LOGICAL AND EMPIRICAL PROBABILITY

A critical supplement on Professor's Ayer's paper "Induction and the Calculus of Probabilities"

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The problem investigated by Professor Ayer concerns the question, whether we are allowed to infer from apriori suppositions of the calculus of chances or of the logical theory of probability some statements about reality, especially some general statements which have the character of natural laws. The attemps to derive generalizations about natural phenomena from the principles of the calculus of chances make use, as shown by Professor Ayer, of the socalled "Law of Large Numbers". This "Law" is a purely mathematical principle and within the mathematical theory of chances it is logically justified to conclude that a given property, if it occurs with the frequency m/n in a large sample, will occur with a high probability with approximately the same frequency in the parent population. But here "probability" is nothing else than a relation between the extensions of defined concepts and thereby the question arises how is it possible to induce from a system of formal relations regularities of empirical phenomena ? Ayer analyzes examples of such derivations as given in the theories of Sir Roy Harrod and Donald Williams. He comes to the conclusion that the calculations in themselves are correct. Surely we can logically derive from the supposed relations of sets that the majority of possible samples match the population. But that does not imply anything, as Professor Ayer pointed out, about the probability that the samples which we actually obtain belong to this majority. By analyzing his examples Ayer shows that we need to make two rather large empirical assumptions, in order to achieve statements about the probability of empirical events by application of the calculus of chances.

The second of the two assumptions is nothing else than the postulation of the uniformity of nature in a rather extensive degree. And we can not avoid this supposition, as Ayer demonstrates, even if we confine our examples to relatively narrow realms of the space-time continuum. So Ayer is right, I believe, maintaining that the attempt to justify induction on the basis of the calculus of chances fails and is fallacious. The derivation can be reached only by using empirical principles which are problematic in their content and validity. It is even hard to find a satisfactory formulation for them.

An other attempt to justify inductive generalizations in empirical science by apriori logical elements is Carnap's theory of "initial probabilities". Carnap's suppositions are discussed by Professor Ayer from the point of view whether the derivation (by which Carnap's tries to show that the stipulations of this theory are a guide of our empirical expectations) does not make use of tacit presuppositions. "Initial probability" is, as defined by Carnap, the probability given to a statement by a tautology. That is, however, a purely formal assertion, as Ayer rightly comments. The further assumption of Carnap that equal initial measures of probability are to be assigned to the socalled "structural descriptions" implies the supposition that it is equally probable, that any given individual has any given primitive property. Applied to empirical reality, this supposition means (as I believe Ayer is right in maintaining) "that whatever the proportion in which a primitive property is distributed among the total population, we are safe in assuming that it is distributed in roughly the same proportion in any reasonably large sample". But this again means nothing else than the assumption of the uniformity of nature, so that Carnap's derivation of the inductive generalization in empirical science from the apriori theses of his inductive logic, implies tacit empirical assumptions too.

These results of Ayer's investigation lead to the general question, what is the reason for the failure of the attempts to derive empirical induction and inductive generalization from logical systems of probability ? None of derivations discussed by Ayer can avoid empirical assumptions, especially the assumption of the uniformity of nature, which contains even the thesis to be derived. We have to ask here, has this failure of the derivations the character of logical necessity, or is it just that the special examples investigated by Professor Ayer contain mistakes which could possibility be corrected ? I think in relation to these questions the explanations of Professor Ayer should be supplemented.

The belief that inductive generalizations as practised in empirical science can be justified by the calculus of chances or by the apriori principles of inductive logic is a result, in my opinion, of the confusion of two different meanings of "probability" or of the statements about probability respectively. There are many possibilities of construing axiom-systems of inductive logic. These systems will contain the traditional calculus of chances and the modern alterations of this calculus too. From the epistemological point of view such systems are nothing else than systems of analytical relations between the formal extensions of selected and defined concepts. They can be represented in the form of special set-theories or in the form of possible mathematical extra- and interpolations. All the modern systems of inductive logic are systems of special forms of mathematical extra- and interpolation. These conclusions often used in mathematics are not deductive derivations, but the prototypes of logical induction. It is, however, logically impossible to derive any statements about reality from analytically selected and stipulated relations. The concept of "logical probability" as used in inductive logic and in the calculi of chances expresses analytically constructed and stipulated relations between the extensions and parts of the extensions of defined classes, sets, or concepts in general. This "probability" can be represented also as relations between the statements of the constructed language systems concerned. But from these analytical constructions nothing can be derived about the statistical order of real phenomena. It is an error to believe that the empirical probability of events, as it is recognized by observation, by statistical counting, could be derived from the stipulated formal relations of a system of inductive logic, on the ground that it is possible to apply the formulae and the rules of logical or mathematical systems of chances to the empirical probability values. We are concerned here with two concepts of probability and it is a matter of principle that it is not possible to derive from the suppositions, the selected and stipulated axioms and rules of any analytically constructed system, its validity for reality.

The situation resulting from the distinction of logical and empirical probability can be illustrated by comparison with an analogous example wellknown in natural philosophy. The opinion that geometry describes the "space", in which all events happen, had to be dropped when the possibility of different geometrical systems incompatible with each other was recognized. Epistemological analysis showed that the geometrical systems of mathematics are purely analytical constructions and that it is therefore impossible to derive from the relation system of any of the mathematical geometries that it must be valid for the physical space. The description of the geometrical metric of the real space is attained in physics by measuring observation. Besides this empirical method it is possible to fix a geometrical metric for the physical space by convention and then to investigate by observation whether the objects and events correspond in their spatial order to the metric selected by convention. Both methods show that the geometry of the real space expresses a specific order of empirical phenomena and can be controlled by the methods of empirical investigation. Consequently we have to discriminate the analytic concept of mathematical spaces and the empirical concept of physical space.

Transfering this discrimination to the problem of probability we are now in a position to specify the meaning of the concepts of probability. As mentioned above "probability" as defined in inductive logic and the calculus of chances means nothing else than formal syntactical relations between the analytically constructed extensions of concepts or the corresponding statements respectively. In opposition to that, "probability" in empirical description characterizes an objective order of events which can be expressed in form of natural laws. The type of the order of phenomena described by probability laws in physics is formally different from the form of the order described by the socalled causality-laws, e.g. the "proximity-effect-laws". The latter ones maintain univocal, in their strict form, one-to-one relations between the empirical states, whereas probability-laws describe non-univocal (one-to-many, many-to-one, many-to-many) relations between the phenomena. Both types of order can be expressed by mathematical functions. From the epistemological point of view, probability-laws are therefore natural-laws of the same kind as the strict causality-laws. And the values of "empirical probability" ascribed to the phenomena are predicates of the same kind as the value of length, duration, mass, weight, energy and so on. They can be controlled equally by methods of observation.

Applying the calculus of chances or the formulae of inductive logic to real phenomena it is of course possible to join the analytical relations to certain empirical predicates. Thereby it is possible to treat the probabilities (empirically ascribed to the real states) according to the rules of logical or mathematical systems of chances. But in every case the validity of the (non-logical) statements about the probability of real events obtained by this method can be controlled and defined only by empirical probation. The application of logical and mathematical theories of probability in empirical description does not entitle us to abolish the duality of the two probabilities and to unite the different meanings of the two concepts in one notion of probability. If we do so, as is done when trying to derive the principle of the uniformity of nature from formal premises apriori, then we make a logical fault, called "equivocation", and the result is a false conclusion. That, I believe, was explained in an impressive manner by the examples discussed by Professor Ayer.

I think it must be mentioned, in order to complete our investigations, that Carnap has distinguished between two concepts of probability in a certain analogy to our logical and empirical probability. He calls them "probability$_1$" and probability$_2$". "Probability$_1$" he defines by the same criteria as we characterize the "logical probability". By "probability$_1$" we have to understand analytical relations between the extensions of the selected and defined concepts, sets, classes, or the corresponding statements respectively. Inductive logic concerns "probability$_1$" alone. "Probability$_2$" is called by Carnap "statistical probability" also, but he characterizes it by criteria which do not agree with the meaning of our "empirical probability". If series of average frequencies have the tendency to converge to limit values, then these convergent sequencies or their limit values, respectively, shall be called "probability$_2$", i.e. "statistical probability". At this point it must be objected that convergent series of average frequencies are defined by mathematical functions in the same manner as any other mathematical metric of probability. Therefore, according to the definition of "probability$_2$" given by Carnap, this concept is again an analytical relation between constructed extensions, and is only a specific form of "probability$_1$". In opposition to that our "empirical probability" is not a formal relation between constructed concepts but characterizes an order of real phenomena and can be controlled by empirical observation.

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