LOGIC OF NORMS AND LOGIC OF NORMATIVE PROPOSITIONS (*)

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I. *Introduction*

In this paper I present a logical system which attempts to reconstruct some important and frequently used concepts of obligation, permission, and other so-called normative characters. This system must not be considered as a new deontic logic, *i.e.*, as a logic of norms, but as a system concerning the logic of normative propositions, *i.e.*, propositions to the effect that a norm has been issued.

I use the expression "deontic logic" to identify the logical properties and relations of norms, and "normative logic" to identify the logic of normative propositions. Normative logic presupposes, but is not identical with, deontic logic. I think that many difficulties related to the interpretation and acceptability of several existent systems of deontic logic result from the lack of distinction between them.

In order to show their differences I shall present first in a rather dogmatic way two formal systems: one for deontic logic and another for normative logic. Next I shall provide an intuitive justification for what I call "normative logic" and try to show that it enables us (1) to detect some ambiguities in the use of such terms as "obligatory", "permitted", etc. in ordinary (and especially in legal) discourse; (2) to characterize some important properties of normative systems such as consistency and completeness, which cannot be adequately formulated in deontic logic. (This shows that normative logic may be regarded as the logic of normative systems and as such is especially fit

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to deal with the so called problem of legal "gaps"); (3) to show that whereas the interpretation of iterated operators in deontic logic is highly problematic, iterated modalities in normative logic represent concepts which are frequently used in legal discourse.

The difference between deontic and normative concepts is at least partially concealed by the fact that under certain assumptions (consistency and completeness) both calculi are isomorphic. This fact may explain why most philosophers have not paid sufficient attention to normative logic.

II. *Deontic logic (system -O)*

(1) *Signs*:

(i) An infinite list of *variables*:

$p, q, r, s, p_1, q_1, r_1, s_1, p_2 \ldots$.

(ii) Left and right parenthesis, and the usual propositional signs for negation, conjunction, inclusive disjunction, material implication, and material equivalence.

(,), $\sim$, . ,V, $\supset$, $\equiv$.

(iii) The deontic operator for obligation

O.

(2) *Well-formed-formulas (O-Formulas)*

There are two kinds of O-Formulas:

(a) *Content formulas (C-Formulas)*:

(i) All variables are C-Formulas.

(ii) If α is a C-Formula, $\sim\alpha$ is a C-Formula (1).

(iii) If α and β are C-Formulas, then $(\alpha . \beta)$, $(\alpha \vee \beta)$, $(\alpha \supset \beta)$, and $(\alpha \equiv \beta)$ are C-Formulas.

(b) *D-Formulas (Deontic Formulas)*:

(i) If α is a C-Formula, $O\alpha$ is a D-Formula. (Moreover an atomic D-Formula.

(1) Greek letters are used as metalinguistic variables for formulas. Propositional connectives and deontic operators, as well as parenthesis, are use autonymously.

(ii) If α is a D-Formula, $\sim\alpha$ is a D-Formula.
(iii) If α and β are D-Formulas, then $(\alpha . \beta)$, $(\alpha \vee \beta)$, $(\alpha \supset \beta)$, and $(\alpha \equiv \beta)$ are D-Formulas.

(3) *Axiom Schema*:

Ax-1) : $\vdash O(\alpha \supset \beta)) \supset (O\alpha \supset O\beta)$ (Where α and β are C-Formulas) (2).

(4) *Rules of Inference*:

R-1): If α is a tautology (in propositional logic), then $\vdash \alpha$. (Where α is an O-Formula).
R-2): $\vdash \alpha$, $\vdash (\alpha \supset \beta) \rightarrow \vdash \beta$ (Modus Ponens) (where α and β are both C-Formulas or D-Formulas).
R-3): $\vdash \sim\alpha \rightarrow \vdash \sim O\alpha$ (where α is a C-Formula (*Principle of Deontic Contradiction*).
R-4): $\vdash \alpha \rightarrow \vdash O\alpha$ (where α is a C-Formula) (*Principle of Deontic Tautology*).

(5) *Definitions*:

Permitted: Df.P: $P\alpha = \sim O \sim \alpha$ (where α is a C-Formula).
Prohibited: Df.Ph: $Ph\alpha = O \sim \alpha$ (where α is a C-Formula).
Facultative: Df.F: $F\alpha = (P\alpha . P \sim \alpha)$ (where α is a C-Formula).

In the intended interpretation the variables stand for propositions; propositional connectives are supposed to be understood in their usual meaning. Deontic operators are to be read in the usual way: "It is obligatory (permitted, prohibited) that". The operator "F" may be read as "It is facultative that" or as "It is optional that".

As the System-O is substancially the same as *von Wright*'s well known system [9] — the only important difference being the rule R-4), which is explicitely rejected in [9] by means of what *von Wright* calls The Principle of Deontic Contingency — I shall not develop it further, but only list those theorems which

(2) I use "$\vdash$" as an abbreviation for "is the form of a thesis of the system in question".

I shall use later in order to stress the differences between the System-O and the System-NO.

T-1) $\vdash O(p.q) \equiv (Op.Oq)$
T-2) $\vdash P(pvq) \equiv (PpvPq)$
(Distributive laws)

T-3) $\vdash (Op \supset Pp)$
T-4) $\vdash (Php \supset P \sim p)$
T-5) $\vdash (Op \equiv \sim P \sim p)$
T-6) $\vdash (\sim Op \equiv P \sim p)$
T-7) $\vdash (Pp \equiv \sim O \sim p)$
T-8) $\vdash (\sim Pp \equiv O \sim p)$
T-9) $\vdash (PpvP \sim p)$
T-10) $\vdash \sim (Php.Ph \sim p)$
T-11) $\vdash (PpvPhp)$
(Laws of opposition and subalternation)

T-12) $\vdash (PhpvOpvFp)$
T-13) $\vdash Php \supset (\sim Op. \sim Fp)$
T-14) $\vdash Op \supset (\sim Php. \sim Fp)$
T-15) $\vdash Fp \supset (\sim Op. \sim Php)$
(These last laws show that "O", "Ph" and "F" are jointly exhaustive and mutually exclusive.)

III. *Normative logic (System-NO)*

Normative logic, *i.e.,* the logic of normative propositions, presupposes (= is an extension of) deontic logic, in the same sense in which deontic logic presupposes propositional logic.

The most elementary propositions of normative logic are propositions to the effect that some agent has issued a norm. In order to formalize them it is necessary to introduce a set of variables for agents and a relational expression which correlates the agent with the issued norm. "x" will be provisionally used as a symbol that stands for a specific agent, *i.e.,* it will be used not as a variable but as a parameter.

For the normative relation I introduce the symbol "N" in such a way that "NxOp" means "x has ruled (issued a norm to the effect) that it is obligatory that p".

The System-NO has the following structure:

(1) *Signs*

(i) All the signs of System-O (Deontic Logic),
(ii) The norm operator "Nx".

(2) *Well-formed-formulas (NO-Formulas)*

There are three kinds of NO-Formulas:

(a) *The content formulas — C-Formulas.* They are the C-Formulas of System-O.
(b) *The Deontic formulas — D-Formulas.* They are the D-Formulas of System-O.
(c) *The normative formulas — N-Formulas.*
   (i) If α is a D-Formula, $Nx\alpha$ is an N-Formula (moreover an atomic N-Formula).
   (ii) If α is an N-Formula, $\sim\alpha$ is an N-Formula.
   (iii) If α and β are N-Formulas, then $(\alpha . \beta)$, $(\alpha \vee \beta)$, $(\alpha \supset \beta)$ and $(\alpha \equiv \beta)$ are N-Formulas.

(3) *Axioms schemata*

Ax-1) $\vdash O(\alpha \supset \beta) \supset (O\alpha \supset O\beta)$ (Where α and β are C-Formulas),
AxN-2) $\vdash Nx(\alpha \supset \beta) \supset (Nx\alpha \supset Nx\beta)$ (Where α and β are D-Formulas).

(4) *Rules of Inference*

R-1) If α is a tautology (in propositional logic), then $\vdash \alpha$. (Where α is an NO-Formula.)
R-2) $\vdash \alpha, \vdash (\alpha \supset \beta) \rightarrow \vdash \beta$ (Where α and β are both of them C-Formulas, D-Formulas or N-Formulas) (Modus Ponens).
R-3) $\vdash \sim\alpha \rightarrow \vdash \sim O\alpha$ (Where α is a C-Formula).
R-4) $\vdash \alpha \rightarrow \vdash O\alpha$ (Where α is a C-Formula).
RN-5) $\vdash (\alpha \supset \beta) \rightarrow \vdash (Nx\alpha \supset Nx\beta)$ (Where α and β are D-Formulas).

It is obvious that all theorems and derived rules of System-O remain valid in System-NO. So I shall present next, some characteristic rules for N-Formulas.

(5) *Some Derived Rules*

RN-6) A C-Formula may be substituted for a variable in a N-Formula.

RN-7) N-Formulas may be substituted for all variables of a C-Formula.

RN-8) $\vdash (\alpha \equiv \beta) \rightarrow \vdash (Nx\alpha \equiv Nx\beta)$ (where α and β are D-Formulas) (*Rule of Extensionality for "Nx"*).

(6) *A Theorem-Schema*

TN-1) $\vdash Nx(\alpha \,.\, \beta) \equiv (Nx\alpha \,.\, Nx\beta)$ (where α and β are D-Formulas).

IV. *A method of decision for the System-NO*

In order to decide whether a given formula is a theorem in the System-NO I propose the following method:

Decision rules for N-Formulas (DN-Rules)

DN-1): Construct a truth-table for the considered formula, taking its atomic N-Formulas as the elements for each arbitrary valuation. If the truth-table of the formula shows a "T" for every possible valuation of its atomic constituents, then it is a theorem. If for some possible valuation the truth-table shows an "F" for the whole formula, apply the following elimination rule:

DN-2): Eliminate those valuations in which one or more atomic N-Formulas are valuated "T", and are such that the conjunction of the D-Formulas contained in them immediately after the operator "Nx" propositionally or deontically implies the D-Formula contained immediately after the "Nx" operator in an (at least one) atomic N-Formula which is valuated "F".

If after one or several applications of DN-2) all the valuations that show an "F" in the truth-table are eliminated, then the formula in question is a theorem. But if there is at least one of such valuations which can not be eliminated by DN-2), then and only then is the formula not a theorem.

Clearly, the effectivenes of DN-2 presupposes a method of decision for D-Formulas, i.e., for Deontic Logic, and a method of decision for Propositional Logic. For the latter ordinary truth tables are available. For the former, the following is a decision procedure:

Decision rules for D-Formulas. (*DD-Rules*)

DD-1): Construct a truth-table for the D-Formula in question, taking its atomic D-Formulas as the elements for each arbitrary valuation.

If the truth-table shows a "T" for every possible valuation, then the formula is a theorem.

If for some possible valuation, the truth-table shows an "F" for the whole formula in question apply the following elimination rules:

DD-2): Eliminate those valuations in which one or more atomic D-Formulas are valuated "T", and are such that the conjunction of the C-Formulas contained in them immediately after the operator "O" propositionally implies the C-Formula contained immediately after the "O" operator in an (at least one) atomic D-Formula which is valuated "F".

DD-3): Eliminate those valuations in which one or more atomic D-Formulas are valuated "T", and are such that the conjunction of the C-Formulas contained in them immediately after the operator "O" is propositionaly invalid (a contradiction).

DD-4): Eliminate those valuations in which there is an atomic D-Formula valuated "F", an is such that the C-Formula contained in it immediately after the operator "O" is propositionally valid (a tautology).

If after one or several applications of DD-2, DD-3, and DD-4, all those valuations that give rise to an "F" are eliminated, then the formula is a theorem. But if there is at least one such valuation which cannot be eliminated by any of the D-elimination rules, then and only then the formula is not a theorem.

V. *Normative operators — Strong and weak ones*

In many contexts "p is permitted" is not uttered with the intention of issuing a norm permitting that p but only with the intention of informing that someone else has issued such a norm, that a definite and identified agent has permitted that p. In many circumstances, as is frequently the case among lawyers, to say that p is permitted is just to say that a normative proposition is true. In such situations, "p is permitted" means exactly the same as "NxPp". Of course, it would be clearer and more informative to say "x has permitted p", but very often, the reference to x is omitted because in the context x and its identification are too obvious. So they are presupposed and it is just said "p is permitted".

Exactly the same happens with "p is forbidden", "p is obligatory", and with other normative expressions.

In order to formalize such meanings of normative expressions I shall introduce the following definitions:

Strong Permission; Df. $\mathbb{P}s$: $\mathbb{P}s$ = NxP

Strong Obligation; Df. $\mathbb{O}s$: $\mathbb{O}s$ = NxO.

The natural reading of "$\mathbb{P}sp$" is "It is permitted that p" in the same way as "Pp", but, of course, they have different meanings, as is clear from their definitions.

Analogously "$\mathbb{O}sp$" is read "It is obligatory that p", in the same way as "Op".

These definitions may be thought objectionable because in the *definiens* seems to contain a free variable ("x") which does not appear in the *definiendum*. But it must be remembered that "x" is not variable there, but a parameter.

Sometimes it may be interesting to consider "x" as a variable. In such cases the definitions must be changed by the following ones:

$$\mathbb{P}s_x = NxP,$$
$$\mathbb{O}s_x = NxO,$$

where "$\mathbb{P}s_xp$" means x has permitted that p, and "$\mathbb{O}s_xp$" means x has commanded that p.

Sometimes "p is permitted" is not used with the intention of informing that someone else has permitted that p, *i.e.*, has issued a norm permitting that p, but of informing that the agent in question has not forbidden that p, *i.e.*, that it is false that the agent has issued a norm to the effect that it is prohibited that p (or, what is the same, a norm to the effect that it is obligatory that not-p).

In such cases "It is permitted that p" means the same as "$\sim NxO \sim p$". I shall call this *the weak sense of Permission.*

By analogy I shall introduce an operator called *Weak Obligation.*

In symbols:

Weak Permission: Df. $\mathbb{P}w = \sim NxO \sim$

Weak Obligation: Df. $\mathbb{O}w = \sim NxP \sim$

"$\mathbb{P}wp$" expresses another sense of "It is permitted that p", (a sense different from the sense of "$\mathbb{P}sp$" and "Pp"), so that it may be read in the same way as the other two.

The case is different with "$\mathbb{O}wp$". I do not think that it can be read "It is obligatory that p"; this last sentence is sometimes used with the meaning of "$\mathbb{O}sp$", sometimes with the meanings of "Op", and sometimes, probably, with other meanings different from these two; but I do not think it is ever used with the meaning of "$\mathbb{O}wp$".

The following theorems are the immediate consequences of the definitions:

TN-2) $\vdash NxPp \equiv \mathbb{P}sp$

TN-3) $\vdash NxOp \equiv \mathbb{O}sp$

TN-4) $\vdash (\sim NxO \sim p) \equiv \mathbb{P}wp$

TN-5) $\vdash (\sim NxP \sim p) \equiv \mathbb{O}wp$

Some *Laws of opposition* in Normative and Deontic Logic

Normative Logic	*Deontic Logic*
TN-6) $\vdash \mathbb{P}sp \equiv \sim \mathbb{O}w \sim p$ TN-7) $\vdash \mathbb{P}wp \equiv \sim \mathbb{O}s \sim p$	T-7) $\vdash Pp \equiv \sim O \sim p$
TN-8) $\vdash \mathbb{O}sp \equiv \sim \mathbb{P}w \sim p$ TN-9) $\vdash \mathbb{O}wp \equiv \sim \mathbb{P}s \sim p$	T-5) $\vdash Op \equiv \sim P \sim p$
TN-10) $\vdash \sim \mathbb{P}sp \equiv \mathbb{O}w \sim p$ TN-11) $\vdash \sim \mathbb{P}wp \equiv \mathbb{O}s \sim p$	T-8) $\vdash \sim Pp \equiv O \sim p$
TN-12) $\vdash \sim \mathbb{O}sp \equiv \mathbb{P}w \sim p$ TN-13) $\vdash \sim \mathbb{O}wp \equiv \mathbb{P}s \sim p$	T-6) $\vdash \sim Op \equiv P \sim p$

The proofs of these theorems are obvious, so I shall omit them. It must be noticed that, for example, the proof of TN-6) is nothing but the proof of:

$$\vdash NxPp \equiv (\sim \sim Nxp \sim \sim p).$$

It is important to observe the form in which negation affects the normative concepts.

The negation of a strong concept (Permission-Obligation) is a weak concept (Obligation-Permission), and the negation of a weak concept (Permission-Obligation) is a strong concept (Obligation-Permission). Each strong concept affirms the existence of a normative act (the act of issuing a norm) and each weak concept denies the existence of a normative act.

Each strong concept is equivalent to the opposite weak concept with a negation before and after it. (Opposite means here: for Permission, Obligation, and vice-versa).

Each weak concept is equivalent to the opposite concept with a negation before and after it.

V. *Strong (internal) negation*

With respect to the above normative concepts it is possible to introduce a different, stronger form of negation, that may be called *internal negation*. For symbolization I shall use the sign sometimes used for intuitionistic negation "$\urcorner$" but with a different meaning.

The strong (Internal) negation of a normative concept represents the operation of introducing a sign of ordinary negation ("$\sim$" after "Nx" and before the deontic operator, i.e.:

Strong negation of Strong Permission; Df: $\urcorner\mathbb{P}s : \urcorner\mathbb{P}s = Nx \sim P$
" " " " Obligation; Df: $\urcorner\mathbb{O}s : \urcorner\mathbb{O}s = Nx \sim O$
" " " Weak Permission; Df: $\urcorner\mathbb{P}w : \urcorner\mathbb{P}w = \sim Nx \sim O \sim$
" " " " Obligation; Df: $\urcorner\mathbb{O}w : \urcorner\mathbb{O}w = \sim Nx \sim P \sim$

Under these definitions the following laws of opposition hold:

Normative Logic	*Deontic Logic*
TN-14) $\vdash \urcorner \mathbb{P}sp \equiv \mathbb{O}s \sim p$ TN-15) $\vdash \urcorner \mathbb{P}wp \equiv \mathbb{O}w \sim p$	T-8) $\vdash \sim Pp \equiv O \sim p$
TN-16) $\vdash \urcorner \mathbb{O}sp \equiv \mathbb{P}s \sim p$ TN-17) $\vdash \urcorner \mathbb{O}wp \equiv \mathbb{P}w \sim p$	T-6) $\vdash \sim Op \equiv P \sim p$
TN-18) $\vdash \mathbb{P}sp \equiv \urcorner \mathbb{O}s \sim p$ TN-19) $\vdash \mathbb{P}wp \equiv \urcorner \mathbb{O}w \sim p$	T-7) $\vdash Pp \equiv O \sim p$
TN-20) $\vdash \mathbb{O}sp \equiv \urcorner \mathbb{P}s \sim p$ TN-21) $\vdash \mathbb{O}wp \equiv \urcorner \mathbb{P}w \sim p$	T-5) $\vdash Op \equiv P \sim p$

Each strong (weak) normative operator is equivalent to the other strong (weak) operator with a strong negation before and an ordinary negation after it.

The following theorems state the properties of iterated negations:

TN-22) $\vdash \sim \urcorner \mathbb{P}sp \equiv \mathbb{P}wp \quad (\equiv \urcorner \sim \mathbb{P}sp)$
TN-23) $\vdash \sim \urcorner \mathbb{P}wp \equiv \mathbb{P}sp \quad (\equiv \urcorner \sim \mathbb{P}wp)$
TN-24) $\vdash \sim \urcorner \mathbb{O}sp \equiv \mathbb{O}wp \quad (\equiv \urcorner \sim \mathbb{O}sp)$
TN-25) $\vdash \sim \urcorner \mathbb{O}wp \equiv \mathbb{O}sp \quad (\equiv \urcorner \sim \mathbb{O}wp)$

When a normative operator is affected by an ordinary and a strong negation, the order of these two negations is irrelevant.

The double ordinary and strong negation of a strong (weak) operator is equivalent to the corresponding weak (strong) operator. By this double form of negation one can pass from a strong (weak) concept to the corresponding weak (strong) concept.

The law of double (equal) negation is valid for both forms of negation, and the result is the same in both cases. For example: $\vdash (\urcorner \urcorner \mathbb{P}sp) \equiv (\mathbb{P}sp) \; (\equiv \sim \sim \mathbb{P}sp)$.

In general, if $\aleph$ is one of the normative operators ($\mathbb{P}s$, $\mathbb{P}w$, $\mathbb{O}s$, or $\mathbb{O}w$), then it is valid that:

$$\urcorner \urcorner \aleph = \aleph = \sim \sim \aleph$$

VII. *The principles of distribution for normative concepts*

The distribution principles for "O" in Deontic Logic are also valid for "$\mathbb{O}$s" in Normative Logic. And the same analogy holds for "P" and "$\mathbb{P}$w".

Normative Operators		*Deontic Operators*
TN-26) $\vdash \mathbb{O}s(p.q) \equiv (\mathbb{O}sp.\mathbb{O}sq)$	——	T-1) $\vdash O(p.q) \equiv (Op.Oq)$
TN-27) $\vdash \mathbb{P}w(p \vee q) \equiv (\mathbb{P}wp \vee \mathbb{P}wq)$	——	T-2) $\vdash P(p \vee q) \equiv (Pp \vee Pq)$

(For the sake of brevity I omit the proofs of these as well as of most other theorems).

It should be noted that for weak obligation and strong permission the implication holds in only one direction:

TN-28) $\vdash (\mathbb{P}sp \vee \mathbb{P}sq) \supset \mathbb{P}s(p \vee q)$ (The converse is not valid),
TN-29) $\vdash (\mathbb{O}w(p.q) \supset (\mathbb{O}wp.\mathbb{O}wq)$ (" " " " ").

We shall see later that these differences are very important. They help to clarify certain difficulties concerning the interpretation of deontic logic, which sometimes is mistakenly confused with normative logic.

VIII. *The principles of subalternation in normative logic*

"Prohibition" is very frequently used as a normative (not deontic) concept with the same meaning as the strong negation of strong permission.

For the symbolic representation of this notion I shall use "$\mathbb{P}$h", reading it "It is prohibited that".

Definition of normative prohibition; Df. $\mathbb{P}h : \mathbb{P}h = \neg \mathbb{P}s$.

This is a strong normative concept. It is possible to introduce a weak concept of prohibition with the meaning of the strong negation of weak premission. But for this notion I shall not introduce any special symbolism, because I believe that "Prohibition" is never used in ordinary language with such a weak meaning. In this sense "Obligation" and "Prohibition" are alike: they are used as normative concepts, but only in the strong sense.

Immediate consequences of the definition given are:

TN-30) $\vdash \mathbb{P}hp \equiv \rceil \mathbb{P}sp$,
TN-31) $\vdash \mathbb{P}hp \equiv \mathbb{O}s \sim p$.

The principles of subalternation of normative logic are exactly parallel to those of deontic logic.

Normative Logic	*Deontic Logic*
TN-32) $\vdash \mathbb{O}sp \supset \mathbb{P}sp$	T-3) $\vdash Op \supset Pp$
TN-33) $\vdash \mathbb{O}wp \supset \mathbb{P}wp$	T-3) $\vdash Op \supset Pp$
TN-34) $\vdash \mathbb{P}hp \supset \mathbb{P}s \sim p$	T-4) $\vdash Php \supset P \supset p$

Obligation implies permission of the same kind, and prohibition implies the strong permission of the negation.

Many authors have expressed their doubts about the legitimacy of accepting as logically true such principles. They argue in the following way: If obligation logically implies permission then it is logically impossible for something to be obligatory and not permitted, *i.e.*, prohibited. This, however, is not only not impossible but it is frequently found in experience. It is not uncommon that states of affairs are qualified as obligatory and also as prohibited. Of course, this is a regrettable situation, but not an impossible one, and lawyers know how frequent it is.

This argument has been directed against

$$\vdash (Op \supset Pp)$$

which, in deontic logic, is equivalent to

$$\vdash \sim (Op.Php).$$

But if the argument is analyzed, it may be seen that the concepts referred to in it are the normative and not the deontic ones. With respect to the normative operators the incompatibility of obligation and prohibition does not hold.

The following is not a law of normative logic:

$$\sim (\mathbb{O}sp.\ \mathbb{P}hp),$$

and this formula is not equivalent to TN-32).

Strong obligation implies strong permission, but prohibition is compatible with strong permission. The following is not valid:

$$\sim(\mathbb{P}hp.\ \mathbb{P}sp).$$

Sometimes, the preceding argument has not been considered conclusive. The explanation for this opinion must, I think, be sought in the ambiguity of the notion of permission, because when "permission" is used in the weak normative sense, then it is true that it is incompatible with prohibition.

In normative logc this situation is reflected in the validity of the following theorem:

	Normative Logic		*Deontic Logic*
TN-35)	$\vdash \sim(\mathbb{P}hp.\ \mathbb{P}wp)$	————————	$\vdash \sim(Php.Pp).$

By the decision method of section (VII) it is easy to check the validity of the preceding theorem, and it is also easy to see that, for example,

$$\sim(\mathbb{P}hp\ .\ \mathbb{P}sp)$$

is not a theorem. By the definitions of "$\mathbb{P}$h" and "$\mathbb{P}$s", it is the symbolic abbreviation of

$$\underset{F}{\sim}(\underset{T}{\underbrace{Nx\sim Pp}}\ \underset{T}{.}\ \underset{T}{\underbrace{NxPp}}),$$

which turns out to be false for the valuation indicated under it.

That this valuation cannot be eliminated follows from the fact that the only rule for elimination in the case of an N-Formula is DN-2), and this presupposes that at least one of its atomic N-Formulas has received in the valuation the value "F".

I think that all this is in agreement with our intuitive feelings, in the sense that it is perfectly possible for an individual agent to norm some state of affairs as permitted and also prohibited. What is also obvious is that in such case x has issued incompatible norms, in the sense in which "$\sim$Pp" and "Pp" are incompatible.

This consideration suggests the convenience of introducing the following definition of deontic inconsistency of the normation of a state of affairs p by an agent x:

Definition of inconsistent normation: Df. IN: $IN(p) = (\mathbb{P}hp . \mathbb{P}sp)$.

The negation of inconsistent normation is equivalent to the disjunction of weak permission and the strong negation of weak permission, *i.e.*:

$$\text{TN-36)} \vdash \sim IN(p) \equiv (\mathbb{P}wp \vee \rceil \mathbb{P}wp).$$

Proof:

(1) $\vdash IN(p) \equiv (\mathbb{P}hp . \mathbb{P}sp)$	(By Df.IN)
(2) $\vdash \sim IN(p) \equiv \sim(\rceil \mathbb{P}sp . \mathbb{P}sp)$	(From (1) by (PL) and Df. $\mathbb{P}$h)
(3) $\vdash \sim IN(p) \equiv (\sim \rceil \mathbb{P}sp \vee \sim \mathbb{P}sp)$	(From (2) by (PL))
(4) $\vdash \sim IN(p) \equiv (\mathbb{P}wp \vee \rceil \mathbb{P}wp)$	(From (3) by TN-22), TN-10)&TN-15)).

When x has normed p as obligatory and as prohibited, he has inconsistently normed p. What he has done is not impossible but is deontically absurd.

$\text{TN-37)} \vdash (\mathbb{O}sp . \mathbb{P}hp) \supset IN(p)$.

Proof:

(1) $\vdash \mathbb{O}sp \supset \mathbb{P}sp$	(TN-32)
(2) $\vdash (\mathbb{O}sp . \mathbb{P}hp) \supset (\mathbb{P}hp . \mathbb{P}sp)$	(From (1) by (PL))
(3) $\vdash (\mathbb{O}sp . \mathbb{P}hp) \supset IN(p)$	(From (2) by Df. IN).

IX. *The principle of permission and the notion of normation*

In normative logic there is no theorem like the Principle of Permission of Deontic Logic.

The Principle of Permission; T-9) $\vdash (Pp \vee P \sim p)$.

Against this principle it has been argued that it is not logically true that everything is such that it is permitted or its negation is permitted, because, if that were so, there could not be any p not deontically characterized, but, as a matter of fact, there are states of affairs that are not normed in any sense, *i.e.,* that have no deontic status.

This kind of argument has a bearing on the position of those who hold that there are gaps in the Law. Those who are disposed to deny that there are such gaps surely would dismiss the argument.

I think that the whole problem arises from a confusion of the different meanings of permission represented by "P", "$\mathbb{P}$s" and "$\mathbb{P}$w", because those who argue against the Principle of Permission are thinking of "$\mathbb{P}$s", and those who argue in its favor usually think of "$\mathbb{P}$w" and presuppose the absence of inconsistent normation.

For weak permission the analogue to the Principle of Permission holds on the condition of not-inconsistency, *i.e.*:

TN-38 $\vdash \sim IN(p) \supset (\mathbb{P}wp \vee \mathbb{P}w \sim p)$.

Proof:

(1) $\vdash (\mathbb{O}sp \,.\, \mathbb{P}hp) \supset IN(p)$ (TN-37)
(2) $\vdash \sim IN(p) \supset \sim(\mathbb{O}sp \,.\, \mathbb{P}hp)$ (From (1) by (PL))
(3) $\vdash \sim IN(p) \supset (\sim \mathbb{O}sp \vee \sim \mathbb{P}hp)$ (From (2) by (PL))
(4) $\vdash \sim IN(p) \supset (\mathbb{P}w \sim p \vee \sim \neg \mathbb{P}sp)$ (From (3) by TN-12) and Df. $\mathbb{P}$h)
(5) $\vdash \sim IN(p) \supset (\sim \neg \mathbb{P}sp \vee \mathbb{P}w \sim p)$ (From (4) by (PL))
(6) $\vdash \sim IN(p) \supset (\mathbb{P}wp \vee \mathbb{P}w \sim p)$ (From (5) by TN-22)).

The following valuation shows that for strong permission

$$(\underbrace{\mathbb{P}sp}_{F} \underset{F}{\vee} \underbrace{\mathbb{P}s \sim p}_{F}) \quad (= (\underbrace{NxPp}_{F} \underset{F}{\vee} \underbrace{NxP \sim p}_{F}))$$

is not a theorem of normative logic.

This valuation cannot be eliminated because the elimination rule for N-Formulas, *i.e.* DN-2), can be applied only when there is at least one atomic N-Formula which is valued "T".

The proceding formula can be used to identify that situation in which x has issued some norm about p, *i.e.*, when x has normed p in some way. I shall use "N(p)" as the symbolic representation of "p has been normed (by x)" ([5]).

([5]) When in the text "N (p)" is used to identify the situation in which x has issued some norm about p, it must be remembered that I have chosen one among various plausible meanings of "to issue a norm about p". For the meaning here chosen it holds good that when x has issued a norm permitting p-or-q, x has ruled p-or-q, but has neither normed p nor q. This is so, because "$\mathbb{P}s(p \vee q)$" does not imply "N (p)", nor "N (q)", although it implies "N $(p \vee q)$".

Definition of normation; Df. N : N(p) = (ℙsp v ℙs ∼ p).

When it is true that x has issued some norm about p, then "N(p)" is true. This is shown, partially, in the following theorems:

TN-39) ⊢ ℙsp ⊃ N(p), TN-40) ⊢ ℙs ∼ p ⊃ N(p),
TN-41) ⊢ 𝕆sp ⊃ N(p), TN-42) ⊢ ℙhp ⊃ N(p).

Inconsistent normation implies normation, but the negation of inconsistent normation does not imply normation.

TN-43) ⊢ IN(p) ⊃ N(p).

This suggests the following

Definition of Consistent normation; Df. CN : CN(p) = (N(p). ∼ IN(p)).

From this it follows; by TN-38) and the Df. N):

TN-44) ⊢ CN(p) ≡ ((ℙsp v ℙs ∼ p) . (ℙwp v ⌉ℙwp)).

Consistent normation implies normation. Consistant and inconsistant normation are incompatible, and are exhaustive under the condition of normation.

TN-45) ⊢ CN(p) ⊃ N(p), TN-46) ⊢ CN(p) ⊃ ∼ IN(p),
TN-47) ⊢ N(p) ⊃ (CN(p) v IN(p)), TN-48) ⊢ N(p) ⊃ (CN(p) ≡ ∼ IN(p)).

X. *The principle of prohibition and the notion of determination*

It was said in page 255 that in normative logic prohibition is incompatible with weak permission but not with strong permission. Moreover prohibition and weak permission are logically exhaustive. This shows another analogy between normative and deontic logic, *i.e.*

Normative Logic	*Deontic Logic*
TN-49) ⊢ (ℙhp v ℙwp)	TN-11) ⊢ (Php v Pp)

But prohibition and strong permission are not logically exhaustive i.e.:

(ℙhp v ℙsp)

is not a theorem. By the definitions of "$\mathbb{P}$h" and "$\mathbb{P}$s" it is the symbolic abreviation of:

$$(\underbrace{Nx \sim Pp}_{F} \underset{F}{\vee} \underbrace{NxPp}_{F}),$$

which turns out to be false for the valuation in question.

That this valuation cannot be eliminated follows from the fact that the application of DN-2 presupposes that at least one of the atomic N-Formulas has, in the valuation, the value "T".

I think that all this is in agreement with our intuitions, in the sense that it is perfectly possible, for a given authority x that there exists a possible state of affairs p in relation to which x has not issued any norm permitting nor any norm prohibiting it. In such a case it may be said that x has not determined any normative character for p, though perhaps he has normatively characterized not-p.

In the same way when x *has issued* a norm permitting p *or has issued* a norm prohibiting p, then it may be said that x has normatively determined p.

For this idea of normative determination I shall use the symbolism "DN(p)" with implicit reference to x.

Definition of normative determination; Df.-DN: $DN(p) = (\mathbb{P}sp \vee \neg \mathbb{P}sp)$.

Immediate consequences of the definition are:

TN-50) $\vdash \mathbb{P}hp \supset DN(p)$, TN-51) $\vdash \mathbb{P}sp \supset DN(p)$,
TN-52) $\vdash \mathbb{O}sp \supset DN(p)$.

When p is normatively determined (by x), then x has normed somehow p, and the same happens when not-p is normatively determined, *i.e.*:

TN-53) $\vdash DN(p) \supset N(p)$, TN-54) $\vdash DN(\sim p) \supset N(p)$.

When x has somehow normed p, then he has at least determined p or determined not-p.

TN-55) $\vdash N(p) \supset (DN(p) \vee DN(\sim p))$,
TN-56) $\vdash N(p) \equiv (DN(p) \vee DN(\sim p))$.

Consistent normation is the contradictory of inconsistent normation under the condition of normative determination:

TN-57) $\vdash DN(p) \supset (CN(p) \equiv \sim IN(p))$ (From TN-53 and TN-48).

The idea of normative determination is a very important one, because it may be used to characterize the concept of completeness for systems of norms, or, what is the same thing, systems without gaps. Suppose that x is a legislator, and has issued a set of norms. The set of norms issued by x are said to be complete, when x has normatively determined every possible p. This suggests the following

Definition of Absolute Normative Completeness; Df. C1N:C1N $= (p)\ DN(p)$.

Immediate consequences of this definition are:

TN-58) $\vdash C1N \equiv (p)\ (\neg \mathbb{P}sp \vee \mathbb{P}sp)$,
TN-59) $\vdash C1N \equiv (P)\ (\mathbb{P}hp \vee \mathbb{P}sp)$.

Usually this is too strong; what is normally demanded is not the determination of every possible p, but only the determination of all the propositions of a certain set.

This means that what is usually wanted is something like the following

Definition of Relative Normative Completeness; Df. C1ΦN:

$C1\Phi N = (p)\ (\Phi(p) \supset DN(p))$ (Where "Φ" is used to identify the selected set of propositions).

Many confusions lie at the bottom of current discussions about the completeness of systems of norms, but surely one of them depends upon the different meanings of "permission".

One of the points in question is whether what is sometimes called *The Principle of Prohibition* is true or not (6) *i.e.*:

(A) Everything that is not forbidden is permitted.

If it is true, what kind of truth does it have ? Is it necessarily true or just contingently true ?

The Principle of Prohibition must not be confused with the Nullum Crimen Principle, because the latter is not a normative

(6) For a similar approach see [1].

proposition but a norm permitting everything not forbidden by other norms.

Statement (A) is used sometimes with the meaning of

(A′) $(\sim \mathbb{P}hp \supset \mathbb{P}wp)$

when the concept of permission refered to in (A) is understood as weak permission.

(A′) is propositionally equivalent to TN-49), and in this sense it is logically true.

Those who maintain the logical impossibility of gaps interpret (A) with the meaning of (A′), and define normative completeness as the universal quantification with respect to "p" of

$$(\mathbb{P}hp \vee \mathbb{P}wp)$$

or, what is equivalent, of (A′).

In this sense it is right to say that the Principle of Prohibition is true, and moreover necessarily true, and because of this the existence of gaps is logically impossible.

But sometimes statement (A) is used with the meaning of

(A″) $(\sim \mathbb{P}hp \supset \mathbb{P}sp)$.

That is so when the concept of permission refered to in (A) is understood as strong permission.

(A″) is propositionally equivalent to

$$(\mathbb{P}hp \vee \mathbb{P}sp)$$

which is not logically true.

When the concept of normative completeness is defined with the aid of the concept of strong permission, as I have previously done, then the existence of gaps is not logically impossible.

This, or something like this is, I believe, what is said by those who maintain the possible existence of gaps, and the contingent nature of the Principle of Prohibition.

Sometimes (A) has been identified with the deontic theorem:

$$\vdash (\sim Php \supset Pp),$$

and, as this is equivalent to

$$\vdash (\sim O \sim p \supset Pp),$$

i.e., with half of the definition of "O" in terms of "P" or of

"P" in terms of "O", the admissibility of such definitions has been questioned by those who admit the existence of gaps.

I think, that this is not a problem of deontic logic but of normative logic.

XI. *The principles of trichotomy and normative determination*

Let us now introduce the concept of facultative in normative logic by the following

Definition of Facultative: Df. $\mathbb{F}$: $\mathbb{F}p = (\mathbb{P}sp \,.\, \mathbb{P}s\sim p)$.

To say that p is facultative means that p and not $\sim p$ are strongly permitted.

The following law of deontic logic

T-12) $\vdash (Php \vee Op \vee Fp)$

may be called *The Principle of Trichotomy.*

The analogical formula of normative logic, i.e.:

(T) $(\mathbb{P}hp \vee \mathbb{O}sp \vee \mathbb{F}p)$

is not a theorem, but it is interesting to see what is the case when it is true.

(T) is true if and only if p and not-p are both normatively determined.

This is reflected in the following theorems:

TN-60) $\vdash (\mathbb{P}hp \vee \mathbb{O}sp \vee \mathbb{F}p) \supset (\mathbb{P}sp \vee \urcorner \mathbb{P}sp)$,
TN-61) $\vdash (\mathbb{P}hp \vee \mathbb{O}sp \vee \mathbb{F}p) \supset (\mathbb{P}s\sim p \vee \urcorner \mathbb{P}s\sim p)$,
TN-62) $\vdash ((\mathbb{P}sp \vee \urcorner \mathbb{P}sp) \,.\, (\mathbb{P}s\sim p \vee \urcorner \mathbb{P}s\sim p)) \supset (\mathbb{P}hp \vee \mathbb{O}sp \vee \mathbb{F}p)$,
TN-63) $\vdash ((\mathbb{P}sp \vee \urcorner \mathbb{P}sp) \,.\, (\mathbb{P}s\sim p \vee \urcorner \mathbb{P}s\sim p)) \equiv (\mathbb{P}hp \vee \mathbb{O}sp \vee \mathbb{F}p)$.

It is interesting to notice that using propositional quantification the following equivalence holds:

$$(p)\,(\mathbb{P}sp \vee \urcorner \mathbb{P}sp) \equiv (p)\,(\mathbb{P}hp \vee \mathbb{O}sp \vee \mathbb{F}p),$$

which shows that the notion of normative completeness can be defined using the universal quantification of (T).

XII. *Iterated modalities*

(i) "Can norms themselves be the content of norms ?" This is the question with which *von Wright* opens his chapter on norms of higher order in [12] (cf. also [13], pp. 91 ff.). This is a straight-forward way of asking for the very possibility of iterated deontic modalities. We face here a very difficult question for it is not at all clear what meaning (if any) can be ascribed to expressions like "OOp". I shall not discuss this problem here, but I think that the distinction between deontic and normative operators may throw some light on it, because it enables us to ask two different though related questions, and to provide a clear answer to them. These questions are:

(ii) Can normative propositions be the content of norms ?

(iii) Can normative operators be iterated ?

The answer to the question (ii) is, I think, clearly in the affirmative. When a normative proposition is the content of a norm the subject of the norm is the authority of the normative proposition. Formally it means that we must draw a clear distinction between, say, OOp and ONxOp (= O$\mathbf{O}$sp). This last formula has a clear meaning: it states that it is obligatory for x to rule (= to issue a norm to the effect) that it is obligatory that p.

The distinction between (i) and (ii) enables us to give a new (normative) interpretation of some deontic formulas.

For instance, *A. N. Prior* considers in [6] p. 225 that

$$O(Op \supset p)$$

is a deontic law, and he interprets it "We ought to do-what-we-ought-to-do".

Using *von Wright*'s symbolism for hypothetical norms, Prior's thesis seems to require that

$$O(p/Op)$$

be considered as a law of dyadic deontic logic.

The meaning of these two formulas is not very clear and I doubt whether a logical system deontically interpreted should contain any thesis of such a kind (cf. *Lemmon* [4]).

But if we replace the formula "O(p/Op)" by

$O(p/\mathbb{O}sp) \quad (= O(p/NxOp))$

(*i.e.* if we interpret the second O normatively as $\mathbb{O}s$), we obtain a very suggestive and interesting norm, which looks very much like *Kelsen*'s "Basic Norm" when "x" stands for the supreme authority in a certain community. *Kelsen* seems to think that his Basic Norm has no specific content but only conceptual import, being something like a logical law (cf. [3] p. 200 ff.). In this he is very close to *Prior*. (This is, of course, not the only way of interpreting *Kelsen*'s Basic Norm. For this interpretation and the difficulties that spring from it for *Kelsen*'s theory see *Ross* [7] p. 156 ff.).

It seems to me that the question (iii) may be answered in the affirmative as well.

Consider the following propositions:

$\mathbb{O}s_y.\mathbb{O}s_x p$ (= NyONxOp),
$\mathbb{P}s_y.\mathbb{O}s_x p$ (= NyPNxOp),
$\mathbb{P}h_y.\mathbb{O}s_x p$ (= NyO ~ NxOp) (= $\mathbb{O}s_y.\mathbb{P}w_x \sim p$).

These formulas seem to have a clear meaning. The first one, *e.g.*, may be read as "y has ruled that it is obligatory for x to rule that it is obligatory that p".

I have put subscripts below the normative operators in order to distinguish the different authorities of the different normative propositions. It might be questioned whether there can be iterated normative modalities with the same susbcript, *i.e.*, whether there can exist something like self-referring normation. I think that the answer may be affirmative, provided that norms are not identified with orders (cf. Hart [2] p. 18 ff.).

XII. *The isomorphism of deontic and normative logic*

Very frequently the symbolism of deontic logic has been read with the meaning of normative logic. This, of course, may produce many confusions, but not necessarily always, because the two calculi are isomorphic under two assumptions which often are, I think, tacitly accepted.

When we are referring to a complete and consistent set of norms, the calculus for deontic operators and the calculus for normative operators are isomorphic.

This means that, under the hypothesis:

(C1-CN) (p) (ND(p) . ∼IN(p)) (Completeness and not-Inconsistency),

the following theorems can be proved:
(I shall put "(C1-CN)" before the next theorems, to remind that they are proved under this assumption)

C1-CN-1) ⊢(ℙsp ∨ ⌉ℙsp) (From (C1-CN) by Df.DN),
C1-CN-2) ⊢(ℙwp ∨ ⌉ℙwp) (From (C1-CN) by TN-36),
C1-CN-3) ⊢(ℙsp ≡ ℙwp),
C1-CN-4) ⊢(𝕆sp ≡ 𝕆wp),
C1-CN-5) ⊢(⌉ℙsp ≡ ∼ℙsp).

These theorems show that under the assumption of completeness and consistency there is just one kind of permission, one kind of obligation and only one kind of negation. The distinction between the weak and the strong forms vanishes.

In order to prove the isomorphism I shall prove that the axioms and rules of inference of deontic logic for "O" can be proved now for "𝕆s", and by (C1-CN-4) for "𝕆w" also, and that ℙs" can be defined in terms of "𝕆s" and "∼".

The analogue of Ax-1) is:

C1-CN-6) ⊢𝕆s(α⊃β) ⊃ (𝕆sα⊃𝕆sβ) (Where α and β are C-Formulas).

Proof:

(1) ⊢O(α⊃β) ⊃ (Oα⊃Oβ) (Ax-1)
(2) ⊢NxO(α⊃β) ⊃ Nx(Oα⊃Oβ) (From (1) by RN-5))
(3) ⊢Nx(Oα⊃Oβ) ⊃ (NxOα⊃NxOβ) (AxN-2)
(4) ⊢NxO(α⊃β) ⊃ (NxOα⊃NxOβ) (From (2) and (3) by (PL))
(5) ⊢𝕆s(α⊃β)⊃(𝕆sα⊃𝕆sβ) (From (4) by Df.𝕆s).

It is clear from the proof, that this theorem does not depend upon the hypothesis (C1-CN), so that it is a theorem of the System-NO. But for the replacement in it of "𝕆s" by 𝕆w" (C1-CN-4) is needed.
The analogous of R-3) is:

C1-CN-7) ⊢∼α → ⊢∼𝕆sα (Where α is a C-Formula).

Proof:

(1) $\vdash \sim\alpha$ (Hypothesis)
(2) $\vdash \sim O\alpha$ (From (1) by R-3)
(3) $\vdash (O\alpha \supset \sim O\alpha)$ (From (2) by (PL))
(4) $\vdash (NxO\alpha \supset Nx\sim O\alpha)$ (From (3) by RN-5))
(5) $\vdash (\mathbb{P}w\sim\alpha \vee \rceil\mathbb{P}w\sim\alpha)$ (From (C1-CN-2))
(6) $\vdash (\sim NxO\sim\sim\alpha \vee \sim Nx\sim O\sim\sim\alpha)$
(From (5) by Df. $\mathbb{P}$w) and Df. $\rceil\mathbb{P}$w))
(7) $\vdash (\sim NxO\alpha \vee \sim Nx\sim O\alpha)$ (From (6) by RN-8))
(8) $\vdash (NxO\alpha \supset \sim Nx\sim O\alpha)$ (From (7) by (PL))
(9) $\vdash \sim NxO\alpha$ (From (4) and (8) by (PL))
(10) $\vdash \sim\mathbb{O}s\alpha$ (From (9) by Df. $\mathbb{O}$s)).

The analogous of R-4):

C1-CN-8) $\vdash \alpha \rightarrow \vdash \mathbb{O}s\alpha$ (Where α is a C-Formula).

Proof:

(1) $\vdash \alpha$ (Hypothesis)
(2) $\vdash O\alpha$ (From (1) by R-4)
(3) $\vdash (\sim O\alpha \supset O\alpha)$ (From (2) by (PL))
(4) $\vdash (Nx\sim O\alpha \supset NxO\alpha)$ (From (3) by RN-5))
(5) $\vdash (\mathbb{P}w\sim\alpha \vee \rceil\mathbb{P}w\sim\alpha)$ (From (C1-CN-2))
(6) $\vdash (\sim NxO\sim\sim\alpha \vee \sim Nx\sim O\sim\sim\alpha)$
(From (5) by Df. $\mathbb{P}$w) and Df. $\rceil\mathbb{P}$w))
(7) $\vdash (\sim NxO\alpha \vee \sim Nx\sim O\alpha)$ (From (6) by RN-8))
(8) $\vdash (Nx\sim O\alpha \supset \sim NxO\alpha)$ (From (7) by (PL))
(9) $\vdash \sim Nx\sim O\alpha$ (From (4) and (8) by (PL))
(10) $\vdash \sim\rceil\mathbb{O}s\alpha$ (From (9) by Df. $\rceil\mathbb{O}$s))
(11) $\vdash \mathbb{O}w\alpha$ (From (10) by TN-24))
(12) $\vdash \mathbb{O}s\alpha$ (From (11) by (C1-CN-4))

The analogous of the definition of "P" by "O" is:

C1-CN-9) $\vdash \mathbb{P}sp \equiv \sim\mathbb{O}s\sim p$.

Proof:

(1) $\vdash \mathbb{P}sp \equiv \sim\mathbb{O}w\sim p$ (TN-6)
(2) $\vdash \mathbb{P}sp \equiv \sim\mathbb{O}s\sim p$ (From (1) by (C1-CN-4)).

XIII. *Lemmon and Stenius on deontic and normative logic*

In [4] E. J. Lemmon advances a system for deontic logic, called D2, whose characteristic deontic principles are:

L.Ax-1) $\vdash O(\alpha \supset \beta) \supset (O\alpha \supset O\beta)$,
L.Ax-2) $\vdash O\alpha \supset \sim O \sim \alpha$,
L.R- $\vdash (\alpha \supset \beta) \rightarrow \vdash (O\alpha \supset O\beta)$.

It is essentially an equivalent axiomatization of the systems of Prior and von Wright contained repectively in [6] and [9].

Later on Lemmon, in [5], rises severe critics against the admisibility of L.Ax-2) as a principle of the logic of obligation. This same law is rejected by Stenius in [8]. But, from the context of their papers, I think, it is clear that they are interpreting "O" as a normative operator like "$\mathbb{O}$s" and not as a deontic one.

Notice that the characteristic laws for "$\mathbb{O}$s", in system-NO, can be derived from the following principles:

$\vdash \mathbb{O}s(\alpha \supset \beta) \supset (\mathbb{O}s\alpha \supset \mathbb{O}s\beta)$ (Corresponding to L.Ax-1)
$\vdash (\alpha \supset \beta) \rightarrow \vdash (\mathbb{O}s\alpha \supset \mathbb{O}s\beta)$ (Corresponding to L.R-).

This are the axiomatic principles accepted by Lemmon for the logic of obligation, and are coincident to those suggested by Stenius in [8]. In this sense the ideas of Lemmon and Stenius are reflected in system-NO. But, it must be observed that in their systems strong permission ($\mathbb{P}$s) cannot be defined — although weak permission ($\mathbb{P}$w) can. So they lack the possibility of expressing some important normative concepts.

I consider the paper of Stenius, and in a certain sense that of Lemmon also, as a sound exposition of the logic of normative obligation ($\mathbb{O}$s), but I think that such logic is only a province of the logic of normative propositions.

On the other side, I believe that their critics to deontic logic comes down to nothing when we recognize the differences between deontic and normative concepts, because they spring from reading the symbolism of deontic logic with the meaning of normative logic.

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