MATHEMATICAL ORGANISMS

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1. PURPOSE. This paper presents an axiomatic theory of organisms which is in partial contrast to axiomatic set theory. Herewith, are two of the distinctive characteristics. (A few words on over-all motivation are offered in the final remarks). The principal objective of the axiom of foundation in axiomatic set theory is to eliminate formulas of the form $x \varepsilon x$. The reason, so it is said, is that such formulas serve no useful purpose. This is an atomistic prejudice. One of the properties of organisms is that they have overlapping extensions which in many cases include for each organism the organism itself as an additional member. In physiology the division between part and whole, member and aggregate, is not so easy to draw as in macroscopic physics — organisms are not simply built out of elements. The axiom system presented in the next section excludes the axiom of foundation as well as any of its alternative forms.

The axiom of extensionality is a mathematical convenience derived from our understanding of macroscopic physics. But again, in physiology organisms can be coextensive and at the same time distinguishable from one another. There are already nonextensional set theories in the literature: for example, Fraenkel-Mostowski's system, originally devised to prove the independence of the axiom of choice, in which so-called "atoms" are coexistensive and distinguishable. Also, Paul J. Cohen's generic sets for the proof of the independence of the axiom of choice in more general set-theoretic settings, although not atoms in the Fraenkel-Mostowski sense, share some of the characteristics of nonextensionality in the relationships between the generic sets and the forcing conditions that determine them. In the next section some mathematical organisms are subjected to the axiom of extensionality, while others are not.

2. MATHEMATICAL ORGANISMS. We shall describe a first-order theory **MO**. **MO** has a finite number of predicate letters $=, \sigma, \sigma_1, \ldots, \sigma_k$ and a finite number of individual constants, but no function letters. We shall use capital letters $X_1, X_2, \ldots$ as variables and $X, Y, Z, \ldots$ to represent arbitrary variables. Intuitively, $\sigma, \sigma_1, \ldots, \sigma_k$ are thought of alternatively as internal or external relations, according to the terms related. $X\sigma Y$ will be read "X stands in Y", $X\sigma_i Y$ $(i = 1, \ldots, k)$ will be read "X i-stands in Y".

In the interpretations we have in mind the variables take organisms as values. We define an organism to be an *organ* if and only if it stands in itself, otherwise organisms which are not organs are called *proper organisms*.

Definition 1. $\mathrm{org}(X)$ for $X\sigma X$.

Definition 2. $\mathrm{Org}(X)$ for $\sim\mathrm{org}(X)$.

Let us introduce small letters $x_1, x_2, \ldots$ as special, restricted variables for organs, and boldface small letters $\mathbf{x}_1, \mathbf{x}_2, \ldots$ as special, restricted variables for proper organisms ($x, y, z, \ldots$ represent arbitrary organs, and $\mathbf{x}, \mathbf{y}, \mathbf{z}, \ldots$ represent arbitrary proper organisms). In other words, $(x_i)A(x_i)$ stands for $(X)(\mathrm{org}(X) \supset A(X))$, i.e., A holds for all organs; $(Ex_i)A(x_i)$ stands for $(EX)(\mathrm{org}(X) \& A(X))$, i.e., A holds for some organ; $(E\mathbf{x}_i)A(\mathbf{x}_i)$ stands for $(EX)(\mathrm{Org}(X) \& A(X))$, i.e., A holds for some proper organism.

We now present the first proper axioms of **MO**, the axioms concerning the relative symmetry of σ and the internal relatedness of proper organisms.

A1. $x\sigma X \equiv X\sigma x$.

A2. $x\sigma\mathbf{x} \; \& \; y\sigma\mathbf{x} \supset x\sigma y$.

Notice that the extension of each organ includes — apart from the organ itself (*Def. 1*) — all the proper organisms in which the organ stands (*A1*), plus all the organs of these proper organisms (*A2*).

A3. $(EX)(Y)\sim Y\sigma X$ (*Null Organism Axiom*).

Definition 3. $(Y)\sim(Y\sigma\emptyset)$.

Theorem 1. $\mathrm{Org}(\emptyset)$.

A1 and *A2* obstruct the usefulness of a pairing axiom for organs as well as the usefulness of an algebra of organs similar to the algebra of sets. Given our lack of definite motivation for selecting any specific axiom system for organs, and in order to preserve freedom of choice, the existence of organs and proper organisms will be provided here by proper axioms chosen more to keep various possibilities open than to formalize regular, specific relationships between organs and proper organisms — relationships whose regularity would make them more interesting from a mathematical viewpoint. As an example of a particular finite system of organisms that shows the possibilities we have in mind, let us consider the following axiom system $\mathbf{MO}_1$, where a_i, b_i ..., m_i, p_i, q_i are individual constants for organs, and $\mathbf{a}$, $\mathbf{b}$, ..., $\mathbf{m}$, $\mathbf{n}$ are individual constants for proper organisms.

S1. $a_i\sigma\mathbf{a}$ $(i = 1, \ldots, k_1)$.
S2. $b_i\sigma\mathbf{b}$ $(i = 1, \ldots, k_2)$.
...
St. $m_i\sigma\mathbf{m}$ $(i = 1, \ldots, k_t)$.
St + 1. $a_j\sigma\mathbf{b}$ $(j = 1, 3)$.
St + 2. $a_1\sigma\mathbf{c}$.
St + 3. $a_j\sigma\mathbf{n}$ $(j = 1, 3)$.
St + 4. $b_2\sigma\mathbf{c}$.
St + 5. $\mathbf{b}\sigma\mathbf{a}$.
St + 6. $\mathbf{d}\sigma\mathbf{a}$.
St + 7. $\mathbf{a}\sigma\mathbf{b}$.
St + 8. $\mathbf{b}\sigma\mathbf{n}$.
St + 9. $\mathbf{e}\sigma\mathbf{n}$.
St + 10. $p\sigma p$.
St + 11. $q_1\sigma q_2$.

Here, between curved brackets, are the extensions (as far as σ is concerned) of a few organisms from $\mathbf{MO}_1$ (the bracket notation does not designate an univocal function for organs — see next section).

a_1: $\{a_1, \ldots, a_{k1}, \mathbf{a}, b_1, \ldots, b_{k2}, \mathbf{b}, c_1, \ldots, c_{k3}, \mathbf{c}, \mathbf{n}\}$.
a_2: $\{a_1, \ldots, a_{k1}, \mathbf{a}\}$.
a_3: $\{a_1, \ldots, a_{k1}, \mathbf{a}, b_1, \ldots, b_{k2}, \mathbf{b}, \mathbf{n}\}$.
b_1: $\{b_1, \ldots, b_{k2}, \mathbf{b}, a_1, a_3\}$.
b_2: $\{b_1, \ldots, b_{k2}, \mathbf{b}, a_1, a_3, c_1, \ldots, c_{k3}, \mathbf{c}\}$.
c_1: $\{c_1, \ldots, c_{k2}, \mathbf{c}, a_1, b_2\}$. (Same extension for $c_2, \ldots, c_{k3}$.)
p : $\{p\}$.
q_1: $\{q_1, q_2\}$.
$\mathbf{a}$: $\{a_1, \ldots, a_{k1}, \mathbf{b}, \mathbf{d}\}$.
$\mathbf{d}$: $\{d_1, \ldots, d_{k4}\}$.
$\mathbf{n}$: $\{a_1, \ldots, a_{k1}, \mathbf{b}, \mathbf{e}\}$.

3. FAMILIES, COLONIES, SPECIES AND HYPERSPECIES. We define a proper organism to be a *family of organs* if and only if it 1-stands in itself, otherwise proper organisms which are not families of organs are called *colonies.*

Definition 6. $\mathrm{Fam}(\mathbf{x})$ for $\mathbf{x}\sigma_1\mathbf{x}$.

Definition 7. $\mathrm{Col}(\mathbf{x})$ for $\sim\mathrm{Fam}(\mathbf{x})$.

We define a colony to be a *proper colony* if and only if it 2-stands in itself, otherwise colonies which are not proper colonies are called *2-species.*

Definition 8. $\mathrm{col}(X)$ for $\mathrm{Col}(X) \,\&\, X\sigma_2X$.

Definition 9. $2\text{-}\mathrm{Spec}(X)$ for $\mathrm{Col}(X) \,\&\sim\mathrm{col}(X)$.

Inductively, we define an i-species ($2 \leqslant i \leqslant k$) to be an (i + 1)-*proper species* if and only if it (i + 1)-stands in itself, otherwise i-species which are not proper species are called (i + 1)-*species.*

Definition 10. $(i+1)\text{-}\mathrm{spec}(X)$ for $i\text{-}\mathrm{Spec}(X) \,\&\, X\sigma_{i+1}X$.

Definition 11. $(i+1)\text{-}\mathrm{Spec}(X)$ for $i\text{-}\mathrm{Spec}(X) \,\&\sim(i+1)\text{-}\mathrm{spec}(X)$. ($\mathrm{Col}(X)$ and $(i+1)\text{-}\mathrm{Spec}(X)$ are, of course, abbreviations for $\mathrm{Org}(X) \,\&\sim \mathrm{Fam}(X)$ and $i\text{-}\mathrm{Spec}(X) \,\&\sim (i+1)\text{-}\mathrm{spec}(X)$ respectively.)

The following three axioms are the *axioms of extensionality.*

A4a. $x = x$.

A4b. $\mathbf{x} = \mathbf{y} \equiv (X)(X\sigma\mathbf{x} \equiv X\sigma\mathbf{y}) \,\&\, (X)(X\sigma_1\mathbf{x} \equiv X\sigma_1\mathbf{y}) \,\&\, \ldots \,\&\, (X)(X\sigma_k\mathbf{x} \equiv X\sigma_k\mathbf{y})$.

A4c. $X = Y \supset (\mathrm{org}(X) \,\&\, \mathrm{org}(Y)) \vee (\mathrm{Fam}(X) \,\&\, \mathrm{Fam}(Y)) \vee (\mathrm{col}(X) \,\&\, \mathrm{col}(Y)) \vee \ldots (k\text{-}\mathrm{spec}(X) \,\&\, k\text{-}\mathrm{spec}(Y)) \vee (k\text{-}\mathrm{Spec}(X) \,\&\, k\text{-}\mathrm{Spec}(Y))$.

Notice that equality of organs is *individual identity* — for organs, some extension does not guarantee equality.

Definition 12. $X \subset Y$ for $(Z)(Z\sigma X \supset Z\sigma Y) \,\&\, (Z)(Z\sigma_1 X \supset Z\sigma_1 Y) \,\&\, \ldots \,\&(Z)(Z\sigma_k X \supset Z\sigma_k Y)$. (*Inclusion*).

Definition 13. $X \subset Y$ for $X \subset Y \,\&\sim X = Y$. (*Proper Inclusion*).

Notice that unequal organs with the same extension are properly included in one another.

Theorem 2. **MO** is a first-order theory with equality.

Theorem 3. $\mathrm{org}(X) \,\&\, X = Y \supset \mathrm{org}(Y)$.

Thorem 4. $\mathrm{Org}(X) \,\&\, X = Y \supset \mathrm{Org}(Y)$.

Theorem 5. $\mathbf{x}\sigma\mathbf{y} \supset \sim \mathbf{x} = \mathbf{y}$.

The following are axioms for proper organisms which establish for them properties that are to a certain extent similar to those of sets.

A5a. $(\mathbf{x})(\mathbf{y})(E\mathbf{z})(\mathbf{u})(\mathrm{Fam}(\mathbf{x}) \supset (\mathrm{Fam}(\mathbf{y}) \supset (\mathrm{Fam}(\mathbf{z}) \,\&\, (\mathrm{Fam}(\mathbf{u}) (\mathbf{u}\sigma\mathbf{z} \equiv \mathbf{u} = \mathbf{x} \vee \mathbf{u} = \mathbf{y})))))$.

A5b. $(\mathbf{x})(\mathbf{y})(E\mathbf{z})(\mathbf{u})(\mathrm{col}(\mathbf{x}) \supset (\mathrm{col}(\mathbf{y}) \supset (\mathrm{col}(\mathbf{z}) \,\&(\mathrm{col}(\mathbf{u}) \supset (\mathbf{u}\sigma_1\mathbf{z} \equiv \mathbf{u} = \mathbf{x} \vee \mathbf{u} = \mathbf{y})))))$.

A5c. $(\mathbf{x})(\mathbf{y})(E\mathbf{z})(\mathbf{u})(i\text{-}\mathrm{spec}(\mathbf{x}) \supset (i\text{-}\mathrm{spec}(\mathbf{y}) \supset (i\text{-}\mathrm{spec}(\mathbf{z}) \,\&\, (i\text{-}\mathrm{spec}(\mathbf{u}) \supset (\mathbf{u}\sigma_i\mathbf{z} \equiv \mathbf{u} = \mathbf{x} \vee \mathbf{u} = \mathbf{y})))))$. $(2 \leqslant i \leqslant k)$.

These are the *pairing axioms*. Notice that the "pair" determined by two proper organisms of the same kind — say, two proper colonies — may include an unlimited number of organisms of other kinds — organs, families of organs and various kinds of hyperspecies. Axiom systems can easily be given in which it is not possible to separate set-theoretically organisms of one kind from organisms of other kinds. Still, this does not preclude the definition of ordered "pairs" of proper organisms of the same kind in the usual manner. But first, the definition for *unordered "pairs"*.

Definition 14. $\mathrm{Fam}(\mathbf{x})\ \&\ \mathrm{Fam}(\mathbf{y})\ \&\ (\mathbf{z})(\mathrm{Fam}(\mathbf{z}) \supset (\mathbf{z}\sigma[\mathbf{x},\mathbf{y}] \equiv \mathbf{z} = \mathbf{x} \vee \mathbf{z} = \mathbf{y}))) \vee$.

$\vee\ (\mathrm{col}(\mathbf{x})\ \&\ \mathrm{col}(\mathbf{y})\ \&\ (\mathbf{z})(\mathrm{col}(\mathbf{z}) \supset (\mathbf{z}\sigma_1[\mathbf{x},\mathbf{y}] \equiv \mathbf{z} = \mathbf{x} \vee \mathbf{z} = \mathbf{y}))) \vee \ldots$

$\vee\ (\text{k-spec}(\mathbf{x})\ \&\ \text{k-spec}(\mathbf{y})\ \&\ (\mathbf{z})(\text{k-spec}(\mathbf{z}) \supset (\mathbf{z}\sigma_k[\mathbf{x},\mathbf{y}] \equiv \mathbf{z} = \mathbf{x} \vee \mathbf{z} = \mathbf{y}))) \vee$

$\vee\ (\text{k-Spec}(\mathbf{x})\ \&\ \text{k-Spec}(\mathbf{y})\ \&\ {\sim}\mathbf{x} = \emptyset\ \&\ {\sim}\mathbf{y} = \emptyset\ \&[\mathbf{x},\mathbf{y}] = \emptyset)$.

Notice that the value of the function $[\mathbf{x},\mathbf{y}]$ is unique for any given pair of non-null proper organisms of the same kind (*A4b*), althought $[\mathbf{x}, \mathbf{y}]$ does not necessarily indicate the extension of the pairing organism thus formed.

Definition 15. $[\mathbf{x}] = [\mathbf{x},\mathbf{x}]$.

Theorem 6. $(\mathbf{x})(\mathbf{y})([\mathbf{x}] = [\mathbf{y}] \supset \mathbf{x} = \mathbf{y})$.

Definition 16. $\langle \mathbf{x},\mathbf{y} \rangle = \big[[\mathbf{x}]\,,[\mathbf{x},\mathbf{y}]\big]$.

Theorem 7. $(\mathbf{x})(\mathbf{y})(\mathbf{u})(\mathbf{v})(\langle \mathbf{x},\mathbf{y} \rangle = \langle \mathbf{u},\mathbf{v} \rangle \supset \mathbf{x} = \mathbf{u}\ \&\ \mathbf{y} = \mathbf{v})$.

Definition 16 can be extended inductively in the usual manner to ordered n-tuples, for which we shall use the standard notation $\langle \mathbf{x}_1, \ldots, \mathbf{x}_n \rangle$.

Additional axioms similar to *S1-St+11* are now in order to introduce further relationships between different kinds of organisms. The choice is, again, more heuristic than systematic (r_i,s_i

are individual constants for organs, $\mathbf{r},\mathbf{s}$ are individual constants for proper organisms).

St+*12*. $\mathrm{Fam}(\mathbf{a})\ \&\ \mathrm{Fam}(\mathbf{b})\ \&\ \ldots\ \&\ \mathrm{Fam}(\mathbf{n})$.
St+*13*. $a_i\sigma_1\mathbf{r}\ \&\ \mathrm{col}(\mathbf{r})$.
St+*14*. $a_i\sigma_1\mathbf{s}\ \&\ \mathrm{col}(\mathbf{s})$.
St+*15*. $r_i\sigma_2 s_i$.
St+*16*. $\mathbf{r}\sigma_2\mathbf{s}$.
St+*17*. $\mathbf{s}\sigma_3\mathbf{r}$.
St+*18*. $c_i\sigma_2\mathbf{r}$.
St+*19*. $[\mathbf{a},\mathbf{b}] = \mathbf{r}$.
St+*20*. $[\mathbf{r},\mathbf{s}]\sigma\mathbf{n}$.
St+*21*. $\emptyset\sigma_1\mathbf{n}$.

With these new axioms the extension of each a_i is increased with $\mathbf{r}$ and $\mathbf{s}$. The extension of each c_i is increased with $\mathbf{r}$. The "pair" $[\mathbf{a},\mathbf{b}]$ is a proper colony whose extension is $\{\mathbf{a}, \mathbf{b}, a_1, \ldots, a_{k1}, \mathbf{s}, c_1, \ldots, c_{k3}\}$. Etc.

We now complete **MO** with an axiom schema similar to the general class existence theorem in Gödel's axiomatic set theory. (Like the theorem, the axiom schema can be obtained from a finite number of proper axioms.)

A6. Let $\varphi(\mathbf{x}_1, \ldots, \mathbf{x}_n, Y_1, \ldots, Y_m)$ be a wf whose variables occur among $\mathbf{x}_1, \ldots, \mathbf{x}_n, Y_1, \ldots, Y_m$ and in which only organisms up to k-proper species are quantified. We call such a wf *predicative*. Then

A6a. $$(E\mathbf{z})(\mathbf{x}_1)(\mathbf{x}_2)\ldots(\mathbf{x}_n)(\sim \text{k-Spec}(\mathbf{x}_1) \supset (\sim \text{k-Spec}(\mathbf{x}_2) \supset (\ldots \supset (\sim \text{k-Spec}(\mathbf{x}_n) \supset (<\mathbf{x}_1, \ldots, \mathbf{x}_n>\sigma\mathbf{z} \equiv \varphi(\mathbf{x}_1, \ldots, \mathbf{x}_n, Y_1, \ldots, Y_m))\ldots).$$

A6b. $$(E\mathbf{z})(\mathbf{x}_1)(\mathbf{x}_2)\ldots(\mathbf{x}_n)(\sim \text{k-Spec}(\mathbf{x}_1) \supset (\sim \text{k-Spec}(\mathbf{x}_2) \supset (\ldots \supset (\sim \text{k-Spec}(\mathbf{x}_n) \supset (<\mathbf{x}_1, \ldots, \mathbf{x}_n>\sigma_i\mathbf{z} \equiv \varphi(\mathbf{x}_1, \ldots, \mathbf{x}_n, Y_1, \ldots, Y_m))\ldots). \quad (1 \leqslant i \leqslant k).$$

Boolean operations are now definable for proper organisms.

Paradoxes are avoided with the following line of reasoning. Let φ be $\sim\mathbf{x}\sigma\mathbf{x}$ or $\sim\mathbf{x}\sigma_i\mathbf{x}$ $(1 \leqslant i \leqslant k)$. Then $(\mathbf{x})(\sim \text{k-Spec}(\mathbf{x}) \supset$

($\mathbf{x}\sigma_i\mathbf{z} \equiv \sim\mathbf{x}\sigma_i\mathbf{x}$)), which leads to contradiction and implies k-Spec(z).

Definition 17. Given any predicative wf $\varphi(\mathbf{x}_1, \ldots, \mathbf{x}_n, \mathrm{Y}_1, \ldots, \mathrm{Y}_m)$, we shall use $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n\varphi(\mathbf{x}_1, \ldots, \mathbf{x}_n, \mathrm{Y}_1, \ldots, \mathrm{Y}_n)$ to denote the proper organism of all tuples $<\mathbf{x}_1, \ldots, \mathbf{x}_n>$ satisfying φ, such that $\sim$k-Spec($\mathbf{x}_1$) & ... & $\sim$k-Spec($\mathbf{x}_n$).

Definition 18. V stands for $\hat{\mathrm{x}}(\sim$k-Spec($\mathbf{x}$) & $\mathbf{x} = \mathbf{x}$).

Theorem(8. $\sim$k-Spec($\mathbf{x}$)$\supset \mathbf{x}\sigma \mathrm{V} \sim \mathbf{x}\sigma_1\mathrm{V}\vee\ldots\vee\mathbf{x}\sigma_k\mathrm{V}$.

Theorem 9. k-Spec(V).

4. THE CATEGORY OF MATHEMATICAL ORGANISMS. *Organisms* can be be described as forming a modified category system in the following way. The *category of mathematical organisms* is an organism A, not a k-Species, together with an organism C that is a disjoint union of the form $C = \cup [X,Y]$ for all (X,Y) in $A \times A$. Each $[X,Y]$ is not an organ, nor is it a k-Species, and can be void. When it is not void it is composed of a single organism, namely, the correspondence composed of all pairs (U,V) such that $U\sigma_i X$ and $V\sigma_j Y$, and such that $U\sigma_h V$ for some σ_i, σ_j, σ_h, where the latter are any of the predicate letters $\sigma, \sigma_1, \ldots, \sigma_k$. In addition, for each triple (X,Y,Z) of members of A we are to have a function from $[Y,Z]$ to $[X,Y]$, eventually void. The image of this function is denoted by $[Y,Z] \cdot [X,Y]$ and called the *composition* of $[Y,Z]$ by $[X,Y]$. Composition functions satisfy two conditions.

(i) Whenever compositions make sense,

$$([Z,W] \cdot [Y,Z]) \cdot [X,Y] = [Z,W] \cdot ([Y,Z] \cdot [X,Y]).$$

(ii) For each X in A not a k-Species we have an element 1_x in C, namely $[X,X]$, such that $[X,X] \cdot [Y,X] = [Y,X]$ and $[X,Y] \cdot [X,X] = [X,Y]$ whenever the composition makes sense.

Given a member $[X,Y]$ in C, X is called the *domain* and Y the *codomain* of $[X,Y]$. Since C is composed of single organisms,

A is somewhat similar to what in category theory is called an *ordered class*. Also, organisms in A can be called *objects* while organisms in C can be called *correspondences*. ("Morphism", a truncation of homomorphism, suggests too stronlgy a condition of single-valuedness that is not intended.)

The elimination of individual objects (k-Species always excluded, of course) is now possible in a manner similar to the so-called non-objective approach in category theory. Objects can be defined in terms of correspondences and retrieved through appropriate identities.

Saunders MacLane considers Paul J. Cohen's proof of the independence of the continuum hypothesis a startling indication that current axiomatic set theories do not appropriately describe the entities a mathematician needs to match his intuitive idea of set as it derives from mathematical usage. MacLane is at present attempting (and he is not alone in this) to reconstruct mathematically useful fragments of set theory from the standpoint of pure category theory ("categories without sets"). This is indeed a considerable project and it is still too early to predict where it may lead. However, it is perhaps to the point to remark that categories, given the great generality with which they are now defined, allow all kinds of unexpected structures with or without slight modifications — and mathematical organisms are a case in point. It seems as if category theoreticians, in their desire for a better set theory (or something that replaces it), may be getting more than they bargained for — which might be all to the good.

5. FINAL REMARKS. Although this is not the place to elaborate on biological motivations, nevertheless a few comments are indispensable. Our logic is the logic of solids, said Bergson, which is why the application of logical concepts to biology is always ridden with innumerable exceptions and qualifications, and worst of all, why it fails to convey two important biological facts (i) Although organs have simple location anatomically conceived, physiologically they influence one another to such a degree that they must be thought of as having the same extension as the whole organism and often a greater one; further, the general state of the organism constitutes an additional term in the phy-

sociological composition of each organ. (ii) Environment and the hierarchy of life are intrinsic to each organism, and although organisms associate in colonies, colonies in species, etc., this does not in any way imply that organisms are built with completely separated individuals or any other elementary abstractions. Indeed, set-theoretic conceptions and hierarchies have a tendency to hide and blur biological significance. They are suitable to describe collections of macroscopic solids but not life. We need a logic of fluids.

In this logic of fluids a certain amount of ambiguity must find formal expression, which requires beginning with what we may call a relativization of the individual. (In biology one must distinguish an organ without completely separating it in a set-theoretic sense, lest the organism vanish altogether). This relativization is of course implicit in **MO** and $\mathbf{MO}_1$. (For another, different, approach to such relativization see [2].) It is our belief that only through a relativization of the individual can we give to networks of relations the primary descriptive role that they should have, a role that is now overshadowed by excessive emphasis on final components. Ultimately, one should be able to describe organisms as pure relations, that is, as systems of correspondences in which events are mere changes in the system.

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REFERENCES

[1] F. G. ASENJO, *El Todo y las Partes: Estudios de Ontología Formal.* Madrid: Editorial Martínez de Murguía, 1962.

[2] F. G. ASENJO, Theory of multiplicities, *Logique et Analyse,* Vol. VIII (1965), pp. 105-110. (Misprint: On page 106, line 7 from bottom, *w*-distinguishable should be *u*-distinguishable).

[3] Ludwig von BERTALANFFY, *Problems of Life.* London: C. A. Watts Co., 1952.

[4] Kurt GOLDSTEIN, *Der Aufbau des Organismus.* The Hague: Martinus Nijhoff, 1934. (*The Organism.* New York: American Book Co., 1938).