest is *not* primarily in finding applications for the formalisms considered; what I am interested in is trying to bring to bear the most sophisticated tools I can on the analysis of certain concepts which seem of fundamental importance in legal reasoning. And at this point it might be of use to look at applications in a couple of other areas.

It has been shown as a theorem of number theory that every even number can be expressed as the sum of at most four prime numbers. I cannot imagine how this piece of information could be put to use by a bookkeeper or an accountant; i.e., it seems to me to have no application to counting, which is one of the things elementary number theory is about. This does not preclude the possibility, of course, that *some* of the results of the abstract mathematical theory might have applications in accounting. I suppose for example that various useful procedures for checking the accuracy of computations ("casting out nines," and the like) might be thought of as applications of number theory to accounting, but the interest or importance of number theory does not depend on its applications.

In a similar way, I doubt that Artin's contributions to the theory of braids would be of much use to a person who was trying to tidy up his small daughter's hair. True, it might suggest novel, or ingenious, or more beautiful ways of braiding hair — but such "applications" are very remote from the motivation for the mathematical treatment.

Equally, if deontic logic should have no applications which might assist a lawyer in preparing briefs or analyzing decisions, this fact would not detract from the interest I have in the topic. But it should be pointed out that there is a regular quarterly publication more than ten years of age (*Modern Uses of Logic in Law* (or "M.U.L.L."), more recently published under the title of *The Jurimetrics Journal,* sponsored by the Electronic Data Retrieval Subcommittee of the American Bar Association, and edited by Layman E. Allen), which devotes many of its pages to the detailed application of techniques of mathematical logic to legal problems. Whether these studies are of use to practising lawyers is of course for them to say; I am certainly not competent to do so. (But I cannot forbear adding that if I *were* a practising lawyer, I would want to get a firm grasp of the techniques before deciding what they were worth — more especially, I would want to get a firm grasp on the topic before my competition did.)

The paper on Hohfeldian propositions which I contributed to this volume was intended as a step toward a mathematical treatment of certain important concepts which one runs across in legal practice and theory, — as such it is meant to be a gift from one discipline to another. As a gift it may be worthless, but that is simply one of the hazards of basic research, and one never knows when something useful *might* pop up. It seems to me that the legal profession might sensibly (and inexpensively) adopt the attitude that many commercial industries take in sponsoring basic research: "You go ahead and play your research games; just show us the results, and leave to us the problem of making money out of them."

M. O. WEINBERGER

Je crois que Monsieur le Professeur Anderson souligne seulement une partie du travail du mathématicien. Il le caractérise bien. Mais quand il ne voit pas tout l'ensemble du travail intellectuel du mathématicien et du logicien, il trace le tableau déformé de notre travail. L'essentiel n'est point seulement la diction spécifique et la méthode constructive, que le logicien utilise, mais c'est toujours, dans toutes les branches de la mathématique et de la logique appliquée, qu'on commence par une analyse approfondie du domaine de l'exploration. Il y a toujours, dans toutes les applications de la logique à la jurisprudence, aux sciences humaines, aux sciences naturelles, une activité qui n'est pas seulement le développement d'un système, mais qui consiste en l'analyse de ce domaine. Il s'agit de la reconstruction rationalisée du domaine étudié par un système, qui exprime ce qu'on a gagné par l'analyse. Ensuite on vérifie l'analyse primaire par une confrontation du système avec le domaine étudié. Le travail analytique est une partie nécessaire de l'activité du mathématicien, du philosophe, du juriste.

Le rôle de la logique déontique et de l'analyse logique dans la sphère du droit concerne surtout la science juridique, beaucoup moins le travail quotidien du juge ou de l'avocat. L'analyse logique forme la base de la science juridique analytique. On ne peut pas, à mon avis, construire un système de Rechtswesensbegriffe, on ne peut pas expliquer le dynamisme du droit et comprendre les processus de l'application de la loi sans recours à la logique et surtout à la logique déontique.

Cette thèse n'est pas en contradiction avec la conception de Monsieur Perelman, qui souligne la nécessité de compléter l'image du raisonnement juridique par un raisonnement non déductif. Je crois bien que Monsieur le Professeur Perelman sera d'accord, que la structure fondamentale du raisonnement juridique est logique et que les raisonnements rhétoriques jouent un rôle de complément.

M. M. MORITZ

Professor Sosa hat einen Schluss konstruiert, wo man von einem konditionalen Imperativ auf einen assertorischen Imperativ schliesst. Ich will hier nicht die Frage diskutieren, ob solch ein Schluss von einem nicht-theoretischen Satz auf einen anderen nicht-theoretischen Satz überhaupt (logisch) korrekt ist. Ich selbst bin geneigt zu verneinen, dass ein solcher Schluss korrekt ist. — Dagegen will ich auf einen anderen Umstand hinweisen: unabhängig davon, ob «praktische Syllogismen» korrekt sind oder nicht, kann jedenfalls der Schluss von einem konditionalen Imperativ auf einen assertorischen nicht korrekt sein.

1. Zuerst eine Bemerkung über die Unterscheidung zwischen konditionalen und assertorischen Imperativen. Ein Beispiel für einen assertorischen Imperativ ist «Gib mir das Buch !». Ein Beispiel für einen konditionalen Imperativ ist «wenn es regnet: Gib mir das Buch !». (Dagegen sind Kants hypothetische Imperative nicht als konditionale Imperative aufzufassen, ja mehr als dies: sie sind nur ihrer grammatischen Form nach Imperative. Der Sache nach handelt es sich um theoretische Sätze. Der hypothetische Imperativ «wenn Du im Alter nicht Not leiden willst: Spare solange du jung bist !» kann in das theoretische Urteil übersetzt werden «ein geeignetes Mittel um zu vermeiden, dass man im Alter Not leidet, besteht darin zu sparen, solange man jung ist».)

Dass ein Imperativ «wenn es regnet: Ruf mich an !» vorliegt, kann in einem Urteil konstatiert werden, und eine solche Konstatierung schreibe ich in folgender Weise: «es regnet//k-Geb (du rufst mich an)».

Hier ist angegeben, was man zu *tun* hat, wenn der positive Fall vorliegt (dass es nämlich regnet). Für den negativen Fall (dass es nicht regnet) können drei verschiedene Handlungen k-geboten sein.

a) es *ist* auch für diesen Fall k-geboten, mich anzurufen,

b) es *ist* k-geboten, mich *nicht* anzurufen (= es ist k-verboten, mich anzurufen),

c) es ist weder k-geboten, mich anzurufen, noch ist es k-geboten, mich nicht anzurufen. (Es ist weder k-geboten noch k-verboten, mich anzurufen. M.a.W.: es ist erlaubt, mich anzurufen.)

Ich werde dies schematisch in der Weise schreiben, dass ich in einer Zeile schreibe, was k-geboten ist, wenn die *positive* Kondition vorliegt, und in der Zeile darunter, was zu tun ist, wenn die *negative* Kondition vorliegt. Das sieht so aus:

a es regnet//k-geboten (Du rufst mich an)

$\neg$(es regnet)//k-geboten (Du rufst mich an)

b (es regnet)//k-geboten (Du rufst mich an)

$\neg$(es regnet)//k-geboten (Du rufst mich nicht an).

c
$$\frac{\text{(es regnet) // k-geboten (Du rufst mich an)}}{\neg\text{(es regnet) // -k-geboten (Du rufst mich an) \& -k-geboten (Du rufst mich nich an).}}$$

Generell kann man dies in folgender Weise schreiben:

a
$$\frac{k /\!/ \text{k-Geb } (p)}{\neg k /\!/ \text{k-Geb } (p)}$$

b
$$\frac{k /\!/ \text{k-Geb } (p)}{\neg k /\!/ \text{k-Geb } (\neg p)}$$

c
$$\frac{k /\!/ \text{k-Geb } (p)}{\neg k /\!/ \neg \text{k-Geb } (p) \;\&\; \neg \text{k-Geb } (\neg p)}$$

Dies sind also die drei Möglichkeiten, einen konditionalen Imperativ zu konstruieren, wenn festliegt, welche Handlung k-geboten ist, wenn die *positive* Kondition erfüllt ist. — Selbstverständlich kann — wenn die positive Kondition vorliegt — auch k-geboten sein, die Handlung H zu unterlassen, oder es kann — wenn die positive Kondition erfüllt ist — weder k-geboten noch k-verboten sein, die Handlung H auszuführen.

Man erhält demnach für die positive Kondition k ebenfalls drei Möglichkeiten:

I $k /\!/ \text{k-Geb } (p)$

II $k /\!/ \text{k-Geb } (\neg p)$

III $k /\!/ \neg \text{k-Geb } (p) \;\&\; \neg \text{k-Geb } (\neg p)$.

Komplette konditionale Imperative, d.h. solche konditionale Imperative, bei denen sowohl für die positive Kondition als auch für die negative Kondition angegeben ist, wie man zu handeln hat, können in neun verschiedenen Formen auftreten. Diese neun verschiedenen Kombinationen erhält man dadurch, dass man die drei Fälle I-III für die positive Kondition mit den drei Fällen für die negative Kondition (welche unter dem Strich

		k// A	k// B	k// C
	k \ ¬k	k — Geb(p)	k — Geb(¬p)	¬k — Geb(p) & ¬k — Geb(¬p)
¬k//	I k—Geb(p)	$\frac{k - \mathrm{Geb}(p)}{k - \mathrm{Geb}(p)}$	$\frac{k - \mathrm{Geb}(\neg p)}{k - \mathrm{Geb}(p)}$	$\frac{\neg k - \mathrm{Geb}(p) \& \neg k - \mathrm{Geb}(\neg p)}{k - \mathrm{Geb}(p)}$
¬k//	II k—Geb(¬p)	$\frac{k - \mathrm{Geb}(p)}{k - \mathrm{Geb}(\neg p)}$	$\frac{k - \mathrm{Geb}(\neg p)}{k - \mathrm{Geb}(\neg p)}$	$\frac{\neg k - \mathrm{Geb}(p) \& \neg k - \mathrm{Geb}(\neg p)}{k - \mathrm{Geb}(\neg p)}$
¬k//	III ¬k—Geb(p) & —k—Geb (¬p)	$\frac{k - \mathrm{Geb}(p)}{\neg k - \mathrm{Geb}(p) \& \neg k - \mathrm{Geb}(\neg p)}$	$\frac{k - \mathrm{Geb}(\neg p)}{\neg k - \mathrm{Geb}(p) \& \neg k - \mathrm{Geb}(\neg p)}$	$\frac{\neg k - \mathrm{Geb}(p) \& \neg k - \mathrm{Geb}(\neg p)}{\neg k - \mathrm{Geb}(p) \& \neg k - \mathrm{Geb}(\neg p)}$

in den Fällen a bis c angegeben sind) kombiniert. Man kann also die drei Fälle I bis III mit den drei Fällen a bis c (für die negative Kondition) kombinieren. In tabellarischer Form kann dies in folgender Weise dargestellt werden: (S. 252).

Ich kommentiere jetzt einige Momente diese Tabelle. (1) Interessant ist das Feld AI und das Feld BII. In Feld AI wird k-geboten, p auszuführen, sowohl wenn die Kondition k vorliegt als auch wenn sie nicht vorliegt. Das ist aber dasselbe wie die Behauptung, dass ein assertorischer Imperativ vorliegt; welche Kondition auch vorliegt: die Handlung ist (k-)geboten. Ob ich sage: «Gib mir das Buch !» oder ob ich sage «wenn es regnet oder wenn es nicht regnet: Gib mir das Buch !», das Resultat ist immer dasselbe: wenn mir das Buch nicht (von «ihm») gegeben wird, hat er das Gebot übertreten. M.a.W.: man braucht die Unterscheidung zwischen assertorischen Imperativen und konditionalen Imperativen nicht zu machen. Assertorische Imperative können als konditionale Imperative aufgefasst werden. Sie machen einen Spezialfall von konditionalen Imperativen aus. Das Spezielle dieser Imperative liegt darin, dass sowohl für die positive als auch für die negative Kondition die *gleiche* Handlung (k-)geboten ist. (2) Es kann auch vorkommen, dass für die positive und für die negative Kondition k-geboten ist, die Handlung p zu unterlassen. D.h.: sowohl, wenn die positive als auch wenn die negative Kondition vorliegt, ist die Handlung p verboten. Eine solche Kombination liegt im Feld BII vor. — Ich verzichte auf weitere Kommentare.

Der Schluss von einem konditionalen Imperativ (und der zweiten Prämisse, dass die Kondition erfüllt ist), kann nicht zu einem Schlusssatz führen, in welchem behauptet wird, dass die Handlung («nun», wo die Kondition vorliegt) assertorisch geboten ist. Das ist aus einem einfachen Grunde nicht der Fall. In der ersten Prämisse wird ein konditionaler Imperativ genannt, bei dem nur für eine Kondition bestimmt ist, wie man handeln soll. Liegt ein konditionales Gebot für die positive Kondition vor, und ist es ausserdem wahr, dass die Kondition erfüllt ist, so kann man daraus nicht schliessen, dass nun ein assertorisches Gebot vorliegt. Denn damit wird behauptet, dass nicht nur wenn die positive Kondition, sondern *auch* wenn die negative Kondition vorliegt, die Handlung p ausgeführt werden soll. Wenn im Obersatz nur gesagt ist, dass bei positiver Kondition die Handlung k-geboten ist, kann im Schlusssatz nicht stehen, dass in beiden Fällen (also sowohl bei positiver als auch bei negativer Kondition) die Handlung p geboten ist.

Bei dieser Kritik habe ich vorausgesetzt, dass im Schlusssatz ein assertorischer Imperativ vorliegt (resp. dass im Schlusssatz behauptet wird, dass ein assertorischer Imperativ vorliegt (= erlassen worden ist)). Vielleicht wird man diese meine Deutung bestreiten. Tut man dies, so bleibt die Frage positiv zu beantworten, wie der Schlusssatz zu deuten ist. Einen Vorschlag habe ich in meinen obigen Diskussionsbeitrag zu Professor Weinbergers Vortrag gemacht. Dort habe ich den Schlusssatz so gedeutet, dass in ihm behauptet wird, dass die k-gebotene Handlung nun aktual-geboten ist (wenn die Kondition erfüllt ist). Jedenfalls muss man genau zwischen assertorisch-geboten-sein und aktual-geboten-sein unterscheiden. — Was hier verwirrend sein kann, ist folgender Umstand: wenn gesagt wird, dass eine Handlung konditional-geboten ist, so kann man daraus allein nicht schliessen, dass die Handlung auch aktual-geboten ist. Anders verhält es sich bei assertorischen Imperativen. Deutet man diese als konditionale Imperative, so wie ich es oben gemacht habe, so ist stets und notwendig eine der Konditionen erfüllt. «Ob es regnet oder ob es nicht regnet: Gib mir das Buch !». Dadurch, dass bei beiden Konditionen die gleiche Handlung k-geboten (resp. k-verboten) ist, ist diese Handlung auch stets aktual-geboten. Zwar ist eine Handlung, welche in einem assertorischen Imperativ geboten ist, stets aktual-geboten. Das Umgekehrte gilt jedoch nicht: daraus, dass eine Handlung aktual-geboten ist, folgt nicht, dass die Handlung assertorisch geboten ist. Auch wenn sie k-geboten ist und die fragliche positive (oder negative Kondition) erfüllt ist, ist die Handlung aktual-geboten. Man muss deswegen zwischen konditional- und assertorisch-geboten-sein einerseits und aktual-geboten-sein andererseits unterscheiden.

M. Ch. PERELMAN

I am very glad to be able to repeat, at the end of our meetings, something which I said at the beginning, because repetition may be a useful rhetorical device.

I said that there is an essential difference between theoretical and practical reasoning, that theoretical reasoning ends with a conclusion and practical reasoning ends with a decision. If we identify theoretical and practical reasoning or the reasoning of the lawyer and that of the mathematician and we forget this difference, or hide it, or lose it, then we have done nothing that may help the lawyer.

It may be so that you don't want to help the lawyer and I am sure that many deontic logicians are not interested in helping laweyrs. But this meeting was planned with the purpose of finding a common language and to give a conceptual analysis of what lawyers and judges are doing, so I'll speak to both of you. Mr. Sosa draws a parallel between philosophy of science and philosophy of law and the word *decision* is completely absent from what he wrote on the blackboard and from his whole speech, and if the logician is interested only in inferences, then legal reasoning is different from logic, because it is concerned with some kind of reasoning or of arguments that lead to decisions. The idea of justification of decisions is essential for law, but justification is something which, in the legal sense, cannot be reduced to inference or deduction. Surely if you have a valid inference, you are justified in accepting the conclusion if you accept the premises; but the idea of justification is interesting especially in the case when it is not backed up by a deductive inference. Terminology may play a misleading role, just like analogy. Thus, for example, if you say: I call inductive reasoning or inductive logic, anything that is not deductive logic. Then you say there is nothing else, but what is justification ? Is it deductive or inductive ? It is neither one nor the other, because there is justification of a decision and that is neither deduction nor induction.

When we speak of analogy or analogical reasoning, it is not deductive, it is not inductive, it is something else. So if, as a logician, you don't make some space in your mind for other kinds of reasoning than deductive and inductive, then you cannot tackle legal reasoning.

And I must say the same to Prof. Anderson. You may play games, but when lawyers look at your games, they say: I prefer football ! Lawyers, as such, are not interested in games. Lawyers look to logic to help them solve problems, their problems, not yours. Are you of some help for this ? If yes, what you are building up should enlighten them in what they are doing, not in what you are doing. If it doesn't, they don't listen to you.

What was the meaning of this meeting ? Last year, there was a meeting in Vienna and there were deontic logicians and almost no lawyers. So I asked the logicians: to whom are you speaking ? You are playing games among yourselves. But, in 1971, there will be a Congress of lawyers and people interested in legal philosophy and I was afraid that there we would have no logicians, only lawyers ! So, I tried to bring together logicians and lawyers in this meeting and I hoped, and I continue to hope, that we will speak *to* each other so that there'll be a kind of communication, so that not only you can explain what you are doing, what kind of game you are playing, but what is the interest of your game to the others, and what is the interest of what the lawyers are doing to you. I hope there may be some kind of common enlightenment. So, if we begin by saying that there is a difference between a decision and a conclusion, and that the difference is lost in your presentation, then I say there is no communication.

M. O. WEINBERGER

Professor Sosa's concept of a "defeasible conditional, the one that does not obey *modus ponens*" defined "If p, then you, a, φ !" $=_{Df}$ "If p, then, other things being "equal" (or, other things being "normal") you, a, φ !"
is very questionable:

1. In my mind the idea of a conditional not yielding a cate-

gorical conclusion when the condition is fulfilled, is not a conditional at all. Some kind of modus-ponens-rule is necessarily connected with every conditional.

2. The proposed definition is logically imperfect as the condition "other things being equal (normal)" may change the truth-value of the definiens without changing the truth-value of the definiendum.

PROF. E. SOSA

Prof. Perelman wants to distinguish between the logic that is applied in theoretical matters and what happens when one reasons in a practical context. He wants to emphasize that whereas in the case of theoretical reasoning you have a conclusion, in the case of practical reasoning you have a decision.

I completely agree that one should distinguish between decisions and conclusions but I would add that one should also distinguish between beliefs and conclusions, for there are conclusions in theoretical reasoning and conclusions in practical reasoning. Once one has drawn one's conclusions, then one goes on either to accept a new belief, or (in the case of a *reductio*) to reject an old belief. Similarly, in practical reasoning once one reaches one's conclusion one may see that there is reason to modify a policy, to go back and reject some of the principles from which the conclusion has been drawn about a particular case. You see that those principles are not so good after all, in view of what they imply about the particular case, and you go back and reject the policy. So conclusions should be distinguished *both* from beliefs and from decisions.

Professor Weinberger objects to the definition:

'If p, then you, *a*, φ !' = Df. 'If p, then, other things being equal (or, other things being «normal»), you, *a*, φ !'.

As I tried to make clear in the text, the conditional being defined, the one on the left, is the defeasible conditional, the one that does not obey *modus ponens*. The conditional on the right is a «material» conditional, one that does obey *modus ponens*, etc. Professor Weinberger objects that there is a free variable hidden in the *ceteris paribus* clause and absent from the left hand side of the definition. I agree that if this were so my definition would be objectionable. But it isn't so. To see that it isn't so I need only further symbolize the definition as follows.

'If p, then you, *a*, φ !' = Df. 'If p, then, if (q)[((q≠p) & (q is true)) only if (q is normal)], then you, *a*, φ !'.

This shows that the only free variables on the right appear free on the left also.

Professor Moritz questions my view that «...one can reason from a conditional imperative to an assertoric imperative». I suppose that he has in mind my endorsement of argument form (F2) ([1]):

$$\frac{\begin{array}{c}A\\A/B\end{array}}{*/B}$$

(Where 'A/B' is read «If A, then let it be the case that B,» and where '*' stands for an arbitrary tautology, so that '*/B' is tantamount to a categorical direction to let it be the case that B). A problem is supposed to arise because (as I would grant) one may replace the asterisk with (Av∼A). But then, since one may derive each of A/B and ∼A/B from (Av∼A)/B, it follows that ∼A/B is derivable from A and A/B together, which is supposed to be counterintuitive. Thus consider the declarative that it rains and the directive that if it rains you are to close the window; from these together we may supposedly derive the directive that if it *doesn't* rain, you are to close the window, a derivation which certainly appears counterintuitive enough.

The question raised by Professor Moritz is most interesting

([1]) See p. 227.

and ingenious, and a full answer to it would require a long paper in itself. Here I must restrict myself to a few brief points.

It seems to me that the grammar of directives, no less than the grammar of declaratives, requires a distinction between two types of conditionals, one strong and one weak. The weak declarative conditional is, of course, the «material» conditional, and its logic has been formalized in various systems of declarative logic. If we symbolize the weak «if p then q» as '$p \supset q$', the following argument form turns out valid:

$$(F_3) \qquad \begin{array}{l} p \\ p \supset q \\ \hline * \supset q \end{array}$$

Again, as in (F_2), we may replace the asterisk, here with ($p \vee \sim p$). It also turns out that q is logically equivalent to $(p \vee \sim p) \supset q$. But then, since one may derive each of $p \supset q$ and $\sim p \supset q$ from $(p \vee \sim p) \supset q$, it follows that $\sim p \supset q$ is derivable from p and $p \supset q$ together. Thus consider the declaratives that it rains and that if it rains, it pours; from these together we may derive the declarative that if it *doesn't* rain, it pours, a derivation which certainly appears counterintuitive enough.

Appearances are deceptive, I think, both here and earlier. Declarative conditionals of ordinary English are normally interpreted as strong. This is vouched for by the quickness and intensity of our puzzlement when told that if it doesn't rain, then it pours. Interpreted as a weak conditional this is perfectly acceptable, and reduces to the disjunction that either it rains or it pours, which is akin to the remark that someone is either tall or very tall. Interpreted as a strong conditional, the quickness and intensity of our puzzlement is well justified, for then it reduces to the subjunctive that if it should not rain, then it would pour. In view of all this, it is important to remember that $p \supset q$ represents the *weak* conditional, especially when we consider that $\sim p \supset q$ is derivable from p and $p \supset q$ together.

Similarly, as I tried to make clear in my paper, (2) my A/B

(2) See the middle of p. 214.

represents a *weak* directive conditional. Suppose we interpret the directive «If it rains, you are to close the window» as a weak conditional. In that case its force is that of the disjunction «Either it doesn't rain or you are to close the window». And it is no surprise that this should follow from the directive that you are to close the window simpliciter, or from a combination of the declarative that it rains and the directive that if it rains you are to close the window.

What misleads us, here as in the case of declarative conditionals, is our normal tendency to impose a strong interpretation on ordinary conditionals. Thus «If it rains, close the window» is normally understood as having the force of the subjunctive «If it should rain, you should close the window». If it doesn't rain, later we can still say that, according to the earlier directive, if it *had* rained, you should then have closed the window. Consider the following argument:

(A_1) She will rise
If she rises, rise !

If she doesn't rise, rise !

If the conditionals here are interpreted in the way in which they are normally interpreted, this amounts to the following.

(A_2) She will rise.
If she should rise, you should rise.

If she should not rise, you should rise.

But this is clearly absurd ! Anyone who affirms the two premisses does *not* thereby commit himself to the conclusion. The absurdity reflects on the following argument form:

(F_4) p
If p. then rise !

If $\sim$p, then rise !

For (A_2) is an instance of this form, provided the conditional schemata of (F_4) are given the strong interpretation. Given the strong interpretation of directive conditionals, it is therefore clear that we must give up *either* (i) argument form (F_2) *or* (ii) the equivalence of «Rise !» to «If pv ~ p, then rise !» *or* (iii) the implication of «If ~ p, then rise !» by «If pv ~ p, then rise !» For, as we saw at the outset, accepting all of these would force us to grant the validity of (F_4). The apparent problem for me is that each of (i), (ii), and (iii) is part of the system in the last half of my paper (3). That the problem is only apparent follows from the fact that it is based on a misconception. For it is based on the assumption that the conditional directives of that system were supposed to be strong (or at least non-weak) whereas in fact they were meant to be weak or «material».

The longer answer to Professor Moritz, to which I alluded earlier, would involve providing an account of the strong directive conditional, perhaps in terms of its weak partner. For this, however, I lack the space on the present occasion, and also the insight (4).

M. H. HUBIEN

There is a most gratifying way of playing heads and tails. You play it in the usual way but for this rule: heads I win, tails you lose. I wonder whether Professor Anderson is not playing the game of deontic logic in such a way.

If I understand him correctly, his position is as follows: you shouldn't expect deontic logic to help lawyers any more than you would expect Artin to help you to braid your daughter's hair.

But then I'd like to ask you, Professor: why did you call your game «deontic logic», why not, for example, «fantastic logic» ? Secondly, if I remember well, yesterday you supported Mr. Peczenik when he proposed to use free-choice permission in deontic logic.

(3) If I understand it correctly, the solution recommended by Professor Moritz is to reject (i). And indeed, on the strong interpretation of directive conditionals, I believe that both (i) and (ii) should be rejected.

(4) Compare the problem of contrary-to-fact conditionals.

(Interruption from Mr. Peczenik who says that he has never proposed to use it)

Yes, but am I right in thinking that Professor Anderson gave us some argument to support anyone who would wish to deal with such a concept in deontic logic ?

(Professor Anderson answers: I do not think that I intended to do so)

Then my mistake. I thought you gave some example in ordinary English which seemed to support such a case.

(Professor Anderson: O. K. I did)

What I ask you then is: what were you doing if not trying to show that there are empirical, presystematic reasons to justify the introduction of such a concept as free-choice permission into deontic logic ?

So, perhaps, we heard you yesterday in your capacity as a philosopher and to-day in your capacity as a mathematical logician, but do you think that such a splitting of scientific personality is desirable ?

M. A. R. ANDERSON

(in reply to Prof. Perelman and Mr. Hubien):

At the beginning of our discussion Prof. Perelman distinguished between coming to a *conclusion,* in the theoretical sciences, and coming to a *decision,* in practical reasoning. This is certainly a distinction worth making, so long as it does not obscure the fact that we sometimes make decisions in theoretical scientific work, and that we sometimes reach conclusions in practical reasoning. We may, in the study of physics, *decide* to discard certain results in spite of the fact that they tend to disconfirm a theory; no one takes the fact that stones fall faster than feathers as evidence against Galileo's Law of Falling Bodies. Of course we *now* have an explanation for the difference in rate of fall, but in Galileo's time that explanation was not available; he simply had to discount some evidence which was available to him.

And just as we might make decisions in theoretical reasoning, so we might reach conclusions in practical reasoning, at least if we construe the latter term broadly. In the absence of extenuating circumstances, a married man is under an obligation (in many societies) to provide financial support for his wife and children (if he has any). From this it *follows logically* that in the absence of extenuating circumstances, a married man is under an obligation (in those same societies) to provide financial support for his wife. Now of course it may be difficult to *decide* whether or not extenuating circumstances are present, or whether a particular married man has fulfilled his obligation; but the logical point is independent of these difficulties, in just the same way that the correctness of the equation $E = mc^2$, (viewed as a consequence of Einsteinian physics) is entirely independent of the difficulties attendant on measuring the speed of light.

Deontic logicians are concerned with those aspects of legal reasoning to which conclusions-drawing of a logical sort is relevant — which is of course far from being the whole of legal reasoning. If practising lawyers can use the analytic tools of deontic logic, well and good; if not, my colleagues and I can continue to play games with symbols, with *some* confidence, anyway, that we are probably not doing anyone any harm.