A TRANSLATION THEOREM FOR TWO SYSTEMS OF FREE LOGIC

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I

During the past decade and a half philosopher-logicians on the western side of the Atlantic have shown an increasing interest in languages which are free of existence assumptions (i) with respect to their *terms* and/or (ii) in the sense that their theorems are true in all domains *including the empty one*. On the western side of the Atlantic, logics free of existence assumptions in sense (i) are called *free logics*; logics free of existence assumptions in both senses are called *universally free logics*. For the purposes of the present paper the distinction is not important. So we shall use the expression "free logic" to refer to languages satisfying (i) and perhaps (ii).

These languages have proved useful in many areas. For example, free logic has been used by van Fraassen[17] to explore certain epistemological questions, by McCall[11] in metaphysics, by Hintikka[1,2,3] in epistemology, ontology and more recently in the area of modal logic, and by Lambert[4,5,6] in the theory of definite descriptions, ontology and to some extent, though informally, in set theory. Further intensive (though as yet unpublished) work, using free logic, is being done by Thomason[15] in modal logic, by Scott[14] in set theory and by Lambert[7] in the philosophy of science. A completeness proof for one version of free logic has been published by van Fraassen[16] and has been extended by van Fraassen and Lambert[18] to a language with definite descriptions. Two other papers, one by Leblanc and Thomason[8] and the other by Meyer and Lambert[12], presenting completeness proofs for still other formulations of free logic are forthcoming. In addition, Schock[13] has recently presented a novel formulation of free logic, complete with completeness proofs, and uses it in the philosophy of science.

A general feature of the languages mentioned above is the following. Some of the theorems of classical first order predicate logic are altered or dropped but the sense of the quantifiers found in most logic textbooks is retained. That is, systems of free logic developed (especially) in North America interpret the quantifiers as follows. Suppose a universe of exactly two objects a_1 and a_2. Suppose, further, a language to contain both referential names and at least one nonreferential name such that 'a_1' designates a_1, 'a_2' designates a_2 and 'b' designates nothing, i.e., 'b' is nonreferential. Then the sense of the quantifiers is conveyed in the following equivalences:

$$(x)Fx. \equiv .Fa_1.Fa_2,$$
$$(Ex)Fx. \equiv .Fa_1 v Fa_2.$$

In the 1920's and 1930's, on the eastern side of the Atlantic, Leśniewski[10] developed a language for the foundations of mathematics which in part was concerned with eliminating the same existence assumptions. Within this tradition, Lejewski[9] quite recently has constructed a language, L4, whose first order fragment, L4′, will concern us in this paper. This language departs in some important ways from the languages mentioned earlier. First, the classical predicate logic is retained. Second, the sense of the quantifiers in L4′ departs from that in the usual presentations of mathematical logic. Specifically, in L4′ the interpretation of the quantifiers can be expressed heuristically by means of the following equivalences. We suppose the universe, and the constants of the language, as above. Then

$$(x)Fx. \equiv .Fa_1.Fa_2.Fb,$$
$$(Ex)Fx. \equiv .Fa_1 v Fa_2 v Fb.$$

A noteworthy consequence of these distinct senses of the quantifiers is that in North American formulations of free logic a statement like '(Ex)(x does not exist)' is not provable, but in languages like L4′ it is.

Another difference between languages like L4′ and North American formulations of free logic, a difference of importance so far as our purpose in this essay is concerned, is the following. L4′ contains a nonclassical formulation of identity theory, whereas the languages with which it is being contrasted contain a classical formulation of identity theory. Specifically, in L4′ the binary predicate '=' is not reflexive; (x)(x=x) does not hold in L4′. It does, of course, in the contrasting formulations of free logic.

A natural question then is the following. Is it possible to find a translation of L4′ into some North American formulation of free logic such that if P is a formula in L4′ and P* is its North American transform (or vice versa), then $\vdash P \equiv P^*$ in some North American version of free logic (or vice versa) ? The main purpose of this essay is to answer the question in the affirmative.

The translation to be presented is important if for no other reason than that it provides for the first time, so far as we know, a way of interpreting at least the first order fragment of one version of Leśniewski's logical system called *Ontology* in more conventional parlance. Since Ontology is regarded as a general theory of objects the suggestion that the more widely known North American formulations of free logic can also be so construed is tempting indeed. Hence the translation has considerable philosophic interest.

II

Since we are here concerned only with the first-order fragment of Lejewski's L4, certain strategic though inessential simplifications and alterations require mention. One obvious change is from a system with axioms and rule of substitution to one with axiom schemata. The equivalence of these approaches is well known.

A second point concerns the matter of definitions. In L4′ the only important definition is that of the constant non-referential name 'O'. We introduce 'O' as primitive, and in the place of its definition we put axiom LA4′. Its correlate could be introduced in FL (1), the North American system of free logic we shall use,

(1) The system, FL, is described in Lambert's study "Free Logic and the concept of existence". See [6] in the bibliography to the present essay.

by means of description theory, as $(\iota x)(x \neq x)$. However, we will not consider languages with descriptions in this paper. Therefore, we extend FL by adding the primitive term 'O', and the axiom FA5, $\neg$E!O. We call this system FL^c.

Another principle common to Leśniewskian languages is extensionality. The form which we will need is provable as a metatheorem, so we will eliminate it from the basis of the system L4′. Other minor changes of no technical import (such as notation) will be clear as we proceed.

For those notions which will correspond one for one under our proposed translations, we will use the same signs. Thus 'x', 'y', 'z', etc. are the individual variables, 'O' is the individual constant (the non-referring name) and the propositional connectives are '$\supset$', '$\neg$', etc., with the usual meaning.

Since there are two notions of equality, we use '$=$' in FL and '$\doteq$' in L4′, and the two approaches to quantification are represented by (x), (Ex) in FL^c, [x], [Ex] in L4′. The usual relations

$$\neg(x)\neg Fx \equiv (Ex)Fx$$
$$\neg[x]\neg Fx \equiv [Ex]Fx$$

obtain under both interpretations of the quantifiers.

We will base both systems on the classical propositional calculus. The definition of well-formed formulas is as usual. Expressions such as 'Fx', 'Fy' indicate arbitrary wffs which contain the variables 'x' and 'y' in the same free positions (so that the usual restrictions on quantification apply). Both systems have the rule of modus ponens. L4′ contains the usual rule of Universal Generalization but FL^c contains the Hilbert-Ackermann version of Universal Generalization.

The axioms for L4′ are:

LA1 $[x]{:}P \supset .Fx{:} \supset {:}P \supset .[x].Fx$ (where 'P' is any wff not containing free 'x')

LA2 $[x].Fx. \supset .Fy$

LA3 $[xy]{:}x \doteq y. \equiv .[Ez].\, z \doteq x.z \doteq y$

LA4 $\neg O \doteq O$

The axioms for FL^c are:

FA1 $E!x. \supset :Fx. \supset .(Ey)Fy$

FA2 $(Ex)Fx. \supset :(Ex).E!x.Fx$

FA3 $x=x$

FA4 $x=y. \supset :Fx. \supset .Fy$

FA5 $\neg E!O$

The translations which concern us in this paper, to be denoted by * and °, are given by these inductive definitions:

Translation *:

The individual variables, constants, and the propositional connectives are identical under the translation. For any individual variables or constants α and β, the atomic formulas are translated by

$$(\alpha \doteq \beta)^* \rightarrow \alpha = \beta.E!\alpha$$

Given the translation of $F\alpha$ as $F^*\alpha$ (for any individual variable α),

$$([\alpha]F\alpha)^* \rightarrow (\alpha)F^*\alpha.F^*O$$
$$([E\alpha]F\alpha)^* \rightarrow (E\alpha)F^*\alpha.v.F^*O$$

Translation°:

The individual variables, constant, and the propositional connectives are identical under the translation. For any individual variables or constants α and β, the atomic formulas are translated by

$$(\alpha = \beta)^\circ \rightarrow \alpha \doteq \beta.\neg\alpha \doteq \alpha.\neg\beta \doteq \beta$$
$$(E!\alpha)^\circ \rightarrow \alpha \doteq \alpha$$

Given the translation of $F\alpha$ as $F^\circ\alpha$ (for any individual variable α),

$$((\alpha)F\alpha)^\circ \rightarrow [\alpha]:\alpha \doteq \alpha. \supset .F^\circ\alpha$$
$$((E\alpha)F\alpha)^\circ \rightarrow [E\alpha].\alpha \doteq \alpha.F^\circ\alpha$$

To show the equivalence of L4′ with FL^c by these translations is a four part task:

1) The axioms of FL^c, when translated, are theorems of L4′.
2) The axioms of L4′, when translated, are theorems of FL^c.
3) The equivalence of the two translations is provable in L4′, ie, $\vdash_{L4'} P \equiv P^{*\circ}$.
4) The equivalence of the two translations is provable in FL^c ie, $\vdash_{FL^c} P \equiv P^{\circ *}$.

It is easily verified that the definition of 'free' and 'bound' in respect to variables in both systems are identical under the translations, and that the rules of the systems are provable under the translations. Therefore, Part (1) (part (2)) is sufficient to establish that the theorems of L4′(FL^c) are theorems of FL^c(L4′), when translated. Parts (3) and (4) are necessary to show that the two translations are the inverses of one another, so that the systems L4′ and FL^c are isomorphic. That is, from (1) through (4) we could prove

	$\vdash_{FL^c} P$ implies $\vdash_{L4'} P^\circ$ by (1)
	$\vdash_{L4'} P^\circ$ implies $\vdash_{FL^c} P^{\circ *}$ by (2)
	$\vdash_{FL^c} P^{\circ *}$ implies $\vdash_{FL^c} P$ by (4)
hence	$\vdash_{FL^c} P$ if and only if $\vdash_{L4'} P^\circ$
and	$\vdash_{L4'} P$ implies $\vdash_{FL^c} P^*$ by (2)
	$\vdash_{FL^c} P^*$ implies $\vdash_{L4'} P^{*\circ}$ by (1)
	$\vdash_{L4'} P^{*\circ}$ implies $\vdash_{L4'} P$ by (3)
hence	$\vdash_{L4'} P$ if and only if $\vdash_{FL^c} P^*$

Under the translations given, the axioms of our systems become:

LA2.*	$(x)Fx.FO.\supset.Fy$
LA1.*	$(x):P.\supset.Fx:P\supset.FO:\supset:P\supset.(x)Fx.FO$
	$O=y.E!O$
LA3.*	$(xy).:x=y.E!x.\equiv:(Ez).z=x.E!z.z=y.E!z.v.O=x.E!O.$
LA4.*	$\neg(O=O.E!O)$

FA1.° $x \doteq x. \supset :. Fx. \supset :[Ey].y \doteq y.Fy$
FA2.° $[Ex].x \doteq x.Fx. \supset :[Ex]:x \doteq x.x \doteq x.Fx$
FA3.° $x \doteq x.v. \neg x \doteq x. \neg x \doteq x.$
FA4.° $x \doteq y.v. \neg x \doteq x. \neg y \doteq y: \supset Fx. \supset .Fy$
FA5.° $\neg O \doteq O$

To prove parts (3) and (4), we will use an inductive argument. If in L4′ we have the theorem

(i) $x \doteq y. \equiv :x \doteq y.v. \neg x \doteq x. \neg y \doteq y.x \doteq x$

which is $x \doteq y. \equiv (x \doteq y)^{*\circ}$
then all atomic formulas of L4′ obey (3).

If P is molecular, then P is of the form Q.R or ¬Q or [x].Fx. Since the propositional connectives are identical under translation, we consider only the quantifier case; assuming

$$Fx \equiv (Fx)^{*\circ}$$

then $[x]Fx \equiv ([x]Fx)^{*\circ}$

becomes (ii) $[x]Fx. \equiv :[x]:x \doteq x. \supset .Fx:FO$
Hence, if (i) and (ii) are theorems of L4′, part (3) is proven.

Similarly, in FL_e, we need

(iii) $x=y. \equiv :x=y.E!x.v. \neg (x=x.E!x). \neg (y=y.E!y)$

which is $x=y. \equiv .(x=y)^{\circ *}$

and

(iv) $E!x. \equiv :x=x.E!x$

which is $E!x \equiv (E!x)^{\circ *}$

to show that (4) holds for atomic wffs, and for molecular wffs, that

(v) $(x)Fx. \equiv .:(x):x=x.E!x. \supset .Fx:O=O.E!O. \supset .FO$

which corresponds to

$(x)Fx \equiv ((x)Fx)^{\circ *}$

To complete parts (1) and (3) of the program, we will prove the following theorems in L4′:

LT1.	$x \doteq y. \supset .x \doteq x$	[LA3]
LT2.	$x \doteq y. \supset .y \doteq x$	[LA3]
LT3	$x \doteq y.y \doteq z. \supset .x \doteq z$	[LT2, LA3]
LT4.	$x \doteq y. \supset :z \doteq x. \equiv .z \doteq y$	[LT3, LT2]
LT5.	$\urcorner x \doteq x. \urcorner y \doteq y. \supset :z \doteq x. \equiv .z \doteq y$	[LT1]
LT6[a].	$x \doteq y. \vee . \urcorner x \doteq x. \urcorner y \doteq y: \supset :[z]:z \doteq x. \equiv .z \doteq y$	[LT5, LT4, LA1]
LT6[b].	$x \doteq y. \vee . \urcorner x \doteq x. \urcorner y \doteq y: \supset :[z]:x \doteq z. \equiv .y \doteq z$	[Similarly to LT6]
LT6[c]	$x \doteq y. \vee . \urcorner x \doteq x. \urcorner y \doteq y: \supset :x \doteq x. \equiv .y \doteq y$	[LT1, LT2]
LT6[d]	$x \doteq y. \vee . \urcorner x \doteq x. \urcorner y \doteq y: \supset :O \doteq x. \equiv .O \doteq y$	[LA4, LT2]
LT6[e]	$x \doteq y. \vee . \urcorner x \doteq x. \urcorner y \doteq y: \supset :x \doteq O. \equiv .y \doteq O$	[LA4]

The theorems LT6[a—e] show that any atomic formula, say Fx, is extensional, i.e., $x = y. \vee . \urcorner x = x. \urcorner y = y. \supset :Fx. \equiv .Fy$

To prove extensionality in general, i.e. LT7, by an inductive argument (²), we use these theorems of propositional logic:

$Fx. \equiv .Fy: \supset : \urcorner Fx. \equiv . \urcorner Fy$
$Fx. \equiv .Fy:Gx. \equiv .Gy: \supset :Fx.Gx. \equiv .Fy.Gy$

and this theorem of quantifier theory:

$[z]:Fz. \equiv .Gz: \supset :[z]Fz. \equiv .[z]Gz$

The details of the proof are left to the reader.

LT7. $x \doteq y.v. \urcorner x \doteq x. \urcorner y \doteq y: \supset :Fx. \supset .Fy$

(²) Inspection of the axioms of L4′ shows that a noninductive proof of LT7 below is impossible. For some purposes, an inductive proof may be inadequate, for example, where L4′ is extended by adding new symbols. Under these circumstances, one is required to introduce FT7 by Leśniewski's rule of extensionality ([10] p. 258).

LT8. $x \doteq x. \supset :Fx. \supset .[Ey].y \doteq y.Fy$ [LA2]
LT9. $[Ex].x \doteq x.Fx. \supset .[Ey].y \doteq y.Fy$ [LA1, LA2]
LT10. $x \doteq x. \vee . \neg x \doteq x. \neg x \doteq x$ [P.C.]

LT8, LT9, LT10, LT7, and LA4′ are the translations of the axioms of FL^c. Part (1) is completed.

LT11. $x \doteq y. \equiv :x \doteq y. \vee . \neg x \doteq x. \neg y \doteq y:x \doteq x$ [LT1]
LT12. $\neg x \doteq x. \supset :FO. \supset .Fx$ [LA4, LT7]
LT13. $[x]:x \doteq x. \supset .Fx:[x]: \neg x \doteq x. \supset .Fx. \supset .[x]Fx$ [LA1, LA2]
LT14. $[x]:x \doteq x. \supset .Fx:FO: \supset .[x]Fx$ [LT12, LT13]
LT15. $[x]:Fx. \supset :[x]:x \doteq x. \supset .Fx:FO$ [LA1, LA2]
LT16. $[x]:Fx. \equiv :[x]:x \doteq x. \supset .Fx:FO$ [LT14, LT15]

LT11 and LT16 show that part (3) is complete.

To do parts (2) and (4) we will have to work in FL^c:

FT1. $(x)Fx. \supset :(x):E!x. \supset .Fx$ [FA1, General.]
FT2. $(x)Fx. \equiv :(x):E!x. \supset .Fx$ [FT1, FA2]
FT3. $(x).P \supset Fx. \supset .P \supset (x)Fx$ [FA1, Gen., FA2] (³)
FT4. $(x).P \supset Fx.P \supset FO. \supset :P \supset .(x)Fx.FO$ [FT3] (³)
FT5. $(x)Fx. \equiv .:(x):x = x.E!x. \supset .Fx:O = O.E!O. \supset .FO$ [FT2, FA3, FA5]
FT6. $E!x. \equiv .x = x.E!x$ [FA3]
FT7. $x = y.E!x. \supset .(Ez).z = x.E!z.z = y$ [FA1, FA3]
FT8. $(Ez).z = x. \supset .E!x$ [FA2, FA4, Gen.]
FT9. $(Ez).z = x.z = y. \supset .x = y$ [FA4, Gen.]
FT10. $x = y.E!x. \equiv .(Ez).z = x.E!z.z = y$ [FT7, FT8, FT9]
FT11. $x = y.E!x. \equiv :(Ez).z = x.E!z.z = y.E!z. \vee . O = x.E!O.O = y.E!O$ [FT9, FA5]

FT4, FT6, FT2 are the translations of the axioms LA1, LA3, LA4, respectively. Left unproven to complete part (2) is

(³) In FT3 and FT4, no free 'x' occurs in P.

(☆1) (x)Fx.FO. ⊃ .Fy

which is the translation of LA2.

FT10 and FT11 are the equivalences (iv) and (v) of the program's part (4). Left unproven is

(☆2) x=y. ≡ :x=y.E!x.v.¬E!x.¬E!y

which is the equivalence (iii).

It is clear that (☆2) is equivalent to

(☆3) ¬E!x.¬E!y. ⊃ .x=y

Likewise, from (☆1) we may prove

a.	(x):.E!x. ⊃ :¬E!x. ⊃ .x=y	[P.C.]
b.	(x):¬E!x. ⊃ .x=y	[a, FA2]
c.	(x):¬E!x.¬E!O. ⊃ .x=O	[b]
d.	¬E!O.¬E!O. ⊃ .O=O	[FA3]
e.	¬E!x.¬E!O. ⊃ .x=O	[c,d, 1]
f.	(x):¬E!x.¬E!y. ⊃ .x=y	[b]
g.	¬E!x.¬E!x. ⊃ .x=y	[e,f, 1]

The converse, that is, the deducibility of (☆1) from (☆3), also holds as we shall see shortly.

None of these propositions has been proven in FL^c, so as matters stand, all that has been shown is that every theorem of FL^c is also a theorem of L4′ (under the translation).

We will see in the final section of this paper that these propositions are not provable in FL^c, although they are consistent, for they state the equivalence of all non-referring names.

To complete our program, we will consider the system FL^t, which is Fl^c with the additional axiom:

FA6. ¬E!x.¬E!y. ⊃ .x=y

First, we verify that its translation is a theorem of L4′. Then we prove in FL^t:

FT12.	$x=y. \equiv :x=y.E!x.v.\neg E!x.\neg E!y$	[FA6]
FT13.	$\neg E!x. \supset .x=O$	[FA6, FA5]
FT14.	$\neg E!x. \supset :FO. \supset .Fx$	[FT13, FA4]
FT15.	$E!x. \supset :(x)Fx. \supset .Fx$	[FA1]
FT16.	$(x)Fx.FO. \supset .Fy$	[FT14, FT15]

Therefore, L4′ and FL^t are isomorphic.

III

FL^t is representative of North American versions of free logic minus axioms FA5 and/or FA6. Indeed the system FL, from which FL^t is obtained, consists of axioms FA1-FA4. The translation constructed in Section II makes it clear, therefore, that North American versions of free logic are weaker than formulations in the Leśniewski tradition in the sense that the set of theorems of FL, upon translation, constitute a subset of the theorems of L4′. This fact has interesting philosophical consequences.

First, as has been noted above, if we introduce definite descriptions into the language FL, define 'O' as '$(\iota x)(x \neq x)$', and replace 'O' everywhere in the axioms of FL^t by the proposed descriptional definiens, the resulting language is similar to van Fraassen and Lambert's FD_2[18], a system which is known to be consistent and complete. If we drop the extensionality axiom FA6 from FL^t, but retain FA5 and treat 'O' as above, the resultant system is a fragment of van Fraassen and Lambert's theory, FD[18], a language which may be more appropriate for intentional discourse, as they have pointed out elsewhere. Van Fraassen and Lambert[18] have shown the theory FD to be complete relative to a certain class of models. The statement which results by replacing 'O' with '$(\iota x)(x \neq x)$' in FA6 is not valid in FD; hence it is not derivable in FD. This is the basis of the remark in Section II that statements (☆1) — (☆3), are not provable in FL^c. For FA6 is (☆3). The upshot of these remarks is that North American versions of free logic permit finer inferential distinctions than does L4′.

Another important consequence of the translation in Section II is the following. Consider the language FL^t. As pointed out in Section I, on the intended interpretation of the quantifiers, the statement '(Ex)(x does not exist)' is not provable in FL^t. However, it is possible, given at least one nonreferential constant, to introduce a new set of quantifiers '[x]Fx' and '[Ex]Fx' into FL^t (or even into FL) such that '[Ex](x does not exist)' would be provable. This possibility, of course, has been established in Section II where quantifier contexts of L4′ were translated into FL^t thusly:

'(x)(Fx).FO' for '[x]Fx'
'(Ex)(Fx)vFO' for '[Ex]Fx'

This fact suggests that we can have two kinds of particular quantifier; for example, one of which would be read as 'there exists' and the other as 'there are' or perhaps better 'some'. Accordingly, it is possible in FL^t (or even in FL) to hold consistently that something does not exist but it is false that there *exists* something that does not exist. Clearly, in virtue of the translation in Section II, a like result holds in Lejewski's L4′. This result blunts the charge of imminent if not actual inconsistency brought by Quine, among others, against those philosophers who would like to hold, for example, that there are propositions but they don't exist. The thesis in question can be expressed in FL, FL^t and L4′ as

[Ex](x is a proposition) . ⌉(Ex)(x is a proposition).

In other places both Lambert[6] and Hintikka[3] have pointed out that the theorem of FL (hence of FL^t), QC: $E!x \equiv (Ey)(y=x)$, is a fairly close analogue of Quine's famous dictum that to be is to be the value of a bound variable. Contrary to Lejewski[9], therefore, Quine's thesis does hold in FL^t, and, by virtue of the translation in Section II, in L4′ also. However, we agree with Lejewski that quantifying into a given context is not necessarily ontically committing, as Quine would have it. For given the quantifier '[Ex]Fx' it simply is not true that a sentence of the form "There is something who is so and so" implies that "There exists something who is so and so", and hence, by the theorem above, that "so and so exists".

Separating what is true in Quine's program from what is false (or at least debateable) hinges on the interpretation of the word 'value'. As Quine uses the word 'value' it refers to the purported designata of the *expressions* which are substitutable into the places occupied by free variables; it does *not* refer to the substituends themselves. If the word 'value' is allowed to refer both to substituends and to their designata, then Quine's dictum that to be is to be the value of a variable would be unacceptable. But, of course, this is to interpret Quine's dictum in a way contrary to his intentions. Therefore it is incorrect to offer the truth of '[Ex] (x does not exist)' as counterevidence to Quine's dictum that to be is to be the value of a variable. For the statement in question is true just in case one of the *substituends* of 'x' in 'x does not exist' is a nondesignating singular term. What the statement in question *is* counterevidence to is the claim that quantifying particularly into a context is ontically committing because it can be true independently of any (possible ?) object which would make it true. That is, for example, '[Ex](x does not exist)' does not imply '(Ex) (x does not exist)'. So what appears to be questionable in Quine's program is the assumption that there must be objects of one sort or another to make the quantified statements of a theory true.

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