ON PRACTICAL INFERENCE, WITH AN EXCURSUS ON THEORETICAL INFERENCE

Ernest Sosa

In the course of explaining, describing, narrating, reporting, etc., one normally issues statements, which are true or false and bear logical relations. These logical relations have long been the subject of intensive studies, the results forming various well known systems. Analogously, in the course of requesting, instructing, commanding, etc., one normally issues directions. But the intensive logical study of these is comparatively a very recent phenomenon. In what follows I hope to enhance our understanding of the subject.

I

What one states is always to the effect that something or other is true or is the case, whereas what one directs is to the effect that something or other be made true, or be made the case. Thus I may state that you are at attention, whereas I can direct only that you *be* at attention. And in general one can state that *a* is F but one would wish or direct that *a* be F. Now, of course, it could be true, or false, that *a* is F. It may be thought, however, that it could not be true or false that *a be* F. But this is far from certain. Fortunately, wishes do sometimes come *true*. And, if I am right above, what one wishes and what one directs are of the same form. In some simple cases, again, they are both of the form: that *a* be F.

Hence, although one does not ordinarily speak of requests, commands, prayers, or instructions coming (or being) true, I

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can see no very good reason for not doing so, especially since we do so speak of wishes, whose objects seem the same as those of requests, commands, prayers, and instructions. However, for the sake of those who still may have qualms we could easily define a notion of "satisfaction" such that the direction that *a* be F is satisfied just in case *a* is F.

Several writers have recently used this notion of satisfaction in their attempts to clarify the logic of directives. I myself have used it in a modified form in formulating the following proposals (2). First, allow me to introduce the schema '—/...' for conditional directives, which is to be read: "On condition that —, let it be the case that ..." Instances of our basic direction schemata will have three possible values: A/B is *neutral* iff ~A; *satisfied* iff A & B; and *violated* iff A & ~B. Finally, the criterion of validity for pure directive arguments is as follows:

(C) A directive argument is valid provided it contains a non-empty subset of conjointly satisfiable premisses such that; (i) if its members are satisfied then necessarily the conclusion is satisfied; and (ii) if the conclusion is violated then necessarily at least one of its members is violated (3).

I allow it is an apparently undesirable feature of (C) that it denies the validity of the inference from "If it rains, close the window!" to "If it rains and thunders, close the window!". But I have ventured the opinion that this feature of (C) is no more than *apparently* undesirable. In his recent discussion of my proposals, however, Professor Alf Ross comes to the op-

(2) For simplicity, I have modified them inessentially in the following account.

(3) I defend and develop this approach in several papers. See, for example, "The Semantics of Imperatives," *American Philosophical Quarterly*, Vol. 4 (1967). For an application of similar ideas to the logic of questions, see the paper by Nuel Belnap, "Åqvist's Corrections-Accumulating Question-Sequences," in *Philosophical Logic*, ed. by J. W. Davies, D. J. Hockney, and W. K. Wilson (D. Reidel Publishing Co.: Holland, 1969), pp. 122-134. Belnap is too kind when he says that his work was guided by mine and Rescher's. At least in my case, the guidance was mutual.

posite conclusion (4). Indeed, he concludes by suggesting that my proposals "... should be provided with a warning: not to be used by judges or other persons concerned with the administration of norms !" (5).

Presumably, Professor Ross has reasoned like this. If we cannot infer p&r/q from p/q, then whenever not only p, but also r is true, the categorical directive "Let it be the case that q" need not be binding. Nothing is then easier than to find loopholes in the law. If there is a law to the effect that p/q, and the condition p obtains, one may need only find some irrelevant truth r to escape the burden of the categorical directive "Let it be the case that q." For p&r/q does not follow from p/q, and so, since not only p but also r is true, the categorical directive need not be binding.

There are two points to be made in answer to Professor Ross. In the first place, if the above is indeed his line of reasoning, it makes the unwarranted assumption that from p/q and p the system under discussion would not allow the conclusion "Let it be the case that q." But the inference in question belongs to the *mixed* logic of conditional directives and statements and the system under discussion makes no commitments with respect to such a logic, but limits itself to the pure logic of directives.

There is a more interesting reply, however, which depends on the defeasible character of legal concepts and of much of our practical discourse. If the inference from p/q to p&r/q were valid, then from the law "If the light is red, stop" we could validly infer that if the light is red and there is a tidal wave right behind you, you are to stop. Here one might reply that the inference is valid enough, and that the legal system is hence committed to the conclusion, but that a tidal wave should result in a suspension of the system. Consider, however, cases of legal or moral conflict such as that between the rules "Keep your promises," and "Don't lie" generated when you promise

(4) Alf Ross, *Directives, Norms, and their Logic* (Routledge and Kegan Paul Ltd.: London, 1968), pp. 175-176.

(5) *Ibid.*, p. 176.

to do something which as it turns out would be to lie. If the inference from p/q to p&r/q were valid, then the conflict situation would be one where a directive of the form "If φ'ing would be to lie and to keep a promise, then φ and don't φ" would be in force. If we rule out such directives as unacceptable, then either we give up codes with rules that allow conflicts, such as the rules about promises and lies, or we give up the inference from p/q to p&r/q. Considering the staggering complexities that must surely accompany developing a useful code free of such conflicts (6), we do well at least to explore the second possibility. That is what I have begun to do in the system criticized by Ross (7).

Let me make it clear, however, that I am not suggesting the foregoing as the only reading one may give to conditional directives. I am only suggesting that it may well be one common and natural reading that fits many if not most occasions, especially in the formulation of codes of conduct. Indeed, it seems to me that conditional imperatives are sometimes to be read differently, in a way that requires a logic much closer to that of the "material" conditional indicative. Of course, it may well be possible to define one of our two types of conditional imperatives in terms of the other. Thus the defeasible conditional, the one that does not obey *modus ponens*, may be definable in terms of the material conditional as follows:
"If p, then you, *a*, φ !" $=_{\text{Df.}}$ "If p, then, other things being "equal" (or, other things being "normal") you, *a*, φ !" (This definition would explain why *modus ponens* fails for the defeasible conditional: what is required for the inference to the categorical direction that *a* φ (given the defeasible conditional p/*a* φ's) is not only the truth of p but also the equality or normality of things other than p. The definition would indeed also explain why the inference from "If p, then you, *a*,

(6) Compare Mill's *Utilitarianism*, last paragraph of chapter II:
"There exists no moral system under which there do not arise unequivocal cases of conflicting obligation. These are the real difficulties, the knotty points both in the theory of ethics and in the conscientious guidance of personal conduct."

(7) Compare Sven Danielsson's *Preference and Obligation* (Uppsala, 1968).

φ !" to "If p&r, then you, a, φ !" is invalid for the defeasible conditional. For the truth of r may conflict with the equality or normality of things other than p.)

I want next to sketch a proposal for a logic of the practical that will allow not only for non-defeasible conditional directives but also for categorical directives and for statements.

II

Let us supplement a standard sentential calculus with an operator / to be flanked by statement schemata , and such that A/B is to be read: "On condition that A, let it be the case that B."

Categorical directions will then be expressible by means of our conditional directives, for they are tantamount to directions on an empty condition 'p v $\sim$p'. Let us have a symbol '*' for an arbitrary empty condition [8].

An account of truth-functional implication for a mixed logic of declaratives and directives is then perhaps accessible as follows.

(D_1) A set of directives and declaratives α truth-functionally yields a directive R/S iff α has declarative members D_1, ..., D_k, and directive members P_1/Q_1, ..., P_n/Q_n, such that (a) R & D_1 & ... & D_k is consistent and truth-functionally implies P_1 & ... & P_n, and (b) Q_1 & ... & Q_n truth-functionally implies S.

Let us say that the "instruction" of a directive A/B is that it be the case that B, the "condition" being that A. The "object" of the instruction that it be the case that B, moreover, is that-B. The idea of truth functional yielding is then this. A set of directives and declaratives α yields a directive A/B iff there is a subset α' of α containing only directives, such that the objects of their instructions together truth-functionally imply that B,

[8] This way of expressing categorical imperatives I owe to Nicholas Rescher, down to the '*'. (See his *Logic of Commands* (Dover: N.Y., 1966), p. 39.)

and where, given the declaratives in α, the conditions of the directives in α' are all included in the condition that A.

Truth-functional yielding should not be identified with truth-functional implication, however, because it would not account for the validity of the argument:

If it rains, close the windows.
If it snows, close the windows.

If it rains or snows, close the windows.

It is clear that the condition of the conclusion (that it rains *or* snows) will not include each of the conditions of any subset of the premisses. And yet surely we should allow that if a directive instruction (e.g., that you close the window) follows on each of a set of conditions (e.g., that it rains, that it snows) from a set of premisses, then the directive instruction on the disjunction of the conditions (that it rains or snows) should also follow. This accounts for the following definition

(D_2) A set of directives and declaratives α truth-functionally implies a directive A/B iff α truth-functionally yields Ai/B for each disjunct Ai in any disjunctive normal form of A ([9]).

Note that this would give us a mechanical test of validity for practical arguments. (For simplicity, let us say that a valid formula has $(p \vee \sim p)$ as the one disjunct in any of its disjunctive normal forms, and that $(p \cdot \sim p)$ plays a similar role for inconsistencies.)

If this criterion is acceptable, I would venture that a similar criterion for quantificational implication could be devised with modest technical virtuosity. Taking one step at a time, I will not complicate matters by attempting to do so here. Note also that the present criterion must be supplemented if we wish to account for the validity of arguments where the conclusion is

([9]) Perhaps this criterion of validity could be reinterpreted to fit deontic sentences. Thus we could read 'A/B' as "On condition that A, it ought to be the case that B."

not a directive but a statement. Consider, for example, the argument whose premisses are 'If it rains, make me eat my hat' and (implicitly) 'Don't you make me eat my hat', the obvious conclusion being 'It won't rain'. A supplement to consider is this: A set of directives and declaratives α truth-functionally implies a declarative D iff $\alpha \cup \{\sim D\}$ truth-functionally implies contradictory directives. (Contradictory directives are directives A/B and A/$\sim$B.)

One of the principal requirements on an adequate logic of directives is surely that it account for the validity of such arguments as the following.

1. The neighbors are in, so keep the noise down.
2. You promised to help him, so do so.
3. Writeright is the best pen for the money, so I will buy one (10).
4. Grin and bear it, so (at least) grin.

Only the last of these is not enthymematic. And it falls quickly into place:

4′. */you grin and you bear it

———————

*/you grin

As for the rest, they yield as follows:

1′. The neighbors are in
The neighbors are in/you keep the noise down

———————

*/you keep the noise down.

2′. You promise to help him
You promise to help him/you help him

———————

*/you help him.

(10) I believe my remarks here to be applicable not only to the logic of commands and requests but also to the logic of wishes and even to the logic of decisions, for I see no essential difference between 'I will φ', 'Let me φ' when addressed to myself and 'Let it be the case that I φ' when addressed to myself. But the last variant is deliberately stilted so as to fit directly into our formalism as '*/I φ'.

3'. Writeright is the best pen for the money
Writeright is the best pen for the money/I buy a Write-right

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*/I buy a Writeright.

Can the present proposal meet the problem of contrary-to-duty imperatives ? Suppose "(1) it ought to be that a certain man go to the assistance of his neighbors; (2) it ought to be that if he does go he tell them he is coming; (3) if he does not go, then he ought not to tell them he is coming; (4) he does not go" ([11]). The problem is this: can we find a way of representing these four statements so that they do not yield an inconsistency ? The answer is that we can, if we read the 'ought' as a directive operator (otherwise, of course, there is no problem here for us):

(1') */He goes to the assistance of his neighbors.
(2') He does go/He tells them he is coming.
(3') He does not go/He does not tell them he is coming.
(4') He does not go.

It may be argued that (2') does not adequately render (2), because (2) is meant as a categorical directive (*/He does go only if he tells them he is coming). But in that case my guess is that the original group (1)-(4) is in fact inconsistent, so long as (1), (3), and (4) are to be understood respectively as (1'), (3'), and (4'). The point is that we do have a natural way of understanding (1)-(4) so that they do not reduce to absurdity. And that, I think, should suffice.

([11]) Roderick M. Chisholm, "Contrary-to-Duty Imperatives and Deontic Logic," *Analysis*, Vol. 24 (1963), pp. 34-35. The problem has been widely discussed. See G. H. von Wright's "New System of Deontic Logic," *Danish Yearbook of Philosophy*, Vol. I (1964) and the "Correction ..." by von Wright in the next volume; Mark Fisher, "A Contradiction in Deontic Logic ?" *Analysis*, Vol. 25 (1964), pp. 12-13, and the review by F. B. Fitch in the *Journal of Symbolic Logic*, Vol. 32 (1967), pp. 243-244. See also the last number of the first volume of *Noûs* (1967), with papers by Alan Ross Anderson, Lennart Åqvist, Lawrence Powers, and Wilfrid Sellars.

In fact, it is in part the problem of contrary-to-duty imperatives that led me to adopt the criterion of validity set forth in (D_1) and (D_2). It would have been easier to define truth-functional yielding in accordance with criterion (C) of p. 2 above, and then to define truth-functional implication as follows.

> A set of directives and declaratives α truth-functionally implies a directive A/B iff α truth-functionally yields a directive C/B such that C truth-functionally implies A.

But this would have resulted in the validity of the argument form:

(F_1) */A
A/B
———
*/B

which leads directly to the problem of contrary-to-duty imperatives. At least, it does so if we do not question the validity of:

(F_2) A
A/B
———
*/B

which on the present understanding of conditional directives we certainly do not wish to do. Invoking the two forms (F_1) and (F_2), we could easily show that (1')-(4') above logically imply contradictory directives (*/A) and (*/$\sim$A). Given a choice between (F_1) and (F_2), I have dropped (F_1) and retained (F_2).

It may be objected to the criterion set forth in (D_1) and (D_2) that it does not provide us with a semantical value that is preserved as we move from the premisses to the conclusion of a valid practical argument. For it is commonly assumed by writers on the subject that a satisfactory account of validity is to be sought by locating such a value [12]. Let us call this

[12] Thus the assumption of this "Semantical Value Thesis' '(SVT) clearly underlies Rescher's discussion of the alternative conceptions of validity

the SVT (Semantical Value Thesis) and let us compare it with another widely shared conviction: that apart from simplicity and mathematical desiderata, the principal test of a logical system is its ability to account for our distinctions between valid and invalid arguments. Plainly, the SVT does not follow from this, since (D_1) and (D_2) do give us a way of distinguishing valid from invalid directive arguments (and a correct way, I hope) without locating any one semantical value that must be preserved as we move from the premisses to the conclusion of a valid argument.

"When we infer a proposition from some premisses," our critic may persist, "we infer its truth. But what do we infer when we infer a direction ? Have you not ruled out a satisfactory account of practical inference by refusing to provide us with an analogue of truth ?" The question thus arises as to just what are practical inference and argument [13]. But let us consider first the presumably better understood ideas of theoretical or assertoric inference and argument. It seems to me that "argument" adds to "inference" only the idea of the reasoning being directed at a public. So let us concentrate on inference [14]:

(D_3) S directly infers that-p from a set of assumptions α iff $\alpha = \{m_1, \ldots, m_k\}$ and it occurs to S that that-m_1& ... & m_k logically implies that-p, or that if m_1 & ... & m_k then necessarily p.

from among which he is to choose. (See section 7.10 of *The Logic of Commands*.) All the alternatives satisfy SVT or a close cousin. (On some of them each of two semantical values must be preserved.) The assumption of SVT is quite explicit throughout Castaneda's writings on practical inference and deontic logic, etc. Indeed it accounts for much of what is most interesting, philosophically, in them. (See, for example, his latest paper on the subject, "Actions, Imperatives, and Obligations," in the *Proceedings of the Aristotelian Society* for 1967, especially part. III.2.) A similar assumption lies beneath my own earlier work. (See criterion C on p. 2, above.)

(13) Compare André COMBAY's "What is Imperative Inference ?" *Analysis*, Vol. 27 (1967), pp. 145-152, especially section III.

(14) Actually, our objective is an account of *deductive* inference.

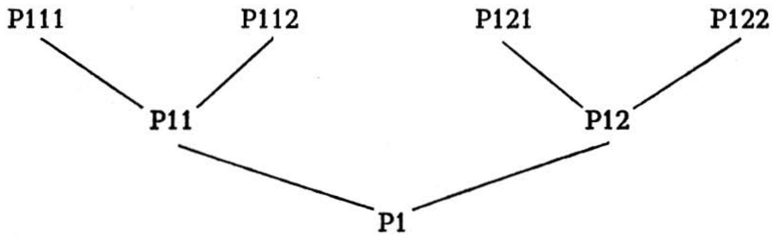
But must one not believe what one infers ? I am not convinced, for one can surely make inferences from mere assumptions, in which case one does not believe what one infers, nor is one even bound to do so. Thus when I make a set of assumptions α and directly infer that p, it seems to me that all I do is to recognize that α logically implies that p.

It is plain, however, that not all inference is direct inference:

(D_4) During interval I, S indirectly infers that-p from a set of assumptions α, modulo m iff there is a set of propositions β for which there is a finite partition whose cells can be arranged in a sequence $C_1, \ldots, C_k$, such that $C_1 = \alpha$, C_k has only that-p as a member, and for each member $\varkappa$ of any cell C_i such that $i>1$, S directly infers $\varkappa$ from some non-empty subset A_x of the union of the predecessor of C_i and α, all within a subinterval I' of I, where the length of I' is m, and where each member of α is a member of at least one such set A_x.

We could strengthen this definition by requiring that at each point, or at least at the conclusion S be aware through memory of each occurrent belief. This requirement would ensure that a thinker is normaly aware of his inferences. But it would surely be too strong, and should hold only for direct inferences. What we could plausibly require is that at the conclusion S be aware through memory that he has made a web or network of direct inferences (such as that described in D_4) with the assumptions in α at the periphery and the conclusion that p at the center. (Note that according to this weaker requirement S need only be aware that he has made a net, and not necessarily that he has made the present one.) We could also require that the occurent beliefs that constitute the net occur in a certain temporal order. (Thus for an indirect inference of P_n from P_1, we could require that the beliefs occur either in the order $P_1 \rightarrow P_2, P_2 \rightarrow P_3, \ldots, P_{n-1} \rightarrow P_n$, or in the reverse order, with few variations.) We could adopt both of these requirements with little difficulty, but for the moment let us forbear.

Definition (D_4) thus gives us a rather weak sense of deductive inference. However, (D_4) will already determine a tree of inference such as the following for each indirect inference:



This may represent a "tree of inference modulo m for S during interval I and the proposition that-p_1." ([15]) Note that each node of such a tree would be a proposition. Thus the "root" node would be the-proposition-that-p_1, and the first "terminal" node (from the left) the-proposition-that P111.

We may now impose the following requirement on such trees of inference:

Let us call a node x a predecessor of a node y provided that there are an i and a j such that x is a proposition that p...ij and y is a proposition that p...i. There must then be some sub-interval I′ of I, of length m, such that each non-terminal node n is directly inferred by S within I′ from the set of predecessors of n.

Thus a tree of inference modulo m for S during interval I and the proposition that-p_1 represents that, within I, S indirectly infers modulo m that-p_1, from the set of terminal nodes. ([16])

A reason for making indirect inference relative to a modulus

([15]) Strictly speaking, what we have here is a *partial tree schema*, for several reasons: for one thing, the nodes are propositional schemata rather than actual propositions as in an actual tree; secondly, we have only P11 and P12, etc., represented, having left out P13 through Pln, etc., and, finally, we have stopped with P111, etc., having left out P1111 through P1111 ....1, etc. Note also that 'p ... i' next to a node dot is elliptical for 'the proposition that p ... i'.

([16]) It would not be difficult to extend the foregoing account of inference so as to cover inference from presumed facts as well as inference from assumptions.

is this. If I make an inference today, I make it in this century. But if I begin the inferential tree during the first day of the century and complete it the last (without reviewing the first part), normally we would not say that an inference had been made, not even an indirect inference. I can explain this by saying that normally we impose a much shorter tacit modulus. The set of occurrent beliefs that constitute an inference must normally be grouped together within a much shorter interval (and must also normally occur in a certain order). However, I propose now to remove the relativity of inference to a modulus in the following way.

It will be recalled that earlier we forbore imposing a further requirement on indirect inference. We are now in a position to impose that requirement.

(D_5) S indirectly infers that-p from a set of assumptions α during interval iff (a) there is some modulus m such that, during I, S indirectly infers that-p from α modulo m, and (b) S remembers that this happened.

This makes inference absolute, and explains the principle of grouping that turns a collection of direct inferences into an inferential net. According to (D_5) what accomplishes this is the memory of the thinker. (17)

On the basis of the present account of inference we can explain the notion of *correct* inference. Since we are restricting ourselves to logical inference, this is to explain the notion of deduction (as opposed to "mere" inference). To deduce that-p from a set of assumptions α you must correctly infer that p from that set of assumptions, i.e., there must be a tree whose terminal nodes are the members of α, and such that for each non-terminal node, S's occurrent belief(s) that it follows logically from the conjunction of its predecessors is (are) correct. (We could also introduce a notion of reasonable or justified inference by replacing 'correct' above with 'reasonable' or 'justified'.)

(17) If this conception of inference is acceptable, fine. If not, we may retreat to the weaker conception of inference relative to a modulus.

On the present conception of inference, moreover, inference is not discovery, even when the inference is correct and justified. But discovery may be based on inference. For a man may note that he has inferred that p from a set of assumptions α, and that he continues to accept the assumptions as actually true even after the inference, in which case he may reasonably and correctly come to believe that p. And this can be a case of discovery so long as the inference and the continued acceptance are reasonable and correct. The connection between S's inference and his coming to believe that p must probably be more than a mere conjunction, if it is really to be a case of discovery based on inference. What more it must be I cannot say, though my guess is that it must be a causal connection.

So much for theoretical inference. To acount for practical inference we need only allow the nodes of our trees to be directions as well as propositions. Thus the direction that you arise (or that you should do so) can be one of the nodes as well as the proposition that you have nothing to lose but your chains. But now the root node must be a direction, so that a tree of inference modulo m for S during interval I and the direction that it be the case that-p_1 (or that it should be the case that-p_1) represents that, within I, S indirecly infers modulo m that it be the case that-p_1 (or that it should be the case that-p_1), from the set of terminal nodes. (D_5) is then also applicable to practical inference. Therefore, to make a practical inference during a certain time interval is to climb down a tree of practical inference, and then to remember that you climbed down such a tree, all during that interval. ([18])

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([18]) Much of what I have said about theoretical inference has application to practical inference, *mutatis mutandis*. Just as discovery can be based on theoretical inference, for example, so correct decision can be based on practical inference.

APPENDIX TO THE PAPER BY E. SOSA

(PROMPTED BY THE DISCUSSION)

It is illuminating, I believe, to draw certain analogies between the philosophy of science and the philosophy of law. In both legal and scientific reasoning deductive logic plays an important role. But in each case there is more to reasoning than can be assessed by the use of deductive logic alone. Thus philosophers of science have long studied the nondeductive principles at work in scientific reasoning. Some of these are said to be systematized rigorously by inductive logicians. Others have not been systematized so thoroughly, but there continues to be progress and hope. (Consider the work on simplicity and its role in theory formation, for example.) Analogously, there is surely a need for philosophers of law to determine the role of, say, equity, or precedence, in the main types of legal system.

Are there any general principles that determine the role of induction (or simplicity) in science ? Are there any general principles that determine the role of equity (or precedence) in law ? The two questions appear analogous to me. And if we let "inductive logic" in a broad sense serve as a rubric for the inquiries of the philosopher of science into the first question, we may also let "juristic logic" serve as a rubric for the inquiries of the philosopher of law into the second.

Of course, all of that goes beyond deductive logic, beyond the realm where premises conclusively entail (rather than "ground" or "support") their conclusions. Inductive premises and considerations of simplicity often support without conclusively entailing. And the same goes for considerations of equity or precedence. But even if we restrict ourselves to deductive logic, I believe that there remain important differences between the practical and the theoretical, between, e.g., law and science. The principles of valid deductive inference needed in the sphere of morality and law go beyond those needed in science or history. Even if the scientist or historian must make value judgments in conducting his research, these would not form part of his results or their grounding. His results would be either factual reports of data or theoretical suggestions supported by factual data.

For the legal mind, however, nontheoretical judgments are needed not only in the choice of questions to treat, but also in their treatment. And here not only quantification theory but also deontic logic and the logic of imperatives appear necessary.