REMARKS ON CHURCH SOLUTION OF THE PARADOX OF ANALYSIS

Thomas M. SIMPSON

In this paper I discuss some special questions which arise from Church's treatment of the paradox of analysis. After a brief statement of a current version of the paradox (I) and Church's solution of it (II), I consider a different version of the paradox, presented by M. White (III) and the solution advanced by White within the framework of Frege-Church semantics (IV); it is then remarked (V) that White's way out depends on an *ad hoc* assumption about obliquity, and (VI) that Church's analysis of psychological contexts implies the very opposite supposition; in (VII) the so-called 'translation test' used by Church in his analysis of psychological contexts is brought into consideration, and it is contended (VIII) that the application of the very same test to the case of the paradox of analysis may reinstate the rejected synonymity of 'the concept brother = the concept male sibbling' and 'the concept brother = the concept brother'. Finally (IX), it is suggested that these difficulties don't imply necessarily an objection to the basic principles of Frege-Church semantics, because they are essentially connected with a special (and disputable) supposition about the nature of analysis which is independent of the mentioned principles.

To avoid misunderstandings, the reader must bear in mind that by "Frege-Church semantics" it is meant Frege's semantics as modified by Church: so, any question related with the differences between Church's and Frege's theories (concerning, for example, the interpretation of predicates) is irrelevant to our discussion.

I. Let us consider the identity

(1) The concept brother = the concept male sibling, assuming that the second member expresses a correct analysis of the first one. From (1), the paradox of analysis arises through

a simple argument, in which the pre-analytical notions of 'synonymity' and 'meaning' are used. If (1) is true, then 'the concept brother' and 'the concept male sibling' are synonymous expressions and hence interchangeable *salva significatione.* But then (1) has the same meaning as

(2) The concept brother = the concept brother.

Consequently, all analysis is either trivial (if true), or false (if non trivial) .

II. Church's solution stands on Frege's distinction between the sense (*Sinn*) and the denotation (*Bedeutung*) of a name, which replaces the undifferentiated notion of 'meaning' that blurs this distinction (In *The Journal of Symbolic Logic,* vol. 11, p. 132). Briefly, once we have distinguished *sense* from *denotation* it is possible to say that although 'the concept brother' has the same denotation as 'the concept male sibling', these names haven't an identical sense; therefore, if synonymity is, strictly, identity of sense, 'the concept brother' and 'the concept male sibling' aren't synonymous; so, they aren't interchangeable *without alteration of sense* and, finally, this commits us to reject the alleged synonymity between (1) and (2).

III. But is the sense-denotation dichotomy really enough to solve this paradox ? It seems possible to show that the assumption that 'the concept brother' and 'the concept male sibling' denote the same concept leads by a different road to the paradoxical synonymity we wanted to avoid. This has been pointed out by M. White ("On the Frege-Church solution of the paradox of analysis", *Phil. and Phen. Research,* vol. XIX, p. 306), whose argument is the following:

> The puzzle begins with the assumption that the name 'brother' expresses the concept of being a brother but doesn't denote it. A similar assumption is made about the name 'male sibling'. It follows from (1) that these two names have the same sense. Church says elsewhere: 'If a name forming part of a longer name is replaced by another having the same sense, the sense of the whole is not altered'. Accordingly, the sense of (1) is the same as the sense of (2),

since sentences on Frege's view are also names whose senses are the propositions they express; therefore, we have a puzzle again (1).

Perhaps it is necessary to explicit why, as the author affirms, 'it follows from (1) that these two names [i.e., 'brother' and 'male sibling'] have the same sense'. This results from the fact (or the supposed fact) that 'the concept brother' and 'the concept male sibling' denote the senses ('concepts', in Church's terminology) expressed respectively by 'brother' and 'male sibling' in their direct (or ordinary) uses, i.e., their direct senses (2); so the truth of (1) implies obviously the identity of the (direct) senses of 'brother' and 'male sibling'.

In order to simplify the following discussion, let us symbolize with '(P)' the principle of interchangeability of names of identical senses quoted in White's argument.

IV. Yet White holds that the Frege-Church theory includes strong enough resources for the solution of this new version of the paradox, by the application of the distinction between the direct (*gewönlich*) and the oblique (*ungerade*) sense of a name, just as the original was solved through the more basic distinction between sense and denotation:

> The point is that the name 'Brother' in (1) is used obliquely and therefore doesn't denote the class of all brothers, but rather the concept of being a brother. Therefore it doesn't

(1) In order to unify the terminology I have substituted 'concept' for White's 'attribute'. This change does not alter the essence of the argument.

(2) The direct sense of a name is the sense the name has when it isn't inside an oblique context, which is syntactically characterized (in the more usual cases) by the presence of certain psychological and modal operators ('believes that', 'It is necessary that', etc.) and obliquying particles like 'the concept'. The direct sense of a compound name such as 'The concept A' is identical with the sense of '*A*' *in* that context, and this latter sense is one of the infinite series of indirect senses of '*A*', series that can be "generated" by the iteration of psychological (modal, etc.) operators and obliquying particles. 'The concept brother' is used in a direct way in (1) and in an oblique way in 'John believes that the concept brother = the concept male sibbling'.

> express that concept. The case of 'male sibling' in (1) is analogous. *It follows* that 'Brother' as used in (1) doesn't have the same sense as 'male sibling' in (1), because both are being used obliquely in that context. It follows that sentences (1) and (2) have not been shown synonymous by this new argument and that the second version of the puzzle is resolved (*loc. cit.*, p. 306. Regarding the word 'concept' in the quoted text, see n. 1. My italics).

V. What does in fact White solution amount to ? We can distinguish two steps in his argument: (a) because of obliquity, 'Brother' and 'Male sibling' have different senses in (1); (b) therefore, (1) and (2) are not synonymous.

Prima facie, one would think that (b) shows the falsity of (P), since it was this principle which allowed White to infer the apparently false statement that (1) and (2) have the same sense. But White doesn't draw the same conclusion:

> Indeed, if one should maintain that (1) and (2) aren't synonymous and yet reject Frege's solution of the puzzle, one [...] will conclude that putting synonymous for synonymous doesn't always yields synonymous. On the other hand, Frege's distinction would seem to preserve this rule in a manner illustrated above, albeit with restrictions of the sort indicated; whether two expressions are synonymous will depend on context (*loc. cit.*, p. 308) ([3]).

([3]) White's formulation of the matter might be in need of an aclaratory remark. He says that the supposition that the synonymity of two expressions "depend on context" (in the Fregean explained way) saves the rule that 'putting synonymous for synonymous yield synonymous'; but the last occurrence of 'synonymous', unlike the two formers, doesn't seem contextually qualified. Thus, to illustrate this with the examples discussed here, White says that 'brother' and 'male sibbling' are not 'synonymous in (1)', and therefore (1) and (2) are not 'synonymous' (without qualification). It is clear that if synonymity 'depends on context', this must hold also for (1) and (2); because of this the word 'context' should cover also the extreme case in which an expression — for instance, (1) — is taken in isolation (in this case the synonymity is usually understood as identity of the direct senses of the names). Under he usual jargon this would be a sort of "null" context,

At this point may be useful to dwell briefly on some known features of Frege's semantics: From a Fregean point of view, in natural languages there's no such a thing as *the* sense of a name; to each name there corresponds an infinite series of senses that can be generated from its direct sense (cf. n. 2). Because of this the principle (P), literally taken, has an univocal meaning only if applied to a language in which obliquity, and hence the distinction between direct and indirect senses, has been eliminated. Now, it is clear from the quoted text that in order to produce his new puzzle White must interpret (P) as

(P_1) If in a compound name we replace a component name by another that has the same *direct* sense, the (direct) sense of the compound name is not altered ([4]).

When he says that (1) implies that 'Brother' and 'Male sibling' have 'the same sense', the only relevant sense is here the *direct* one; with the help of (P_1), that identity leads straightforwardly to White's puzzle.

White's final interpretation of (P), which is assumed to allow to solve the puzzle, might perhaps be formulated explicitly as

(P_2) If a name occupying a position x in a compound name is replaced by another one that has in x the same sense that the first has in this position, then the direct sense of the compound name doesn't change.

In this way, we shouldn't speak generally of *the* synonymity of two terms but rather of their synonymity *in* a position x, or more broadly, *in* a linguistic context C.

We can return now to the first step in White argument, which rises the obvious question: Why does obliquity determine that 'brother' and 'male sibling' have different senses in (1) ? Let's remember once more that 'brother' and 'male sibling' are sup-

with the added proviso that (1) and (2), being both taken in isolation, are in the same context. Furthermore, White's conclusion that (1) and (2) are not synonymous relies on the implicit supposition that the sense of a compound name is a combination of the senses that its components parts have in it.

([4]) We refer here to the direct sense of a compound name because this latter is considered in isolation (cf. n. 3).

posed to have the same direct sense, and that in an oblique context a name doesn't express this sense, but a new (indirect) one. As a matter of fact, nobody has ever offered a rule according to which this change of sense is performed, so that given two names which posses the same direct sense we can't know if their senses will change in a parallel or in a divergent way when we put them in an oblique position x. It is therefore surprising to hear White saying, without further explanation, that from the fact that 'brother' and 'male sibling' are used obliquely in (1) *it follows* that they have in (1) different senses. Why does *it follow* ? Considering the mentioned lack of rules we could say, at most, that the senses of 'brother' and 'male sibling' *might* be different in (1), and this would leave open the question of the synonymity between (1) and (2). But if this is all we can say, then it would seem that this is hardly a solution of the puzzle. On the other hand, the supposition that the senses of 'brother' and 'male sibling' in (1) *are* different looks too much like an *ad hoc* device to solve the puzzle, not having a general scope nor being supported by any independent argument; we will see, furthermore, that Church's analysis of psychological contexts implies indeed the very opposite supposition, revealing thus the above mentioned 'ad-hocness'.

VI. In 'Intensional Isomorphism and Identity of Belief' (*Phil. Studies*, Vol. V, N° 5, pp. 65-73; [Isomorphism]), Church considers a possible artificial language in which obliquity has not been eliminated and supposes this language to have a predicate R which is synonymous to the abstraction expression $(\lambda x)[..x..]$. Then, according to Church,

> R must be interchangeable with $(\lambda x)[..x..]$ in all contexts, including belief contexts, being synonymous by the very construction of the language — by definition, if you choose to call it that (p. 67).

The interchangeability referred to in the quoted passage (to which we will mention as (P_3)) is the interchangeability *salva veritate* of two synonymous terms, and even though Church doesn't explain there his use of 'synonymity', it seems clear from

the discussion that in this case it must be understood as identity of direct sense. After saying that it is necessary to provide a determination of synonymity as a part of the semantical basis of the mentioned language, he goes on: 'This might be done directly, by means of *rules of synonymy* and *rules of non-synonymy*, or it might be done indirectly, by means of *rules of sense*' (p. 67). Concerning these he refers to his paper 'The Need for Abstract Entities in Semantic Analysis' (in L. M. Copi and J. A. Gould, *Contemporary Readings in Logical Theory*, The Macmillan Company, 1967), where three illustrative rules of sense for 'round', 'the world' and 'the world is round' give us in fact, the direct senses of these expressions:

> *Rules of sense:* 'round' expresses the property of roundness; 'the world' expresses the (individual) concept of the world; 'the world is round' expresses the proposition that the world is round (*loc. cit.*, p. 207).

Now it is easy to show that for the validity of (P_3), so interpreted, it is required that terms with the same direct sense be also synonymous *in* all oblique contexts, i.e., precisely the assumption contrary to the one introduced by White in order to solve his puzzle.

To make this exposition clearer, we will call 'obliquity of degree 1' that which is determined by only one obliquying prefix; 'obliquity of degree 2' the one determined by two different obliquying prefixes or by two occurrences of the same (provided that one is within the scope of the other); and, in general, the obliquity of degree n shall be determined by n obliquying prefixes or n occurrences of the same prefix (with the same proviso). Thus the degree of obliquity of 'A' in 'John believes that ..*A*..' is 1, but it is 2 in 'Peter believes that John believes that ..*A*..', provided that '..*A*..' doesn't contain any obliquying prefix. (This formulation should perhaps be qualified in order to allow for the cases, if any, in which an obliquying prefix occurs vacuously, as the prefix 'the individual' in 'the individual *x*'.)

It is then obvious that the identity of the direct senses of '*A*' and '*B*' guarantees their interchangeability *salva veritate* in all

the contexts that have an obliquity of degree 1. The hypothesis that '*A*' and '*B*' have the same direct sense implies that they have the same denotation in

(4) John believes that ..*A*..

and

(5) John believes that ..*B*.. .

Therefore, the proposition denoted by 'that ..*A*..' is identical to the one denoted by 'that ..*B*..', and, as according Church's conception a belief-sentence establishes a relation between an individual and a *proposition* (i.e., the sense of a declarative sentence), it is obvious that (4) and (5) establish exactly the same relation between the same entities.

As soon as we iterate the 'believes that' prefix, we see that the validity of (P_3) requires that also the senses of '*A*' and '*B*' *in* (4)-(5) (and not only their direct senses) be identical. Consider

(6) Peter believes that John believes that ..*A*..;

(7) Peter believes that John believes that ..*B*...

If 'A' and 'B' had respectively different senses in (4) and (5) (even though they have there the same denotation because both denote their direct senses) then the proposition denoted by 'That John believes that ..A..' would not be the same one as the proposition denoted by 'That John believes that ..B..', so that the second term of the relation established by (6) would not be the same as the second term of the relation established by (7). Let us symbolize with '*that p*' the grammatical direct complement in (6) and with '*that r*' the one in (7); then we see from (6) that Peter believes *that p,* and from (7) that Peter believes *that r,* being *that p* and *that r* different entities. This shows that if 'A' and ' B' had different senses *in* (6)-(7), it would be logically possible for (6) and (7) to have different truth-values, which contradicts (P_3). But in [Isomorphism] Church presents an independent argument to show that the failure of (P_3) is impossible.

VII. This problem is related to a well known example that Benson Mates has presented as a radical objection to all possible intent to characterize synonymity (for a natural language) in such a way as to satisfy the principle of universal interchan-

geability *salva veritate.* ('Synonymity', in *Semantics and the Phil. of Language,* ed. by L. Linsky). Let's suppose D and D′ to be two synonymous sentences, or, in Frege-Church terminology, two sentences that express the same proposition. According Benson Mates it is true that

(8) nobody doubts that whoever believes *D,* believes *D,*

but it is false that

(9) Nobody doubts that whoever believes *D,* believes *D′.*

In opposition to Mates, Church holds in [Isomorphism] that this is impossible. His argument relies on an application of the so called 'test of translation', first propunded by C. H. Langford in another connection.

For the sake of the discussion Church supposes that the expressions 'forthnight' and 'period of fourteen days' are synonymous in English, and interprets '*D*' and '*D′*' respectively as 'the seventh consulate of Marius lasted less than a fortnight' and 'the seventh consulate of Marius lasted less than a period of fourteen days'; this interpretation transforms (7) and (8) into

(7′) Nobody doubts that whoever believes that the seventh consulate of Marius lasted less than a fortnight believes that the seventh consulate of Marius lasted less than a fortnight,

and

(8′) Nobody doubts that whoever believes that the seventh consulate of Marius lasted less than a fortnight believes that the seventh consulate of Marius lasted less than a period of fourteen days.

Now Church calls attention to the fact that the German language has no single word which translate 'forthnight', so that the German translations of (7′) and (8′) are exactly the same German sentence *S,* which contains the very same expression 'Zeitraum von vierzehn Tagen' as the German translations of both 'forthnight' and 'period of fourteen days'. Because of this, the German translation of 'Mates doubts that (8′) but doesn't doubt that (7)' results in a direct formal contradiction.

The translation test turns on the usual (but not too clear, however; cf. Quine's misgivings) supposition that translation

preserves meanings, so that the identity of the German translations of (7′) and (8′) reflects a fact relevant to the meaning of the original English sentences: that they are synonymous in the strongest conceivable sense, and that the alleged difference in truth-value between (7′) and (8′) is impossible.

VIII. We may wonder now why not to apply the translation test to the problematic pairs (1)-(2). As Church's argument doesn't depend upon any empirical fact relative to German, being this fact handled as merely illsurative for a logical possibility, we might imagine a language L that, like German in relation with 'forthnight', doesn't have a single word for 'brother', but only a complex expression corresponding to 'male sibling'. Under this supposition the translations of (1) and (2) to L would result in the very same L sentence. Would this entail, contrary to White's conclusion, that (1) and (2) are synonymous in English, restating thus the paradox of analysis ? In footnote 21 of [Isomorphism], Church says that

> the existence of more than one language is not usually to be thought of as a fundamental ground for the conclusions reached by this method. Its role is rather as a useful device to separate those features of a statement which are essential to its meaning from those which are merely accidental to its expression in a particular language, *the former but not the latter being invariant under translation*" (p. 72, my italics).

So our imaginary language L may be a useful device for the same ends and if it is true that only the essential features of the meaning of (1)-(2) are 'invariant under translation', then it would seem that the supposed difference in meaning between (1) and (2) is an illusion arised from a 'merely accidental feature of these expressions in English'. I am not claiming that this is so; only that it seems to result from the application of the translation test as used by Church, who considers 'Zeitraum von vierzehn Tagen' as a correct German translation of 'forthnight' in the given example.

IX. Now this result is disturbing only if we admit the double supposition that an analysis is a non trivial statement of identity between concepts and that (1) is an example of analysis; but this supposition is whclly independent from the principles of Frege-Church semantics, so that our criticism doesn't imply necessarily an objection to the latter. It is possible to maintain, without giving up the Frege-Church's semantics, either (a) that 'the concept *A'* and 'the concept *BC'* have the same direct sense — like '*A'* and '*BC'* — and therefore (1) is as trivial as (2), in which case the concept *BC* is not an analysis of the concept *A*, or (b) that '*A'* and '*BC'* have different direct senses, like 'the concept '*A'* and 'the concept *BC'* and that the concept *BC is* an analysis of the concept *A* but (1) is false, in which case they aren't interchangeable *salva veritate* in the context '... is an analysis of ---'. Either of these possibilities would be more in agreement than White's solution with Church's approach to substitutivity of synonymous in psychological contexts [5]).

Consejo Nacional de Investigaciones Cientificas
Buenos Aires (Argentine) Thomas M. SIMPSON

(5) Different possibilities are taken into account in the current literature. In Carnap's *Meaning and Necessity,* for example, (i) *'A'* and *'BC'* are not synonymous (under the elucidation of synonymity as intensional isomorphism), nor (ii) are synonymous 'the concept *A'* and 'the concept *BC'*; (iii) the concept BC is an analysis of the concept *A,* and (iv) the identity (1) is true. More related with Frege-Church semantics is W. Sellar's proposal ('The Paradox of analysis: A neo-Fregean Approach') where the concept *BC* is an analysis of the concept *A* but (1) is false.