IDENTITY, QUANTIFICATION AND PREDICABLES (*)

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The addition of identity and quantification to a logical calculus with individual constants is a simple matter. Quantification over the predicate place is quite a different matter, and syntax generally prohibits flanking the identity sign with predicate constants ('pious is identical with humble' is just ungrammatical). Frege and others have made much of these differences between individual constants (or singular terms) and predicate constants (or general terms) and the differences they reflect in the kinds of entities which may be referred to and the ones which may be predicated of other things (1).

Professor W.V.O. Quine has led a sustained attack on predicable entities (which he calls 'attributes'), based partly on the asymmetry between singular and general terms.
He says

> General terms, in contrast to singular ones, do not occur in positions appropriate to variables. Typical positions of the general term 'man' are seen in 'Socrates is a man', 'All men are mortal'; it would not make sense to write:
>
> (1) Socrates is an x, All x are mortal.
>
> or to imbed such expressions in quantifications in the fashion:

(1) The standard reference for FREGE is "Über Begriff und Gegenstand," *Vierteljahrsschrift für wissenschaftliche Philosophie*, XVI (1892), 192-205, translated as "On Concept and Object," in *Translations from the Philosophical Writings of Gottlob Frege*, ed. Peter GEACH and Max BLACK (Oxford: Blackwell, 1952), 42-55.

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(2) ($\exists x$) (Socrates is an x),

(3) (x) (all x are mortal $\supset$ Socrates is mortal).

> The 'x' of an open sentence may refer to objects of any kind, but it is supposed to refer to them one at a time; and then application of '(x)' or '($\exists x$)' means that what the open sentence says of x is true of all or some objects taken thus one at a time (2).

Quine continues to develop the asymmetry between singular and general terms by eliminating all singular terms from discourse, replacing each by its analysis according to Russell's theory of descriptions. Where the singular term is a name, it is rewritten as a predicate (true of just the purported referent of the name), and the whole analyzed as a description (3).

His final step is to eliminate variables too, by means of six operators (4). The ultimate in singular term-general term asymmetry has been reached: the first class of terms has no members.

The bulk of this paper is devoted to eliminating general terms and general term variables in a manner symmetrical to Quine's elimination of singular terms and variables. Throughout attention will be paid to maintaining those grammatical differences which exist between singular and general terms; in particular, quantification over general terms will be interpreted in a way which avoids treating them as grammatically singular terms. The result is to deny that any philosophically interesting asymmetry between singular and general terms is illuminated by discussing quantification.

In the course of eliminating predicates an identity sign must be developed for general terms. It will be extensional; two general

(2) Willard Van Orman QUINE, *Methods of Logic* (New York: Henry Holt, 1955), 206. Sections 33 and 34 bear reading in this regard as does Quine's essay "On What There Is" in *From a Logical Point of View* (Cambridge: Harvard, 1953).

(3) *Methods of Logic*, 218-227; *From a Logical Point of View*, 7-8.

(4) W. V. QUINE, "Variables Explained Away," *Proceedings of the American Philosophical Society*, CIV (1960), 343-47.

terms true of just the same things will be said to predicate the same entity. According to Quine, such an entity must be a class: "If someone views attributes as identical always when they are attributes of the same things, he should be viewed as talking rather of classes" (5). The paper will conclude with an attempt to shed some light on this claim.

THE ELIMINATION OF GENERAL TERM VARIABLES

The elimination of general terms must begin with quantification. What of Quine's contention that "General terms... do not occur in positions appropriate to variables" (6) of quantification ? It may be answered by proposing a reading of the quantifier which does not treat the variable as occupying the place of a singular term. Professor Wilfrid Sellars, for example, has suggested that '$(\exists F)(F(x))$' should be read simply as "x is something", and '$(F)(F(x) \supset F(y))$' might be read correspondingly as "y is everything x is" (7). (The 'something' in 'x is something'

(5) W. V. QUINE, *Set Theory and Its Logic* (Cambridge: Harvard at the Belknap Press. 1964), 2.

(6) *Methods of Logic,* 206.

(7) Wilfrid SELLARS, "Grammar and Existence: a Preface to Ontology," *Mind,* LXIX (1960), 499-533. Sellars adds that this interpretation of such quantifiers involves one in no ontological commitment; that question may remain below the surface here.

Mr. P. F. STRAWSON has questioned an aspect of this approach ("Singular Terms and Predication," *The Journal of Philosophy,* LVIII (1961), 393-412). He would allow quantification over a place or term signifying a "principle of grouping" (a predicable) only on the condition that "this term is coupled to another term which signifies a higher principle of grouping such principles of grouping." (405-5) "Suppose," he says, "the term 'Socrates' identifies the philosopher. Then 'There is something that Socrates is' is bound to be true, and 'Socrates is everything' (or 'There is nothing Socrates isn't') is bound to be false." (411) "In general, it will never be to the purpose to quantify over items of a higher type unless some still higher principle of collection is being implicitly used." (411).

If Strawson is right, the whole attempt to explain predicates away is misguided, for the attempt is begun with the notion that predicates can be eliminated through the use of singular terms and operators on may be replaced by a general term, say 'tall', yielding 'x is tall'. Instances of the universal generalization are 'if x is tall then y is tall', 'if x is pious then y is pious'.) Complicated formulas may read artifically, but most complicated quantified formulas do read artificially. A further artificiality, shared with Quine, will be to fit all predications into the mold of a predicate letter (representing a general term) followed by a suitable number of individual constants (in parentheses).

Part of Quine's technique in eliminating individual variables is to employ operators which generate conjunctive and negative predicates (8). We shall create conjunctive and negative subject expressions, treating, say, 'Tom, but not Dick' as the compound subject of the sentence 'Tom, but not Dick is hungry'. Since Quine's operators were defined on predicates only, and sentential operators were required, he treated sentences as no-place predicates (perhaps viewing the whole world as the subject of every complete predication) (9). We shall correspondingly treat sentences as no-place singular terms (that these are the only complete subject expressions reflects, perhaps, that the world is the totality of facts, not of things).

singular terms, just as Quine eliminates singular terms through the use of predicates. Were it necessary to introduce higher-order predicates, the symmetry with Quine's techniques would be lost.

Fortunately, Strawson's claim, which cannot be established by examples, can be defeated by them. Although 'Socrates is everything' is bound to be false, 'Socrates is everything Plato's father is' is not *bound* to be false. Rather its falsity is a condition of their being distinct persons. By contrast, 'Hesperus is everything Phosphorus is' is true, though not *bound* to be true, and its truth guarantees their identity.

Thus it will sometimes be to the purpose to quantify over items of a higher type without using any principle of some still higher type.

(8) "Variables Explained Away." 345.

(9) "Variables Explained Away," 344. Quine's operators are as follows: *Derelativization*: (Der$P$)$x_1 \ldots x_{n-1}$ if and only if there is something x_n such that $Px_1 \ldots x_n$.

Major inversion: (Inv P)$x_1 \ldots x_n$ if and only if $Px_n x_1 \ldots x_{n-1}$.

Minor inversion: (inv P)$x_1 \ldots x_n$ if and only if $Px_1 \ldots x_{n-2} x_n x_{n-1}$.

Reflection: (Ref P)$x_1 \ldots x_{n-1}$ if and only if $Px_1 \ldots x_{n-1} x_{n-1}$.

Negation: (Neg P)$x_1 \ldots x_m$ if and only if not ($Px_1 \ldots x_m$).

Cartesian multiplication: ($P \times Q$)$x_1 \ldots x_m y_1 \ldots y_n$ if and only if $Px_1 \ldots x_m$ and$Qy_1 \ldots y_n$

The technique for eliminating general term variables proceeds in this manner: the usual quantifier and its bound variable, say '$(\exists F)F(s)$', are replaced by the operator 'Dur' in guttural mimic of Quine's derelativization operator 'Der' (read, he remarked, as "Der is"). The result, '(Dur (s))', may be read "s is something". 'Dur' may be applied to a variable preceded only by other variables and quantifiers. So other operators are required to move the predicate letters to a position accessible to 'Dur'. These create the compound subject term. For example, the sentence 'Eve is something Adam isn't, and Eve is something to Adam', might be written in ordinary notation as

(4) $(\exists F)(F(e) \,\&\, —F(a)) \,\&\, (\exists R)(R(ea))$.

The second quantifier may be eliminated immediately.

(5) $(\exists F)(F(e) \,\&\, —F(a)) \,\&\, (Dur(ea))$,

but the variable bound by the first quantifier appears twice, once deeply imbedded in the formula. To begin its extraction, the subject '(a)' of '$—F(a)$' is replaced by its negation (or *nu*-gation), thus creating two positive occurrences of the variable 'F':

(6) $(\exists F)(F(e) \,\&\, F(\mathrm{Nug}(a))) \,\&(\mathrm{Dur}(ea))$.

The two occurrences of the variable 'F' can be coaxed together by creating a conjunctive subject '$((e)*(\mathrm{Nug}(a)))$', which could, if necessary, be read, 'Eve, but not Adam, is'. The result is

(7) $(\exists F)FF((e)*(\mathrm{Nug}(a))) \,\&\, (\mathrm{Dur}(ea))$.

This formula requires only a reflexivity operator, 'Ruf', to reduce the two occurrences of the variable 'F' to one,

(8) $(\exists F)F(\mathrm{Ruf}((e)*(\mathrm{Nug}(a)))) \,\&\, (\mathrm{Dur}(ea))$.

This single occurrence may be eliminated:

(9) $(\mathrm{Dur}(\mathrm{Ruf}((e)*(\mathrm{Nug}(a))))) \,\&\, (\mathrm{Dur}(ea))$.

The last ampersand gives way to an asterisk, conjoining the two-no-place subject terms into one:

(10) ((Dur(Ruf((*e*)*(Nug(*a*)))))*(Dur(*ea*))).

For more complex examples, operators for major and minor inversion may be required to get multiple occurrences of the same variable in place for reduction by 'Ruf'; so six operators in all are necessary. They are given as follows:

Dur: $P_1 \ldots P_{n-1}(\text{Dur}(s))$ if and only if $(\exists P_n)P_1 \ldots P_{n-1}P_n(s)$
Nug: $P_1 \ldots P_n(\text{Nug}(s))$ if and only if Not $P_1 \ldots P_n(s)$
Unv: $P_1 \ldots P_n(\text{Unv}(s))$ if and only if $P_nP_1 \ldots P_{n-1}(s)$
unv: $P_1 \ldots P_n(\text{unv}(s))$ if and only if $P_1 \ldots P_{n-2}P_nP_{n-1}(s)$
Ruf: $P_1 \ldots P_n(\text{Ruf}(s))$ if and only if $P_1 \ldots P_nP_n(s)$
* : $P_1 \ldots P_mQ_1 \ldots Q_n((s)*(r))$ if and only if
$(P_1 \ldots P_m(s)$ and $Q_1 \ldots Q_n(r))$

Eliminating General Terms

Quine suggested treating each name as a predicate true just of the thing named. For symmetry's sake we must treat each simple predicate as a singular term to which only that predicate could be truly attached.

What kind of singular term might this be ? Quine's singular-term-made-predicate might be thought of as the conjunction of all the predicates true of the named object. For example, 'is a horse', 'has wings', 'was captured by Bellerophon', and so on, are all predicates true of Pegasus. If the conjunction of these is symbolized by '*P*', then Pegasus is the one and only thing which is *P*.

If we construct the "conjunction", '*e*', of all the names of horses — 'Man o' War', 'Whirlaway', 'Native Dancer', 'Black Beauty', and so on — then a horse should be the one and only thing which (*e*) is. In the first place, '(*e*)' has as many places, that is, takes as many predicates, as it contains conjoined names. A 'Ruf' (the referential operator) prefixed for each '*' will keep the number of places down to one.

A more serious difficulty appears if we note that a horse is not the only thing which (*e*) is; it is also a mammal, possessor of a heart, and so on, for each one of Whirlaway, Black Beauty and the rest is a mammal, possesses a heart and so on. To construct a subject term of which 'is a horse' alone is true, we must add the names of all the things which are not horses. These names will include that of a dog, say 'Fido'. Being a dog, Fido is a mammal and has a heart; so "Nug(Fido)" does not have a heart and is not a mammal; it is, however, a horse, since Fido is not. Conjoining these nugations with the names of horses will thus give a compound subject expression to which 'is a horse' alone truly applies.

Let '(*h*)' represent this compound subject. Its English rendering might be 'the-exemplary-horse' (10). The predicate constructed from it will be a "definite *as*cription", analogous to Russell's definite *de*scriptions; we may say of something that it is *what the-exemplary-horse is (and is nothing else but)*. The formal version of the definite ascription requires an identity sign, and to keep from treating predicates as singular terms the identity sign for predicates must be another new subject.

Enter the cents-sign, as a two-place subject, governed by the two axiom schemata

(I) $\varphi\,\varphi\,\emptyset$
(II) $\varphi\,\psi\,\emptyset \supset (\varphi(\alpha) \equiv \psi(\alpha))$

Any instances of these are axioms (with 'φ' and 'ψ' replaced by any predicates and 'α' by any one-place singular term. These are usual enough axioms for identity, with the exception that the identity sign is now a singular term).

Saying that something, *b*, is a horse will be as simple as writing

(11) $(\exists F)(F(h)\ \&\ (G)(G(h) \supset FG(\emptyset))\ \&\ F(b))$.

The first clause says that the-exemplary-horse is something, the

(10) Professor R. L. CLARK suggested considering exemplars in this context, as well as some improvements in the exposition.

second that it is only that, and the third that *b* is that too. The whole says that *b* is what the-exemplary-horse is, namely, a horse.

(The claim here is not precisely that the subject term '(*f*)' as described above could be written for every predicate '*F*', but only that a singular term may be mated to each general term with the same plausibility that Quine mates a general term to each singular term. In practice, in fact, the definite ascription may be more easily come by: what the word 'orange' is used to predicate may be predicated by using the definite ascription, 'what our sofa and Hermione Gingold's hair, but not *her* sofa, are'. This is, of course, a contingent ascription, as 'the author of Waverley' is a contingent description of Scott.)

General terms have gone the way of singular terms; either can be eliminated provided that the language is supplied with a sufficient stock of the other; the moves to eliminate the one class of terms are quite symmetrical to those employed in eliminating the other; even the identity axioms are symmetrical. No interesting asymmetry persists through the consideration of quantification. Rather, an interesting and unwanted *symmetry* appears.

IDENTITY

Quine has said, "I deplore the notion of attribute partly because of vagueness of the circumstances under which the attributes attributed by two open sentences may be identified" (11). There is no vagueness in the above calculus, however; the identity axioms are satisfied by any pair of general terms true of just the same things.

Quine also said, "If someone views attributes as identical always when they are attributes of the same thing, he should be viewed as talking rather of classes" (12). Yet coextensive predicates may be substituted for each other without changing the

(11) *Set Theory and Its Logic*, 2.
(12) *Ibid.*

truth-value of any sentence in his language of explaining variables away, or in the one from which predicates have been explained away here. Does this mean that all talk of predicables is talk of classes, even if the syntactical proprieties are observed and a class-name is never introduced ? Or does it mean that these languages, and others Quine advocates, are not rich enough to reflect distinctions which he himself insists on ? (13) The point can be put more directly: is a non-extensional identity condition for predicables possible in a language which eschews higher-order predicates ?

This question is similar to that put forward by nominalists who have wished to explain higher-order predication as some kind of first-order predication, or else meta-linguistic talk in disguise. Their answers are generally unsatisfactory. Such sentences as

(12) Squareness is a shape

(13) Mauve is a color

are not analyzable as generalized conditionals nor in the formal mode. Rewriting 'Squareness is a shape' as

(12′) Whatever is square is shaped

would fail if we allowed the possibility of all and only squares being, say, mauve, for then it would be also true that

(14) Whatever is mauve is shaped.

but still quite false that

(15) Mauve is a shape.

Similarly, 'Mauve is a color' can hardly be replaced by the metalinguistic

(13) Mr. P. T. Geach has argued a similar point in "Class and Concept," *Philosophical Review,* LXIV (1955), 561-70.

(13′) 'Mauve is a color-word.

for what Kim Novak's eyes and hair were in one film is a color (mauve), but it is not true that

(16) 'What Kim Novak's eyes and hair were in a recent film'

is a color word [14]. The formal mode interpretation of higher-order predicates will not do.

It has been suggested that the condition for predicable identity is *necessary coextensiveness,* yet this seems to break down when definite ascriptions are employed. It is generally admitted that the expressions 'red' and 'the color of blood' can be used to predicate the same thing of things, but it seems reasonable to suppose that these predicates could have been used to predicate different things, that they are not *necessarily* coextensive, that blood might have had another color, that it might have been green [15].

[14] Someone may object here that the eyes and hair were not only mauve, but, say, colored and extended. He may amplify the definite ascription by picking some other thing, say her skirt, which was not mauve but was all these other things. The ascription will then read 'what Kim Novak's eyes and hair, but not her skirt, were in a recent film'. Definite ascriptions, like definite descriptions in ordinary language, most often rely on the context to make them truly definite.

[15] The question of necessary coextensiveness is similar to the question of necessary identity in motivation and treatment. Professor A. F. SMULLYAN pointed out long ago that there are *two* possible readings of '$\Box\ (s = (\imath x)Fx)$', that one of these implies that it is necessary that there be some unique thing which is F and identical to s; the other implies only that what is in fact uniquely F is also necessarily identical to s. Should we be very careful about the scope of our definite ascriptions, so that the necessary operator, '$\Box$', did not include in its scope the clause concerning blood's having a color, we should be left with the question of deciding why one predicable, contingently identified as the color of blood, should be necessarily coextensive with another. The modifications wfich follow in the paper seem to provide a partial answer (Smullyan's paper is "Modality and Description," *Journal of Symbolic Logic,* XIII (1948), 31-37. Professor N. L. WILSON has discussed it in the light of recent developments in "Modality and Identity: a Defense," *The Journal of Philosophy,* LXII, 18 (September 23, 1965), 471-77).

How *can* a first-order language be enriched to provide differentia for coextensive predicates without treating predicates as singular terms or importing singular terms correlated with them and higher-order predicates to attach to the singular terms? English provides a clue in the use of adverbs and adverbial phrases.

Suppose that all and only squares are mauve, so that everything square is shaped and everything mauve is shaped. We *can* construct a rubric which has different truth values depending upon whether 'square' or 'mauve' is substituted, namely,

(17) x is F in shape.

Instances are

(18) This patch is square in shape,

which (referring to the patch before me) is true, and

(19) This patch is mauve in shape,

which is false. The rubric

(20) x is colored F

will work the other way, and will be true if a definite ascription of a color is substituted for 'F':

(21) This patch is colored what Kim Novak's eyes and hair were in a recent film.

Among the other modifications of ordinary predication are those expressing purposes or intentions, such as 'for the fire' and 'to get warm' in

(22) Peter is chopping wood for the fire,

and

(23) Peter is chopping wood to get warm.

Some indicate a manner of performing an action, as 'vigorously' in

(24) Peter is chopping wood vigorously.

These adverbial modifications do not fit the standard mold of higher-order predications, that is, higher-order predicates applied to abstract singular terms. They cannot be symbolized in the form

(25) $\mathscr{T}(F^*)$

where 'F^*' is the abstract singular formed from the predicate letter 'F' and '$\mathscr{T}$' is a higher-order predicate. Rather, they force a new rubric, modifying a standard first-order predication, say,

(26) $F\text{-}\mathscr{T}\text{-}(s)$,

with '$\mathscr{T}$' an adverbial modifier. The second identity axiom for general terms might then be modified to read

(II′) $\varphi\psi\varnothing \supset (\varphi\ldots \supset (\psi\ldots))$

where the context indicated by blanks and dots may be a simple predicative context *or* a predicative context with an adverbial modifier (and 'φ' and 'ψ' are syntactical variables for predicates). The result is some subject-predicate asymmetry: the identity conditions for individual constants may be expressed simply in terms of first-order predicates, where now the identity conditions for general terms are not expressed simply in terms of subject expressions.

This suggestion and enrichment of logic brings with it new difficulties. For example, not all of the new predications are only contingently true or false:

(19) This patch is mauve in shape

seems necessarily false. There may be general terms which satisfy these conditions although they are not thought to predicate the same thing. The new adverbial modifiers need identity conditions, too, and the question of quantifying over them might be raised. These are questions for other papers. What the suggestions are meant to provide here is some formal elaboration of two prejudices: first, that coextensive general terms may not ascribe the same thing, and second, that neither quantification nor non-extensional identity-claims require treating general terms as proper names.

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