WHAT IS A PROPOSITION ?

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§ 1. *Introduction*

Certain theories of modern logic have the purpose of defining interesting classes of linguistic expressions, such as the set of sentences of a language, or relations between expressions, such as derivability among formulas. Other theories aim at describing semantic relations between linguistic expressions and nonlinguistic objects, such as the relation of being the meaning of an expression. Yet a third kind of theories may give a direct analysis of non-linguistic objects which could stand in semantic relations to linguistic expressions.

This paper first propounds and discusses certain constructions of the second kind and then attempts an explication of the third kind of the notion of non-linguistic proposition. However, only a limited class of propositions (called "elementary propositions") will be explained, viz., propositions corresponding to the sentences of a language of elementary logic. Admittedly, this explication will have merely a remote connection with the problems of ordinary language. On the other hand, a tradition of logical semantics has accumulated since the 19th century dealing with technical and more or less formalized languages, and it may be worth while to attempt a solution of some problems encountered in such studies.

In writing this paper I have profited from comments and criticism of Professor A. Wedberg, University of Stockholm.

§ 2. *The semantics of elementary logic*

Elementary logic is the theory of classical (two-valued) first-order predicate logic with identity. The language of elementary logic to be constructed in this section will be denoted by "L".

The vocabulary of L consists of the following symbol shapes: (,), / (joint denial), $\exists$ (existential quantification), $=$ (identity); individual constants: $i_1, i_2, \ldots$, and individual variables: $x_1, x_2, \ldots$; and an infinite list of n-ary predicates ($n = 1, 2, \ldots$).

The *atomic formulas* of L are of the following form: $F(s_1, \ldots, s_n)$, where $s_1, \ldots, s_n$ are individual constants or variables and F is an n-ary predicate. In particular an atomic formula containing the identity symbol is of the form: $=(s_1, s_2)$. (We use italicization to indicate syntactic variables, taking symbols of L as values.) The set of *formulas* of L is the intersection of all sets Φ containing all atomic formulas of L and containing (A/B) and $\exists x(A)$ if A and B belong to Φ and x is an individual variable. The *sentences* of L are the formulas of L without occurrences of free individual variables.

By a *domain* D of individuals we understand, as usual, a non-empty set. The notation "$D^{(n)}$" is employed to denote the set of all ordered n-tuples $\langle i_1, \ldots, i_n\rangle$ such that $i_k (k = 1, \ldots, n)$ is a member of D. An *(extensional) interpretation* of L with respect to a domain D is a unary operation E_D, defined for individual constants i and n-ary predicates F, such that $E_D(i)$ is a member of D and $E_D(F)$ is included in $D^{(n)}$. A *valuation* of L with respect to D is a unary operation V_D, defined for individual variables x, such that $V_D(x)$ is a member of D. Now, by a *realization* of L we shall understand an ordered triple $\langle D, E_D, V_D\rangle$ such that D is a domain, E_D is an extensional interpretation of L with respect to D, and V_D is a valuation of L with respect to D.

Let the notation "$H_D(s)$" stand for "$E_D(s)$" if s is an individual constant, and for "$V_D(s)$" if s is an individual variable. The notion of *satisfaction* ("Sat") is that binary relation between formulas of L and realizations of L which always fulfils the following recursive conditions:

$\mathrm{Sat}(=(s_1, s_2), \langle D, E_D, V_D\rangle)$ if and only if
  $H_D(s_1)$ is identical with $H_D(s_2)$:
$\mathrm{Sat}(F(s_1, \ldots, s_n), \langle D, E_D, V_D\rangle)$ if and only if
  $\langle H_D(s_1), \ldots, H_D(s_n)\rangle$ is a member of $E_D(F)$;
$\mathrm{Sat}((A/B), \langle D, E_D, V_D\rangle)$ if and only if not both
  $\mathrm{Sat}(A, \langle D, E_D, V_D\rangle)$ and $\mathrm{Sat}(B, \langle D, E_D, V_D\rangle)$;

$\mathrm{Sat}(\exists x(A), \langle \mathrm{D}, \mathrm{E_D}, \mathrm{V_D} \rangle)$ if and only if it does not hold
for all valuations $\mathrm{V'_D}$ that not Sat $(A, \langle \mathrm{D}, \mathrm{E_D}, \mathrm{V'_D} \rangle)$,
where $\mathrm{V'_D}(y)$ is identical with $\mathrm{V_D}(y)$ for all individual variables y other than x.

On the basis of the notion of satisfaction we can now define some important semantic concepts. We say that the formula A *logically implies* the formula B("$A \Rightarrow B$" for short) if for every D, $\mathrm{E_D}$, $\mathrm{V_D}$, either not $\mathrm{Sat}(A, \langle \mathrm{D}, \mathrm{E_D}, \mathrm{V_D} \rangle)$ or Sat $(B, \langle \mathrm{D}, \mathrm{E_D}, \mathrm{V_D} \rangle)$. Furthermore, A is said to be *logically equivalent* to B ("$A \Leftrightarrow B$") iff $A \Rightarrow B$ and $B \Rightarrow A$.

By a *proper realization* of L we shall understand an ordered pair $\langle \mathrm{D}, \mathrm{E_D} \rangle$ such that D is a domain and $\mathrm{E_D}$ is an extensional interpretation of L with respect to D. The semantic notion of *truth* ("Tr") is defined as that binary relation between sentences P of L and proper realizations which always fulfils the condition:

$\mathrm{Tr}(P, \langle \mathrm{D}, \mathrm{E_D} \rangle)$ if and only if Sat $(P, \langle \mathrm{D}, \mathrm{E_D}, \mathrm{V_D} \rangle)$ for all $\mathrm{V_D}$.

Now, a *statement* is a sentence P paired with a proper realization R. A statement $\langle P, \mathrm{R} \rangle$ is true if $\mathrm{Tr}(P, R)$ and false otherwise.

When applying L, we select a domain D and assign an interpretation $\mathrm{E_D}$ to the individual constants and the predicates. Hence, such an application determines a proper realization. If L embraces names and predicates from a natural language, some of the applications of L may be characterized as "ordinary" or "normal" with respect to a specific group of users. A proper realization determined by a normal usage we call a *normal realization*. By a *normal statement* we understand a statement $\langle P, \mathrm{R_u} \rangle$, where $\mathrm{R_u}$ is a normal realization with respect to the group U of users.

A sentence is an abstract shape which can be represented by concrete linguistic occurrences. Within a given application of L, a sentence may be said to *express* a statement $\langle P, \mathrm{R} \rangle$ if P is the sentence in question and R is determined by the application.

Logical equivalence is an equivalence relation and thus defines a partition of the set $\mathscr{P}$ of elementary sentences into a set of pairwise disjoint, non-empty subsets of $\mathscr{P}$ the union of which equals $\mathscr{P}$. The logical equivalence class of which A is a member is denoted by "$[A]$". Hence, $[P]$ is the set of all elementary sentences

Q such that $Q \Leftrightarrow P$. Within a given application of L, a sentence Q may be said to *express* the abstract linguistic object $\langle [P], \mathrm{R} \rangle$ if Q is a member of $[P]$ and R is determined by the application.

By a *meaning postulate* in a language $\mathscr{L}$ we understand a sentence or formula which restricts the range of possible realizations of $\mathscr{L}$. Obviously, a formula can be a meaning postulate with respect to one group of users without having that status in regard to some other group. If a meaning postulate M in $\mathscr{L}$ can be represented in L as an equivalence, M may (under certain conditions) be called a *definition* in $\mathscr{L}$. For example, if the definition $F(x) \leftrightarrow G(x)$ is adopted in L (where "$A \leftrightarrow B$" abbreviates "$(A/B)/((A/A)/(B/B))$"), the range of realizations of L is confined to realizations wherein $\mathrm{E_D}(F)$ is identical with $\mathrm{E_D}(G)$; and if the meaning postulate $F(x) \rightarrow G(x)$ is added to the axioms of L (where "$A \rightarrow B$" abbreviates "$A/(B/B)$"), all realizations of L must satisfy the condition that $\mathrm{E_D}(F)$ is included in $E_{\mathrm{D}}(G)$.

The notion of *analyticity* for elementary formulas can now be introduced as a ternary relation holding between a formula A, a set Δ of definitions or meaning postulates, and a language $\mathscr{L}$ of elementary logic embracing A and the members of Δ. In particular, we introduce the following scheme of definition:

Analytic $(A, \Delta, \mathscr{L})$ = def. $\Delta \Rightarrow A$, where A and the members of Δ are formulas of $\mathscr{L}$.

Here we presuppose, of course, that the relation of satisfaction has been defined for $\mathscr{L}$.

The relation of *analytic equivalence* will also be useful:

Analytically equivalent $(A, B, \Delta, \mathscr{L})$ = def. $\Delta \Rightarrow (A \leftrightarrow B)$, where A, B, and the members of Δ are formulas of $\mathscr{L}$.

Analytic equivalence is an equivalence relation, and in a specific language the analytic equivalence class (relative to Δ) of which A is a member can be denoted by "$[A]\Delta$".

§ 3. *Requirements for propositions*

In this section we shall formulate certain necessary conditions of adequacy for explications of the notion of nonlinguistic proposition. We shall then consider some possible explications and investigate how they cope with these conditions.

In trying to explicate the nature of propositions, I shall take the following requirements for granted:

(R1) A proposition is an abstract object, i.e., it does not exist concretely (in space-time).

(R2) There are nondenumerably many propositions.

(R3) Not every proposition is atomic. A compound proposition is built up inductively from atomic propositions by means of explicitly stated primitive operations.

(R4) Propositions are independent of language, i.e., not all propositions are to contain linguistic objects as parts.

(R5) If a sentence expresses something, p, then p is a proposition.

(R6) If *A* and *B* both express the proposition p, then *A* is analytically equivalent to *B*.

(R7) If the elementary formulas *A* and *B* express the propositions p and q respectively and have the same truth-value for all assignments of values to the individual symbols, then p need not be identical with q.

(R8) If p and q are expressed by *A* and *B* respectively, and if *A* is analytically equivalent to *B*, then p need not be identical with q.

Propositions with weaker identity conditions will result if (R8) is replaced by its contradictory. Extensionally wider notions of proposition, governed by (R8), may be needed in certain philosophical contexts, however.

Condition (R2) is necessary from the standpoint of classical set theory, because for all sets F of individuals in a denumerable domain there is the proposition (true or false) that i_k is a member of F (where i_k is an arbitrary individual). As a consequence of (R2), the set of formulas cannot be identified with the set of propositions.

Many philosophers have suggested that there is a relation of being a part which somehow describes the inner structure of propositions. Among the relevant references one could mention Bolzano [1837], § 558, Frege [1892], pp. 46 ff., Carnap [1947], pp. 30f., Martin [1963], pp. 137f. Of the cited authors, Bolzano has perhaps given the most profound general analysis of the notion of nonlinguistic proposition in spite of his comparatively rough technical equipment. However, none of these writers have quite clarified how propositions are generated from a set of simple parts, as required by condition (R3).

As an illustration we consider some explications where linguistic objects are essential constituents of propositions. Elementary propositions are identified with:

(I) Elementary statements, $\langle P, \mathrm{R} \rangle$;
(II) Elementary normal statements, $\langle P, \mathrm{R}_{\mathrm{U}} \rangle$;
(III) Analytic equivalence classes of elementary sentence shapes, $[P]\Delta$:
(IV) Ordered pairs consisting of an analytic equivalence class of elementary sentences and a normal realization of L, $\langle [P]\Delta, \mathrm{R}_{\mathrm{U}} \rangle$.

None of the approaches (II)-(IV) would fulfil requirement (R2) (nor (R4), of course). There are nondenumerably many sets of sentences but at most denumerably many logical or analytic equivalence classes of sentences. Under (III) and (IV), too many propositions will coincide. On the other hand, (I) and (II) do satisfy (R8), but possibly too few propositions will then coincide (e.g., $\langle P/Q, \mathrm{R} \rangle \neq \langle Q/P, \mathrm{R} \rangle$). Moreover, neither (III) nor (IV) make the inner structure of a proposition quite clear as required by condition (R3). (An identification similar to (I) is utilized in Kanger [1957], p. 4; method (III), or possibly something like (IV), is hinted at in Russell [1940], p. 209, and in Quine [1943], p. 120).

§ 4. *Concepts and extensions*

In giving an extensional interpretation of the language L we have presupposed the existence of classes. The explicit assumptions concerning classes can be developed in a classical set theory based on L and supplying quantification over relation variables.

We here presuppose such a *set-theoretical* basis containing, inter alia, a principle of extensionality and an axiom of choice. For our later constructions we also need, however, a realm of objects with stronger identity conditions than classes. These entities will be called *concepts*; they could be introduced by postulation in a way more or less similar to a system of set theory.

First we introduce the expression "$C(x_1, \dots, x_n, D, \gamma)$" as short for "the sequence $x_1, \dots, x_n$ of individuals in D comes under the concept γ". We then state explicitly the existence of concepts by the following postulates:

(P1) There is a γ such that for all $x_1, \dots, x_n$, $C(x_1, \dots, x_n, D, \gamma)$ if and only if A and $x_1, \dots, x_n$ belong to D,
where A is an atomic formula of L interpreted with respect to D and containing "x_1", ... , "x_n" as the only free variables.

By the construction of L, A does not contain the constant "C".

Now, every concept must have an *extension*. More precisely, with respect to every domain D:

(P2) There is a relation F such that for all $x_1, \dots, x_n$, (F is included in $D^{(n)}$ and $F(x_1, \dots, x_n)$) if and only if $C(x_1, \dots, x_n, D, \gamma)$.

The uniqueness of the relation F in (P2) then follows from the principle of extensionality of the basic set theory. The unique relation corresponding to the concept γ, with respect to D, we denote "$Ext_D(\gamma)$".

The extension of a concept can vary from domain to domain. If there is at most one individual x such that $C(x, D, \gamma)$ for all D, then γ is called an *individual concept*. Such concepts will be referred to by the symbols "ι", "ι_1", "ι_2", ... Hence the use of the variable "ι", e.g., may be explained thus: "for all ι, - - - ι - - -" stands for the phrase "for all γ, if γ is an individual concept, then - - - γ - - -". A concept which is not an individual concept is called a *relation concept*.

If we should wish to distinguish between individual and relation concepts before a principle similar to (P2) has been introduced, the only method apparently is to say that the former has an individual and the latter a set as an "extension". But then we must provide for this notion of "extension" in a way different from (P2). A further complication would be the need for a "null individual" to guarantee that every individual concept has an "extension".

Next we consider the question of principles of individuation. The usual set-theoretic definition of identity of individuals is presupposed: $=(x, y)$ if and only if for all F, $F(x) \leftrightarrow F(y)$. Coextensiveness of the concepts γ_1 and γ_2 in D of course means that $\mathrm{Ext}_D(\gamma_1)$ equals $\mathrm{Ext}_D(\gamma_2)$. Now if, for all D, the coextensiveness of γ_1 and γ_2 in D is *analytic in T*, where T is the basic set theory enlarged by the theory of concepts, then γ_1 equals γ_2. Hence we adopt the following *principle of individuation for concepts:*

(P3) $\gamma_1 = \gamma_2$ if and only if for all D, $\mathrm{Ext}_D(\gamma_1) = \mathrm{Ext}_D(\gamma_2)$ is analytic in T.

Therefore, two concepts may be coextensive in D without being coextensive in all D, and coextensive in all D without being equal. The principle (P2) establishes a mapping of concepts onto extensions but the converse, then, does not necessarily hold.

§ 5. *A system of elementary propositions*

One of the main obstacles in explicating the notion of nonlinguistic proposition is to analyse the propositional operations which correspond to generalization of formulas and to clarify the notion of variable at the propositional level. Free individual variables in formulas may be considered as vague names on a par with constants referring to specific individuals, and the corresponding parts of propositions could be ordered linearly. On the other hand, bound individual variables of a formula have no denotation at all, except for the general reference to the domain of individuals as a whole. To remove this obstacle in the elementary case we now construct a system of elementary propositions and follow the instructions (R1) through (R8) of § 3.

First we define *atomic propositional functions* as sequences of the form $\langle \gamma, \xi_1, \dots, \xi_n \rangle$, where γ is either an individual concept (n=1) or an n-ary relation concept ($n \geqslant 1$) and ξ_j ($j = 1, \dots, n$) is either an individual variable of L (cf. McKinsey [1949], p. 431) or an individual concept. Next we determine the set of *propositional function* as the least class K such that: (i) all atomic propositional functions are in K, and (ii) all subclasses of C with a cardinal power not higher than that of the domain D are in K. This upper boundary on the power of the members of K is sufficient for our purpose. (Imposing no such boundary could lead to Cantor's paradox.)

Some propositional functions defined in this fashion "contain" linguistic objects. More precisely, we say that a variable x is *contained in* the propositional function b in one of the following two senses:

(i) if $b = \langle \gamma, \xi_1, \dots, \xi_n \rangle$ for some $\gamma, \xi_1, \dots, \xi_n$, then x is contained in b if and only if $x = \xi_j$, for some $j = 1, \dots, n$;
(ii) if $b = \{a_1, a_2, \dots\}$, for some $a_1, a_2, \dots$, then x is contained in b if and only if x is contained in a_j, for some $j = 1, 2, \dots$. (We use braces to indicate sets.)

Now it is natural to define a *proposition* as a propositional function containing no variable. Intuitively, the passage from a concept γ and the individual concepts $\iota_1, \dots, \iota_n$ to the (n + 1)-tuple $\langle \gamma, \iota_1, \dots, \iota_n \rangle$ corresponds to the assertion that $\langle x_1, \dots x_n \rangle$ is a member of $\mathrm{Ext}_D(\gamma)$, where x_j is member of $\mathrm{Ext}_D(\iota_j)$, for some domain D. And the passage from the propositions $p_1, p_2, \dots$ to the set $\{p_1, p_2, \dots\}$ corresponds to the denial of the simultaneous assertion of p_1 and p_2 and etc. In particular, the passage from p to $\{p\}$ corresponds to the denial of p.

As a consequence of the notion of a sequence, we obtain the following *principle of individuation for atomic propositions:*

(P4) If $b_1 = \langle \gamma, \xi_1, \dots, \xi_m \rangle$ and $b_2 = \langle \gamma', \xi', \dots, \xi'_n \rangle$ are propositions, then $b_1 = b_2$ if and only if $\gamma = \gamma'$ and $m = n$ and $\xi_j = \xi'_j$ for $j = 1, \dots, m$.

In case that $b_1 = \{a_1, a_2, \dots\}$ and $b_2 = \{a_1', a_2', \dots\}$ are propositions, their identity of course follows from the axiom of extensionality of the basic set theory.

Instead of creating propositions from relational and individual concepts, one might generate them from classes and individuals in an analogous way. Such a construction would not fit our purpose, however, since it would violate requirement (R7) of § 3 under an intuitively satisfactory mapping of the set of sentences into the set of propositions. For example, if the expression "$F(x) \leftrightarrow G(x)$" is true for all x, but not analytic, with respect to a domain and an extensional interpretation (F and G being classes), and if "$F(i_k)$" and "$G(i_k)$" are mapped onto the "propositions" $\langle F, i_k \rangle$ and $\langle G, i_k \rangle$ respectively (i_k being an individual), we get $\langle F, i_k \rangle = \langle G, i_k \rangle$ since $F = G$. Similar results would obtain for all "propositions" constructed in this manner.

§ 6. *The notion of truth for propositions*

A sentence *P* of L expresses a proposition relative to a certain realization R only. We may then say that p, expressed by *P* under R, is true if and only if Tr(*P*, R). (Carnap [1942], p. 90, construes "p is true" substantially as "*P* is true in the language $\mathscr{L}$ if *P* expresses p in $\mathscr{L}$, for all $\mathscr{L}$ and *P*". But then all propositions without expression in any $\mathscr{L}$ would be true.) This approach is in accordance with the traditional way in modern semantics of first constructing a formal system and then assigning an interpretation under which some formulas may be said to express propositions. It would seem intuitively more natural, though, to consider propositions as primary entities some of which are represented by certain linguistic objects. The notion of truth then has to be defined directly for propositions, and as soon as an acceptable mapping of the sentences of a language $\mathscr{L}$ into the set of propositions has been found, a sentence of $\mathscr{L}$ may be said to be true if the corresponding proposition is true.

The intuitive interpretation of the operations on concepts and propositional functions introduced in § 5 leads to the following definition of *truth for propositions* in a domain D:

(1.1) $\langle \gamma, \iota_1, \ldots, \iota_n \rangle$ is true in D if and only if $\langle x_1, \ldots, x_n \rangle$ is a member of $\mathrm{Ext}_D(\gamma)$, where x_j is member of the non-empty $\mathrm{Ext}_D(\iota_j)$ ($j = 1, \ldots, n$);

(1.2) $\{p_1, p_2, \ldots\}$ is true in D if and only if not all of $p_1, p_2, \ldots$ are true in D.

If γ is an individual concept, the proposition $\langle \gamma, \iota \rangle$ asserts (in relation to D) that γ and ι are coextensive (in D). The definition (1.1) is partial only; if ι_j, for some $j = 1, \ldots, n$, has an empty extension in D, the proposition $\langle \gamma, \iota_1, \ldots, \iota_n \rangle$ is not assigned any truth-value at all.

Our objective now is to define truth for the sentences of L in terms of truth for elementary propositions. To do so we need a mapping of the set of formulas of L into the set of propositional functions.

then $b^{x}_{\iota} = b$;

result of replacing all occurrences of the variable π in the propositional function b by the individual concept ι". The operation of *replacement* is then defined recursively for propositional functions as follows:

(2.1) if $b = \langle \gamma, \xi_1, \ldots, \xi_n \rangle$, where $x \neq \xi_j$ for all $j = 1, \ldots, n$,

First we let the notation "b^{x}_{ι}" abbreviate the phrase: "the

(2.2) if $b = \langle \gamma, \xi_1, \ldots, \xi_n \rangle$, where $x = \xi_j$ for some $j = 1, \ldots, n$, then $b^{x}_{\iota} = \langle \gamma, \eta_1, \ldots, \eta_n \rangle$, where $\eta_i = \xi_i$ for $i \neq j$ and $\eta_j = \iota$;

(2.3) if $b = \{a_1, a_2, \ldots\}$ is propositional function, then $b^{x}_{\iota} = \{a_1{}^{x}_{\iota}, a_2{}^{x}_{\iota}, \ldots\}$.

Now we are able to define the important notion of *existential quantification* into propositional functions relative to a domain D. If b is any propositional function and x is an individual variable, we shall use the notation "$\exists(x, \mathrm{b})$" for the set $\{\{a_1\}, \{a_2\}, \ldots\}$ of all $\{a_j\}$ such that $a_j = \mathrm{b}\,{}^{x}_{\iota_y}$ for some y of D such that $\mathrm{Ext}_D(\iota_y) = y$, where ι_y is a conceptual representative of y selected by an application of the axiom of choice. The formal definition is:

(3) $\exists(x, \mathrm{b}) =$ def. $\{\{\mathrm{b}\,{}^{x}_{\iota_y}\}$: y is a member of D$\}$.

We then map the set of formulas into the set of propositional functions by an operation f with the following properties:

(4.1) if $A = F(s_1, \ldots, s_n)$, then $f(A) = \langle\gamma, \xi_1, \ldots, \xi_n\rangle$, where γ is a concept with $\mathrm{Ext}_D(\gamma) = E_D(F)$ (cf. § 2) and where ξ_j is an individual concept with $\mathrm{Ext}_D(\xi_j) = E_D(s_j)$ if s_j is an individual constant and a variable otherwise;

(4.2) if $A = (B_1/B_2)$, then $f(A) = \{f(B_1), f(B_2)\}$;

(4.3) if $A = \exists x(B)$, then $f(A) = \exists(x, f(B))$.

In case (4.1), the existence of f is always guaranteed by the axiom of choice. Picking out such an f amounts to laying down an *intensional interpretation* of the language L in relation to D.

Now it follows that $f(A)$ is a proposition if A is a sentence of L, and the sentence A may be said to be true in the domain D with respect to the mapping f if and only if the corresponding proposition $f(A)$ is true in D.

Under such a mapping the requirements (R5) through (R7) are readily verifiable. The atomic propositions will contradict (R8), though. For suppose that for some D, A = "$F(i_k)$" expresses $f(A) = \langle\gamma, \iota\rangle$ and B = "$G(i_k)$" expresses $f(B) = \langle\gamma', \iota\rangle$, where $\mathrm{Ext}_D(\gamma) = E_D$("F"), $\mathrm{Ext}_D(\gamma') = E_D$("G"), and $\mathrm{Ext}_D(\iota) = E_D$("$i_k$"). Furthermore, assume that $A \Leftrightarrow B$. Then "$F(x) \leftrightarrow G(x)$" is valid in L and therefore, by (P3), $\gamma = \gamma'$. By (P4) we get $f(A) = f(B)$; and similar cases would obtain for all atomic elementary propositions.

On the other hand, (R8) will be fulfilled for non-atomic propositions under any mapping of sentences into propositions. For example, laws corresponding to the law of double negation of formulas: $A \leftrightarrow \sim\sim A$ (where "$\sim A$" stands for "A/A"), and the law of idempotence for conjunction of formulas: $(A \wedge A) \leftrightarrow A$ (where "$A \wedge B$" stands for "$(A/B)/(A/B)$"), will not hold for propositions, because $p \neq \{\{p\}\}$. Furthermore, if we send the sentences *P, Q, S* into the propositions p, q, s, respectively, the propositons corresponding to $((P \wedge Q) \wedge S)$ and $(P \wedge (Q \wedge S))$ differ, as they should: $\{\{\{\{p, q\}\}, s\}\} \neq \{\{p, \{\{q, s\}\}\}\}$. (A natural way to make the notion of proposition extensionally still wider is to strengthen the identity condition by substituting sequences $\langle a_1, a_2, \ldots \rangle$ for sets $\{a_1, a_2, \ldots\}$ of propositional functions.)

A replacement operation for elementary propositions, sending p into $p^{\gamma}_{\gamma'}$, may be modeled on (2.1)-(2.3). This operation can be extended to a simultaneous replacement of the distinct concepts $\gamma_1, \ldots, \gamma_n$ in a proposition, sending p into $p^{\gamma_1 \ldots \gamma_n}_{\gamma'_1 \ldots \gamma'_n}$. A prop osition p containing no defined concepts is then said to be valid in a domain D if $p^{\gamma_1 \ldots \gamma_n}_{\gamma'_1 \ldots \gamma'_n}$ is true in D for all sequences $\langle \gamma'_1, \ldots, \gamma_n' \rangle$ of concepts, where γ_j' is of the same type as γ_j.

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BIBLIOGRAPHY

B. BOLZANO [1837]. *Wissenschaftslehre* (Sulzbach).

R. CARNAP [1942]. *Introduction to Semantics* (Cambridge, Mass.).

R. CARNAP [1947]. *Meaning and Necessity* (Chicago).

A. CHURCH [1946]. A formulation of the logic of sense and denotation. *Journal of Symbolic Logic*, vol. 11, p. 31.

A. CHURCH [1951]. A formulation of the logic of sense and denotation. *Structure, Method and Meaning. Essays in Honor of Henry M. Sheffer* (New York), pp. 3-24.

G. FREGE [1892]. Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik*, N.F., vol. 100, pp. 25-50. [Reprinted in *Gottlob Frege: Funktion, Begriff, Bedeutung. Fünf logische Studien* (ed. G. Patzig, Göttingen 1962).]

S. KANGER [1957]. The Morning Star paradox. *Theoria*, vol. 23, pp. 1-11.

R. M. MARTIN [1963]. *Intension and Decision* (Englewood Cliffs).

J. C. C. Mc Kinsey [1949]. A new definition of truth. *Synthese,* vol. 7, pp. 428-433.

J. Myhill [1958]. Problems arising in the formalization of intensional logic. *Logique et analyse,* vol. 1, pp. 74-83.

J. Myhill [1963]. An alternative to the method of extension and intension. *The Philosophy of Rudolf Carnap* (ed. P. A. Schilpp, La Salle, Ill.), pp. 299-310.

W. V. Quine [1943]. Notes on existence and necessity. *Journal of Philosophy,* vol. 40, pp. 113-127.

B. Russell [1940]. *An Inquiry into Meaning and Truth* (New York).