CLASSIFYING THE CLASS-MEMBERSHIP RELATION

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Consider the theory 'A saying of the form "x is a member of y" is absurd (i.e. neither true nor false) unless y immediately succeeds x in the series "Non-classes, first-order classes (i.e. classes of non-classes), second-order classes, third-order classes ... n-order classes, n + 1-order classes".' Call this theory 'T'.

It may look like T is incompatible with the claim that the class-membership relation is intransitive, asymmetrical and irreflexive. Say we symbolize 'ε is intransitive' as $(x)(y)(z)[((x\varepsilon y) . (y\varepsilon z)) \supset \sim(x\varepsilon z)]$ (S_1), 'ε is asymmetrical' as $(x)(y)[(x\varepsilon y) \supset \sim (y\varepsilon x)]$ (S_2), and 'ε is irreflexive' as $(x)[\sim(x\varepsilon x)]$ (S_3). The x and z in the consequent of S_1 are not consecutive; nor are the y and x in the consequent of S_2; nor, of course, are x and x in S_3. Consequently, by T, $x\varepsilon z$, $y\varepsilon x$, and $x\varepsilon x$ all yield absurd sayings. It is tempting to conclude, therefore, that $\sim(x\varepsilon z)$, $\sim(y\varepsilon x)$, and $\sim(x\varepsilon x)$ equally yield absurd sayings, which would mean that S_1, S_2 and S_3 are not true and the class-membership relation is not intransitive, asymmetrical or irreflexive.

One way to resolve this conflict is to deny that the absurdity of instances of $x\varepsilon z$, $y\varepsilon x$ and $x\varepsilon x$ entails the absurdity of instances of $\sim(x\varepsilon z)$, $\sim(y\varepsilon z)$ and $\sim(x\varepsilon x)$. To say, for example, that 'Trudeau is a member of the United Nations' is absurd *is* to imply that it and its falsifying negation, 'Trudeau is not a member of the UN', both fail to be true. But it does not imply that the negative saying 'It is not the case that Trudeau is a member of the UN' is not true. On the contrary, the latter is true. Consequently, if we restrict the instantiation of $\sim(x\varepsilon z)$ to sayings like 'It is not the case that Trudeau is a UN member', we can admit that $x\varepsilon z$ is absurd, deny that $\sim(x\varepsilon z)$ is absurd, and insist on the intransitivity of ε. Similarly, we can restrict the instantiation of $\sim(y\varepsilon x)$ to sayings like 'It is not the case that the UN is a member of Trudeau', and the instantiation of $\sim(x\varepsilon x)$ to sayings like 'It is not the case that Trudeau is a member of Trudeau', and in that way protect the asymmetry and irreflexibility of ε.

This move does resolve the clash between T and the desired classification of ε, and it means that the classification cannot be given as a reason for rejecting T. It does not protect T from all objections, however.

In certain cases, instances of $x\varepsilon z$, $y\varepsilon x$ and $x\varepsilon x$ are indeed absurd. For example, they are absurd in the contexts:

1. "Trudeau is a member of the UN. He was just admitted yesterday, along with Communist China and North Vietnam. It's not any too soon either, since his population is starving."
2. "The UN is a member of Trudeau, although I don't think it will be for long, since Trudeau's super-secretary-general is having a difficult time keeping his members united."
3. "Trudeau is a member of himself, and it's rather embarrassing because he keeps voting to give himself more money. I imagine some of his other members are pretty disgruntled."

If one were to claim that instances of $x\varepsilon z$, $y\varepsilon zx$ and $x\varepsilon x$ are false in the above contexts, one would have to say something like:

1'. "Trudeau is not a member of the UN. He was refused admission early today and now his population will really starve."

2'. "The UN is not a member of Trudeau. It considered applying for membership but decided not to precisely because Trudeau's super-secretary-general is having so much trouble."

3'. "Trudeau is not a member of himself. He simply attends meetings as an observer. He exerts a fair amount of influence on his members when he does this, however, and I think some of them are pretty unhappy as a result."

Both 1 and 1' mistakenly imply that Trudeau is a nation and hence both are wrong. Therefore each is absurd. Similarly, 2 and 2' both mistakenly imply that Trudeau is a second-order organization, and 3 and 3' both mistakenly imply that Trudeau is an organization. Moreover, the sayings 'Trudeau is not a member of the UN', 'The UN is not a member of Trudeau', and 'Trudeau is not a member of himself' are themselves absurd in the contexts 1′, 2′ and 3′, just as their affirmative counterparts are absurd in the contexts 1, 2 and 3.

The above facts do not entail, however, that what the sentences 'Trudeau is a UN member' and 'Trudeau is not a UN member' are used to say must be absurd in *every* context. For example consider the context:

4. "Trudeau is a UN member, even though he's obviously not a nation but a man. A newscast said he was elected yesterday. I imagine you've simply got your facts wrong when you say that every UN member is a nation."

4′. "Trudeau is not a UN member. Indeed, he cannot be, since he is a man. It's not just a contingent fact that UN members are nations, subject to the confirmation of newscasters. It's something dictated by the very concept 'UN member', since the UN, conceptually, is a class of nations which satisfy a certain description, and a member must therefore be a nation. The newscast probably reported that Trudeau was appointed UN *representative,* not that he was elected a member."

In the contexts 4 and 4′, the sentences 'Trudeau is a UN member' and 'Trudeau is not a UN member' are used to say something false and true respectively. Consequently, what they are used to say is not absurd. Similar contexts could be found in which what the sentences 'Trudeau is not a member of himself' and 'The UN is not a member of Trudeau' are used to say is true, and what their negative counterparts are used to say is false.

This means that, contrary to T, instances of $x \varepsilon z$, $y \varepsilon x$, and $x \varepsilon x$ need not all be absurd. They may be false. Similarly, instances of $\sim(x \varepsilon z)$, $\sim(y \varepsilon x)$ and $\sim(x \varepsilon x)$ may be true, even though they are not restricted to sayings of the form 'It is not the case that ...'. Consequently, one could preserve the symbolic versions of 'The class-membership relation is intransitive, asymmetrical and irreflexive' by stipulating that instances of $\sim(x \varepsilon z)$ do not presuppose either 'x is a class', or 'x is a class whose order immediately precedes that of class z', that instances of $\sim(y \varepsilon x)$

do not presuppose either 'x is a class' or 'y is a class whose order immediately precedes that of class x', and that instances of $\sim$(xεx) do not presuppose either 'x is a class' or 'x is a class whose order immediately precedes that of class x' ([1]). This is a more complicated restriction to state, but it has the advantage of being less severe than the demand that the appropriate instances all have the form 'It is not the case that ...'. It also allows that some instances of xεz, yεx and xεx will be false and not absurd, since they will be false in cases where they are free of the indicated presuppositions.

Such a restriction on the force of classifying ε as intransitive, asymmetrical and irreflexive might suggest a theory which could serve as a replacement for T, viz. 'A saying of the form "x is a member of y" will be absurd if it presupposes "x is a class whose order immediately precedes that of y" and y is not an immediate successor of x in the series "Non-classes, first-order classes ... n-order classes, n + 1 order classes".' (T_1). But since 'p presupposes q' here simply means that both p and p's falsifying negation entail q, and since 'absurd' means 'not true and not false', then T_1 amounts to nothing more than 'If both "x is a member of y" and its falsifying negation imply that x is a class whose order immediately precedes that of y, then "x is a member of y" will be neither true nor false if y is a class whose order precedes that of x.' And this is hardly very exiting. Thus, T is not true and its only available replacemecnt is uninteresting.

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([1]) Strictly, the first disjunct in each of the disjoined presuppositions is redundant, since (e.g.) 'x is a class whose order immediately precedes that of class z' entails 'x is a class' and therefore anything which presupposes the former also presupposes the latter. I've retained both disjuncts in each case to make things as explicit as possible.