ARISTOTLE ON UNIVERSALITY AND NECESSITY

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Many contemporary writers on modal logic — Prior being the obvious exception — treat the necessity functor as if it might be translated without residue either by the universal quantifier alone or by the universal quantifier with an assertion sign (1). That is to say, they treat it as if being true in every case and being necessarily true were all the same thing. This tendency is not always immediately apparent — as when modal functors and quantifiers appear side by side in a quantified modal logic; but it sometimes does reveal itself, as in attempts to map over modal systems onto Boolean algebra. In these mappings, "It is necessary that p" amounts to "p = V" (where V is the universal element of the Boolean algebra) (2).

The currency of this view, that modality and quantity may in some sense be identified, is perhaps due to Professor Carnap, who, although he holds that the symbol 'N' (which he uses for logical necessity) "cannot be defined on the basis of the ordinary truth-functional connectives and quantifiers for individuals", also holds that "a proposition *p* is logically necessary if and only if a sentence expressing *p* is logically true" (3). According

(1) Cf. A. N. PRIOR, *Formal Logic* (2nd ed.; Oxford: Oxford University Press, 1962), pp. 185-193, wherein Prior finds close parallels between modality and quantity, but resists the temptation to make a "simple identification of modality and quantity". Prior refers to the distinction of "universals of law" from "universals of fact" made by W. E. Johnson in his *Logic* (Part III; Cambridge: Cambridge University Press, 1924), i.5. He also mentions more recent discussion of the distinction of necessary from merely assertoric universals, which had as its starting point R. M. CHISHOLM's article "The Contrary-to-Fact Conditional", *Mind,* IV (1946), pp. 289-307.

(2) Cf., for example, P. HENLE, in Appendix II of C. I. LEWIS and C. H. LANGFORD, *Symbolic Logic* (New York: Dover Publications, 1959).

(3) R. CARNAP, "Modalities and Quantification", *Journal of Symbolic Logic,* XI (1946), p. 34.

to Carnap, a sentence "is usually regarded as logically true or logically necessary if it is true in every possible case" (4). 'Possible' is left unexplained in his formal definition of logical truth, in which a sentence is said to be logically true (in his system) when its range is the universal range of state-descriptions (5). By his own admission, Carnap's notion of logical truth is based on Wittgenstein's, which takes as its criterion the universality of the range (6). Of course, it is well known that, for Wittgenstein, "the only necessity that exists is *logical* necessity", which is the necessity of properly logical propositions, or tautologies, which are those propositions true for all the truth-possibilities of their elementary propositions (7). And on this view, any notion of necessity which departs from universal quantification is precluded from the outset.

The influence of this sort of view among interpreters of Aristotle's modal logic appears prominently in the work of Łukasiewicz and Hintikka. Łukasiewicz contends that Aristotle's notion of syllogistic necessity must be conveyed, and is conveyed adequately, by universal quantifiers. Indeed, he declares in so many words, citing *Analytica Priora* 25a20-26 by way of authority, that "Aristotle uses the sign of necessity in the consequent of a true implication in order to emphasize that the implication is true for all values of variables occurring in the implication", so that "the Aristotelian sign of syllogistic necessity represents a universal quantifier" (8). Professor Hintikka,

(4) CARNAP, p. 50.

(5) CARNAP, p. 51, D7-6.a. A state-description for Carnap is a class of sentences which contains for every atomic sentence either that sentence or its denial, but not both, and no other sentences (p. 50, D7-4).

(6) CARNAP, p. 47. Cf. Carnap's *Introduction to Semantics* (Cambridge: Harvard University Press, 1942), §§ 18-19. Cf. also L. WITTGENSTEIN, *Tractatus Logico-Philosophicus,* trans. D. F. PEARS and B. F. MCGUINESS (London: Routledge & Kegan Paul, 1961), 4.462, 4.463, 5.5262.

(7) *Tractatus,* 6.37, 6.124, 4.46. "Truth-possibilities of elementary propositions" are for Wittgenstein "possibilities of existence and non-existence of states of affairs" (4.3).

(8) J. ŁUKASIEWICZ, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* (2nd ed. enlarged; Oxford: Oxford University Press, 1957), p. 11. Cf. § 41, pp. 143-146. Łukasiewicz does recognise that

who does not confine his attention to syllogistic necessity, takes a slightly different line with Aristotle. Omnitemporality is his concern: "in passage after passage", Hintikka writes, "[Aristotle] explicitly or tacitly equates possibility with sometime truth and necessity with omnitemporal truth" (9).

As Łukasiewicz notes, Aristotle did not use for quantifiers a conventional notation like any of the current ones, so far as is known (10). On the other hand, Aristotle did make some remarks on the premisses of demonstration from which one might gather some information about his views on quantification. These remarks are to be found at *Analytica Posteriora* 73a21-74a3 (11), where Aristotle distinguishes three kinds of predicate assignment to subjects: (1) assignment κατὰ παντός, (2) assignment καθ'αὑτό, and (3) assignment καθόλου. I wish to argue in the remainder of this paper that these remarks belie the historical accuracy of the accounts of Aristotle's treatment of modality given by Łukasiewicz and Hintikka, and that they open a field for modalities which Wittgenstein and Carnap would close off.

Aristotle has a notion of propositional necessity according to which "propositions which ascribe essential properties to objects are ... not only factually, but also necessarily true". Łukasiewicz himself, however, believes this to be an "erroneous distinction", and asserts in his own behalf that "there are no true apodeictic propositions, and from the standpoint of logic there is no difference between a mathematical and an empirical truth" (p. 205).

(9) J. Hintikka, "Necessity, Universality, and Time in Aristotle", *Ajatus,* xx (1957), pp. 65-90; "Aristotle and the 'Master Argument' of Diodorus", *American Philosophical Quarterly,* i (1964), pp. 101-104; "The Once and Future Sea Fight: Aristotle's Discussion of Future Contingents in *De Interpretatione* ix", *Philosophical Review,* lxxiii (1964), pp. 461-492. Professor Hintikka has offered more extensive discussions of the concept of time among the Greeks in "Observations on the Greek Concept of Time", *Ajatus,* xxiv (1962), pp. 39-66; and in "Time, Truth and Knowledge in Ancient Greek Philosophy", *American Philosophical Quarterly,* iv (1967), pp. 1-14.

(10) "Arisotle had no clear idea of quantifiers and did not use them in his works; consequently, we cannot introduce them into his syllogistic". Łukasiewicz, p. 83.

(11) Reference is to Ross's text. *Aristotle's Prior and Posterior Analytics.* A revised text with introduction and commentary by W. D. Ross (Oxford: Oxford University Press, 1965).

1. Assignment κατὰ παντός: Aristotle characterised this kind of assignment negatively: κατὰ παντός μὲν οὖν τοῦτο λέγω ὃ ἂν ᾖ μὴ ἐπὶ τινὸς μὲν τινὸς δὲ μή, μηδὲ ποτὲ μὲν ποτὲ δὲ μή (73a28-29). This kind of assignment seems to correspond to universal quantification as ordinarily understood, since it is assignment of the predicate to all instances of the subject and at all times. It should be noted, however, that this reference to omnitemporality on Aristotle's part is not connected by him to any reference to necessity, even though the distinction drawn among three kinds of predicate assignment is designed to shed light on the nature of premisses for demonstration; and demonstration does, according to Aristotle, proceed ἐξ ἀναγκαίων (73a24). Indeed, at 74a4-12, Aristotle states flatly that only assignment καθόλου suffices for demonstrative premisses. This seems to suggest that assignment κατὰ παντός — universal quantification — is less than Aristotle required for demonstrative premisses.

2. Assignment καθ'αὑτό: After having given several examples of this kind of assignment ([12]), Aristotle proposes the following definition: τὰ ἄρα λεγόμενα ἐπὶ τῶν ἁπλῶς ἐπιστητῶν καθ'αὑτὰ οὕτως ὡς ἐνυπάρχειν τοῖς κατηγορουμένοις ἢ ἐνυπάρχεσθαι δι'αὑτά τέ ἐστι καὶ ἐξ ἀνάγκης (73b16-18). This definition says that one assigns a predicate καθ'αὑτό in virtue of what a subject is and of what it is of necessity. Aristotle goes on to point out that it is formally not possible (οὐ ... ἐνδέχεται) for any predicate assigned καθ'αὑτό, or for one of any pair of opposite predicates so assigned, not to belong to its subject. He then derives the necessity of assignment καθ'αὑτό from premisses given previously and the law of excluded middle: ὥστ'εἰ ἀνάγκη φάναι ἢ ἀποφάναι, ἀνάγκη καὶ τὰ καθ'αὑτὰ ὑπάρχειν (73b23-24). In this discussion, unlike the discussion of assignment κατὰ παντός, clear reference is made to necessity, but no reference is made to quantity or quantification; assignment καθ'αὑτό is distinguished from assignment κατὰ παντός, if not contrasted with it.

([12]) The first two examples are the most important for Aristotle's purposes (Cf. Ross, *APPA*, pp. 519, 521). They are (1) definitions or elements of definitions of subjects, and (2) attributes definable by reference to their subjects.

3. Assignment καθόλου: In *Analytica Posteriora* 73b25-74a3, Aristotle stipulates a technical sense of καθόλου, which led Ross to note: "This strict sense of καθόλου is, perhaps, found nowhere else" in the Aristotelian *corpus* ([13]). To be assigned in this technical sense, a predicate must be both κατὰ παντός and καθ' αὑτό. Further, in Aristotle's definition — καθόλου δὲ λέγω ὃ ἂν κατὰ παντός τε ὑπάρχῃ καὶ καθ'αὑτὸ καὶ ᾗ αὐτό (73b26-27) — a third condition is added. Ross takes Aristotle's ᾗ αὐτό to signify that the predicate assigned "Must be true of the subject ... precisely as being itself, not as being a species of a certain genus", if it is to be assigned καθόλου. The fact that one finds in b28 the remark τὸ καθ'αὑτὸ καὶ ᾗ αὐτὸ ταὐτόν is no cause for difficulty. Ross rightly observes that this is to be explained on the grounds that Aristotle here was making more precise his previous remarks on predicate assignment καθ'αὑτό ([14]). This precision of terminology should be borne in mind in reading Aristotle's succeeding chapter, in which the moral is drawn that a demonstration may fail if predicates are not assigned καθόλου — that is to say, both κατὰ παντός and καθ'αὑτό — in its premisses and in its conclusion.

Now it still might be argued that these distinctions which Aristotle makes do not damage Łukasiewicz's contention, because, as everyone knows, Aristotle's requirements for demonstration are stricter than his requirements for mere syllogism. It might be urged that all Łukasiewicz has claimed here is that for Aristotle all valid moods of the syllogism represent necessary inferences, which is true. Let's have another look, however, at the way Łukasiewicz states his claim.

"...Aristotle [he writes] uses the sign of necessity in the consequent of a true implication in order to emphasize that the implication is true for all values of variables occurring in the implication. We may therefore say 'If A belongs to some B, it is necessary that B should belong to some A', because it is true that 'For all A and for all B, if A belongs to some B, then B belongs to some A'. But we cannot say 'If A does not belong to some B,

(13) *APPA*, p. 523.
(14) *APPA*, *ad loc.*

it is necessary that B should not belong to some A', because it is not true that 'For all A and for all B, if A does not belong to some B, then B does not belong to some A'. There exist, as we have seen, values for A and B that verify the antecedent of the last implication, but do not verify its consequent. In modern formal logic expressions like 'for all A' or 'for all B', where A and B are variables, are called universal quantifiers. The Aristotelian sign of syllogistic necessity represents a universal quantifier and may be omitted, since a universal quantifier may be omitted when it stands at the head of a true formula" ([15]). It can be shown that Łukasiewicz's contention, as stated above, is subject to a number of difficulties. First, there is the difficulty about what Łukasiewicz means by 'variables' in 'where A and B are variables'. Capital A, B, C in this passage apparently correspond to the lower case a, b, c, ... of Chapter IV, "Aristotle's System in Symbolic Form", in which Łukasiewicz avows the purpose of setting out "the system of non-modal syllogisms according to the requirements of modern formal logic, but in close connexion with the ideas set forth by Aristotle himself" ([16]). In that chapter, he remarks:

"By the initial letters of the alphabet *a, b, c, d, ...*, I denote term-variables of the Aristotelian logic. These term-variables have as values universal terms, as 'man' or 'animal' ([17])."

The ordinary propositional variables p, q, r, ... also appear in this chapter. They do not belong to Łukasiewicz's account of syllogistic, however, since he believes that Aristotle did not use them ([18]); they are introduced only for the sake of comparing syllogistic schemata with the schemata of propositional logic. The term-variables a, b, c, ... apparently are themselves not used as propositional variables either. Instead, in combination with one another and as arguments to the logical constants of the system ([19]), they are used to form what Łukasiewicz calls "pro-

([15]) *Aristotle's Syllogistic,* p. 11.
([16]) *Aristotle's Syllogistic,* p. 77.
([17]) *Aristotle's Syllogistic,* p. 77.
([18]) *Aristotle's Syllogistic,* § 16.
([19]) *Aristotle's Syllogistic,* p. 77. Łukasiewicz's logical constants are A, E, I, O, used, as he says, in their mediaeval sense.

positional functions". His example of a propositional function is 'Aab', to read 'All a is b' or 'b belongs to all a' ([20]).

Łukasiewicz chooses to employ constants with a quantificational sense instead of the quantifiers of the ordinary predicate calculus; but what he offers can be mapped over onto the ordinary predicate calculus according to a suggestion of Prior's. In his exposition of Łukasiewicz's axiomatisation of syllogistic, Prior instructs the reader to "regard the terms 'a', 'b', etc., as abbreviations for the predicational functions 'φx', 'ψx', etc., 'A' as an abbreviation for '$\Pi x C$', and 'I' as an abbreviation for '$\Sigma x K$' ([21]).

If, following Prior, we regard Łukasiewicz's term-variables as abbreviations for predicational functions and his constants as abbreviations for quantifiers in combination with truth-functional operators, then the role of both quantifiers and variables in Łukasiewicz's account is clarified. Prior gives an example of his understanding of Łukasiewicz's reconstruction of syllogistic schemata in the following symbolisation of Datisi:

$$CK(\Pi x C \psi x \vartheta x)(\Sigma x K \psi x \varphi x)(\Sigma x K \varphi x \vartheta x)$$ ([22]).

Here the quantifiers range over individual variables. The same is true in the symbolisation employed by Anderson and Johnstone, who give for Darii, for example:

$$\Pi x[M(x) \supset P(x)],\ \Sigma x[S(x) \ \&\ M(x)] /_{**}^{*}\ \Sigma x[S(x) \ \&\ P(x)]$$ ([23]).

Assuming that Łukasiewicz is speaking of universal quantifiers ranging over the individual variables which are arguments in predicational functions, the question remains: does he mean to say that the formulae of syllogistic schemata are true for all values of their variables, or does he mean to say that syllogistic schemata are valid under any interpretation of their variables ?

If Łukasiewicz means to say that the formulae of syllogistic

([20]) *Aristotle's Syllogistic*, pp. 77-78.

([21]) *Formal Logic*, p. 121.

([22]) *Formal Logic*, p. 121.

[23] John M. ANDERSON and Henry W. JOHNSTONE, Jr., *Natural Deduction: The Logical Basis of Axiom Systems* (Belmont: Wadsworth Publishing Company, Inc., 1962), p. 177.

schemata are true for all values of their variables, then, since he regards Aristotelian syllogisms as implications, he must intend that a syllogism be represented

$$(x)\ [\text{———} \cdot \text{———} \supset \text{———}],$$

or

$$([\text{———} \cdot \text{———} \supset_x \text{———}],$$

as it is sometimes written, or with the quantifier omitted but understood, as Łukasiewicz proposes — where the first two blanks are filled with the premisses of the syllogism and the third blank is filled with the conclusion. However, in the standard predicate calculus of first order, an entire implication governed by a universal quantifier implies an implication with each of its members governed by a universal quantifier — as in Church's

$$*333.\ \vdash A \supset_a B \supset .\ (a)A \supset (a)B$$ (24).

It can readily be seen, upon inspecting the formulation of Prior or of Anderson and Johnstone, that, if this is Łukasiewicz's account of syllogistic, it will not do. The premisses of syllogistic schemata need not all be universally quantified. The valid moods with particular premisses, e.g. Datisi and Darii, illustrated above with existential quantifiers, have the same syllogistic necessity as Barbara and Celarent (25).

If, on the other hand, Łukasiewicz means to say that syllogistic schemata are valid under any interpretation of their variables, then his use of the universal quantifier in this connexion is precipitate, since quantifiers come into play only when some interpretation already has been decided upon. But even granting his precipitate usage, his reading cannot be allowed, since Aristotelian syllogistic schemata are not valid syllogisms under every interpretation of their variables. A schema yields a valid syllogism for Aristotle only if these two conditions are satisfied: (1) the conclusion is something different from the

(24) A. Church, *Introduction to Mathematical Logic* (Princeton: Princeton University Press, 1956), I, 187.

(25) *An. Pr.* 47a31-33.

premisses, and (2) the terms of the conclusion are related by a middle term occurring in both the premisses. If the same sentence, for instance, were substituted for all the predicational function variables in one of the syllogistic schemata symbolised above, the result might be a valid inference, but it would not be an Aristotelian syllogism, because it would not satisfy either condition. For example, in the implication

if A is predicated of all A, or $(x)(Ax \supset Ax)$
and A is predicated of all A, or $(x)(Ax \supset Ax)$
then A is predicated of all A, or $(x)(Ax \supset Ax)$,

there is a valid inference, but the inference is warranted rather by conjunction elimination or simplification — depending on what sort of logic one prefers — than by any syllogistic device.

Thus, however Łukasiewicz's statements are construed, neither are they true of Aristotle's doctrine of syllogism nor do they show that "the Aristotelian sign of syllogistic necessity represents a universal quantifier".

Hintikka's contention that Aristotle "explicitly or tacitly equates possibility with sometime truth and necessity with omnitemporal truth" appears to call for rejection also, but on different grounds. In the first place, Hintikka's declaration is inconsistent with an important feature of Aristotle's doctrine which Hintikka himself takes note of. As Hintikka puts it, this is Aristotle's distinction "between saying on one hand that something is of necessity *when it is* and on the other hand that it is of necessity *haplôs*" [26]. According to Hintikka, Aristotle intends to contrast necessary statements having temporal qualifications with necessary statements lacking "temporal qualifications that would limit the scope of a statement to some particular moment or interval of time" [27].

The inconsistency arises on two counts: (1) since Aristotle recognises necessary statements which have temporal qualifications that limit their scope to some particular moment or interval of time, then it cannot be the case that he "equates... necessity with omnitemporal truth"; and (2) since Aristotle does re-

[26] "Sea Fight", p. 473; cf. *De Int.* 19a23-27, which Hintikka cites.
[27] "Sea Fight", pp. 473-474.

cognise temporally qualified necessary statements, then these statements do not differ from merely possible statements if Aristotle "equates possibility with sometime truth".

In the second place, Hintikka's contention that the adverb ἁπλῶς "is often used by Aristotle to indicate the absence of *temporal* qualifications", which Hintikka uses as evidence for the contention that Aristotle equates necessity with omnitemporal truth, appears ill-founded where it might be pertinent. Hintikka cites nine places in the *corpus* in support of his contention ([28]). Of these, (1) *Analytica Priora* 34b7-11, while it indeed contrasts κατὰ χρόνον with ἁπλῶς, contains no mention of necessity. The same is true of (2) *De Interpretatione* 16a18. In the same work, Hintikka's citation (3) — 23a16 — contains no time-expression, although ἤδη occurs in a14. (4) *Analytica Priora* 30b31-40 is about not periods of time but rather τούτων ὄντων (b33), and appears to be a consideration of reference failure. In (5) — *Analytica Priora* 34b17-18 — there is no more than a summary of what 34b7-11, cited above, has to say about the premisses of syllogism.

Hintikka's (6) — *Topica* 102a25-26 — contrasts ἁπλῶς not only with ποτέ but also with πρός τι. Part of a discussion of the difference between mind and desire, (7) *De Anima* 433b9 contrasts ἤδη with ἁπλῶς, but with respect to pleasure, not to necessity. In (8) — *De Partibus Animalium* 639b25 — the contrast of ἁπλῶς is with ἐξ ὑποθέσεως; the contrast of τοῖς ἀϊδίοις and τοῖς ἐν γενέσει πᾶσιν is that between the subjects to which something might belong, respectively, ἁπλῶς or ἐξ ὑποθέσεως. Hintikka's last text, (9) *Metaphysica* 1015b11-14, contains no time expression, but rather a contrast of τὸ ἁπλοῦν with ἄλλως καὶ ἄλλως.

Of Hintikka's nine citations, then, only four contain unambiguous contrasts of ἁπλῶς with some time-expression; and none of these deals with necessity. Further, in the entry in Bonitz which Hintikka cites, contrasts of ἁπλῶς with time-expressions are far outnumbered by contrasts of ἁπλῶς with other adverbial expressions ([29]). When all of this is brought to bear on Hintik-

([28]) "Sea Fight", p. 474.

ka's original assertion, what are the results ? Hintikka declares that "in contexts comparable to the one we have here [*De Interpretatione* 19a23-27], *haplôs* is often used by Aristotle to indicate the absence of *temporal* qualifications that would limit the scope of a statement to some particular moment or interval of time". But, in the first place, this alleged usage of Aristotle's cannot, on the evidence of the texts adduced by Hintikka, be used in support of the contention that Aristotle equates necessity with omnitemporal truth, since none of the texts contains both a contrast of ἁπλῶς with some time-expression and a mention of necessity. Next, if it is the case that Aristotle "often" uses ἁπλῶς to indicate the absence of temporal qualifications, it is also the case, on the evidence of Bonitz, that he more often uses ἁπλῶς to indicate the absence of qualifications other than temporal ones. Last, if by "contexts comparable to the one we have here" Hintikka means contexts in which ἁπλῶς is opposed to some qualification or other, then his contention regarding Aristotle's preoccupation with temporality falls once again to the argument from Bonitz. And if, on the other hand, a comparable context is one in which ἁπλῶς is opposed to a time-expression, then his contention obviously cannot be gainsaid — since it amounts to "Where Aristotle contrasts ἁπλῶς with a temporal qualification, there Aristotle contrasts ἁπλῶς with a temporal qualification"; but just as obviously it furnishes no support to his contention and as little aid to the student of Aristotle.

These considerations aside, Hintikka's contention still falls in light of Aristotle's distinctions in *Analytica Posteriora* 73a21-74a3. Hintikka says, in a note on what he takes to be Aristotle's equation of ἀεί and ἀνάγκη, that "for Aristotle a genuinely universal sentence refers to all the individuals existing at different moments of time (*An. Pr.* I, 15, 34b6ff). Hence if it is true once, it is true always, and therefore necessarily true according to the Aristotelian assumptions" ([30]). But here Hintikka's "genuinely universal sentence" corresponds exactly to assignment

([29]) "Sea Fight", p. 473, n. 16. Cf. BONITZ, 76a61-77b9.

([30]) "Sea Fight", p. 482, n. 26. Cf. "Necessity, Universality, and Time in Aristotle", pp. 66-67.

κατὰ παντός which, it will be remembered, is both μὴ ἐπὶ τινὸς μὲν τινὸς δὲ μή and μηδὲ ποτὲ μὲν ποτὲ δὲ μή (73a28-29) — assignment to all instances of the subject and at all times. Assignment κατὰ παντός is not necessary assignment, whereas assignment καθ'αὑτό is necessary, and assignment καθόλου — the only "genuinely universal" assignment according to the *Posteriora* text — is also necessary, because it includes assignment καθ'αὑτό, along with assignment κατὰ παντός.

The interpretive validity of Hintikka's stand thus may be called into question, since Aristotle gives no hint that necessity can be reduced to an ordinary quantifier notion — his assignment κατὰ παντός. Rather, Aristotle distinguishes from assignment κατὰ παντός a notion of necessity — assignment καθ'αὑτό — which seems not only to be independent of assignment κατὰ παντός, since the two are required for assignment καθόλου, but which seems to be independent of quantificational considerations altogether.

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