APPLYING RULES

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All rules are governed by the Principle of Rules, which can be stated as,

(1) If situation S is of type T, then rule R applies to S.

Moral, legal, administrative rules and others fall under this principle, but I shall here develop only its implications for rules of logic and formal systems. Whenever any rule is used, an argument form is presupposed in which the major premiss is (1), the minor premiss is (2), and the conclusion is (3), below.

(2) Situation S_1 is of type T. (Therefore:)

(3) Rule R applies to S_1.

The need for judgments like (2) is the basis of my criticism of the claim, made by Haskell Curry [1] and others, that intuitive deduction can always be "replaced" by derivation according to formal rules. Curry is right in speaking of the "refinement" and "improvement" of intuition resulting from formalization, but wrong to claim that intuitions are not "ultimate," at least in ways I shall show. Perhaps we can replace by a formal rule any particular intuitive judgment which we can specify in advance. I shall not even deny that numerous judgments like (2) above might be so replaced. To give a concrete meaning to "replaced," let us go further and entertain favorably the speculation that a machine can be designed and programmed to make many judgments like (2). This would annex whole realms to the corpus of logical and mathematical transformations already formalized.

If I admit that machines might be able to "decide" whether a given rule applies to a presented situation, how can I then regard this type of decision-making as requiring intuition ? My answer is that the capability I have conceded to machines is far

[1] Haskell R. CURRY, "Purposes of Logical Formalization", *Logique et Analyse*, Vol. 11, Sep., 1968, pp. 357-66.

from embracing all that is signified by (2). Let us see this for ourselves by means of a few illustrations.

The reader, aided by his intuition or understanding, will be able to see which of the items below are "situations" subject to the rule Modus Tollens. That is, the reader can make judgments of the form (2), where S is an item on the list, T is the logical form of the premisses of Modus Tollens, and where R is the Modus Tollens itself. But how far down this list will the *machine's* capability extend ?

(4) If A then B. Not B.
(5) If B then A. Not A.
(6) If A then B. Not A.
(7) Not C. If D then not C.
(8) Not X. But X if Y.
(9) —(p & q). If r then —(p & q).
(10) $-P_3$. If $-(P_1 \& P_2)$ then P_3.
(11) $-(b \rightarrow a)$. $-(c \& d) \rightarrow (b \rightarrow a)$.
(12) (r or s) $\rightarrow$ —q. —(r or s).
(13) Not both X and Y. But if Z or W then Y and X.
(14) Neither K nor L if M. Not K and not L.
(15) President Kennedy was shot from behind, while the Warren Report implies that he was not.
(16) Einstein's theory implies a red shift in star spectra, which is in fact observed (2).

Are any of the illustrations "unfair" to the machine, or to those who claim that all intuition can be formalized ? Perhaps we should say that varying the alphabet and the typographical fonts was fair play, since machines can be taught several alphabets, but mixing up the negatives and switching the order of the premisses and varying idioms were uncalled for, since a certain amount of standardization is indispensable to any formalization or mechanization. My reply would be that the illustrations of Modus Tollens contain no variation, that is, no variation of *logical* form. The instructions were to judge the logical form,

(2) The fallacy of affirming the consequent is committed by items 7, 9, 14, 16. Items 6, 12, deny the antecedent. Items 4, 5, 8, 10, 11, 13, and 15 yield valid conclusions by Modus Tollens.

not the typographical form, and if the machine or formal system is incompetent to handle *any* of the illustrations, this is because logical form is something to be understood, comprehended, rather than merely checked, card-punched, or pigeon-holed.

Formality is essentially a matter of typography. To the degree that something remains logical, it has not been completely formalized. That this is the current concept of formality among logicians is easily documented. Thus, Quine: "Now all these characterizations are *formal,* in that they speak only of the typographical constitution of the expressions in question and do not refer to the meanings of those expressions" ([3]). And Tarski (discussing formalization): "... the meanings of all expressions are to be disregarded without exception, and we are supposed to behave in the task of constructing a deductive theory as if its sentences were configurations of signs void of any content" ([4]). This absence of meaning and understanding is what makes the analogy so close between formal systems and programmed machines, and makes the question of the machine's capability a fair and interesting test of the replaceability of intuition.

If there are indeed typographical variations of the Modus Tollens situation which the machine or formal system cannot recognize because it lacks understanding of the meanings of the signs and deals only with their typographical shapes, then it would seem that my case has been proved: Understanding or intuition is exactly what is needed; it has not been replaced.

A defender of formalization might claim that the machine's ability has been underestimated. Perhaps there is no theoretical limit to the number of alphabets, type fonts, and other variations which the machine could be taught. In this respect, a machine is actually superior to a printed formal system, since the latter cannot *do* anything or be "taught" to adapt itself to different typographies. My reply to this objection, however, is that the ability of a rational being to invent new forms will always outrun the ability of any mechanical or formal apparatus to absorb these

([3]) W. V. QUINE, *Mathematical Logic,* Rev. ed., New York, Harper & Row, 1962, p. 283.

([4]) Alfred TARSKI, *Introduction to Logic and to the Methodology of tye Deductive Sciences,* New York, Oxford University Press, 1965, p. 133.

inventions, and that in any case, the formal or mechanical apparatus itself is re-adapted or programmed or even created by the rational being by means of his insight. This is a much disputed issue in the discussion of "thinking machines." But it must always be the rational, or let us say the imaginative being, which will comprehend the mechanical, and not the converse. "The light shineth into the darkness; the darkness comprehendeth it not."

A second defense might be that it is absurd to expect a formal system or machine to deal with items that are not in "standard form". To make this defense is incompatible with the previous one, of course. All formal systems do in fact require that their "input" be predigested into standard forms, so this defense amounts to an admission of defeat, since the standard forms themselves can only be created by the rational being capable of imagination and insight. Even after standard forms have been constructed, their use requires further acts of imagination. If the charge is pressed that rational beings are themselves equally handicapped by ignorance of foreign languages, the rebuttal consists in pointing out that they are not helpless in this situation, as are machines and systems. Without outside help they can, through insight and initiative, learn any foreign language they wish.

If some readers feel I owe them some account of what I mean by "rational being", I can here offer only the following partial payment of the debt. First, I appeal to the existence of sciences, philosophies, critical theories of art, etc. Next, I demand not that a being be able to create or to contribute to such enterprises, but only that he be able to understand them. I will accept any dolphin, chimpanzee, computer, or extra-terrestrial creature who meets this qualification. As for how such understanding is to be demonstrated, any of the usual written and oral examinations, in any language, will serve. This criterion by no means excludes children or uncivilized people, since it tests capability, not prior achievement.

There may be those who will deny outright that there is any common form in the illustrations (4)-(16) above. They may take the Wittgensteinian view that such a common form is a myth,

since there is nothing but a family resemblance among the variations. To this I reply, "Typographically yes, logically no." Indeed, even a family resemblance would be difficult to establish among the typographical variations. What is common is *logical,* conceptual form. It is the form of Modus Tollens *itself,* which cannot be given typographically if this were taken as identifying Modus Tollens with any single typographical form. In another sense, however, any one of the typographical forms can be *used* to indicate the logical form, provided the reader possesses understanding or intuition of logical form. But just what is "Modus Tollens itself," then ? It is that valid form of inference in which the denial of the antecedent in an implication is inferred from the denial of the consequent. This is the *concept* of Modus Tollens. But Modus Tollens itself is not to be identified with the particular set of *words* I used, since other words can refer to the same logical form. For example, the principle that any implication having a false consequent has a false antecedent. More briefly, (17) "Whatever implies something false is false." Here the essence of Modus Tollens is expressed by a true statement.

I have shown several points at which intuition or understanding is "ultimate", despite the claim of Haskell Curry that particular instances can usually be formalized. The Principle of Rules shows that before a rule can be applied, judgments of situations are required, and these are often beyond the limits of what has been formalized. Second, the ability of a rational being to perceive a common logical form in variations exceeds and encompasses the ability of a system or machine. Third, intuition or understanding is required to create "standard forms" as well as, fourthly, to translate non-standard materials into the standard forms. Fifth, logical forms exist to be recognized, independently of particular exemplifications in words or symbols.

In conclusion, I shall observe that the Principle of Rules exposes the unavoidable circularity in the foundations of logic. Appeals to formalization will not avail, for there is a "logocentric predicament", as Sheffer called it ([5]). Curry says, "If the

([5]) Henry M. SHEFFER, review of *Principia Mathematica, Isis,* VIII, 1926, p. 228.

subject matter (of formalization) is logic itself, then the idea that we derive the theorems by means of logic is circular. In order to refute the charge that in operating the system we use the very logic we are formulating, we have to pass to full formalization ..." ([6]). But let us suppose that the rule, R, is Modus Ponens, which almost every formal system does in fact employ. Then in order to apply Modus Ponens properly, we must pass through an argument or interpretation having the form of steps (1), (2), (3), above, wherein, lo ! the very form and image of Modus Ponens reappears. This is not an infinite regress like that of Lewis Carroll's Tortoise, but a circle. Formalization cannot break it, but understanding can see across it.

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([6]) *Op. cit.*, p. 360.