ANALYTICITY, INFORMATIVENESS, AND THE INCOMPATIBILITY OF COLORS

Robert W. BEARD

The view that a sharp distinction must be drawn between analytic and synthetic sentences is current among many philosophers, and especially those whose approach depends upon the tools of mathematical logic. However, there are disagreements about how the distinction should be formulated. One of the requirements which an adequate explication of 'analyticity' must satisfy is that all analytic sentences must be devoid of factual content. But at the same time, there are sentences which we would normally want to classify as analytic, and yet they are in some sense or other informative. It is the question of how an analytic sentence may be informative without being factually informative that I want to consider in the present paper.

In order to come to any conclusion on this matter, we shall have to specify an explicatum for the notion of 'analyticity' I shall adopt for the purposes of this paper the proposal by Carnap in his article, "Meaning Postulates." (1) Roughly, Carnap's account comes down to the following. A state-description is a set containing every atomic sentence or its negation (but not both), and all sentences constituting a state-description are logically independent of one another. A sentence will be said to hold in a specified state-description if and only if it is true, given that every sentence in that state-description is true. The range of a sentence is the set of state-descriptions in which it holds and the total range is the set containing every state-description. An L-true (logically true) sentence is one whose range is the total range, i.e., one that holds in every state-description. An A-true (analytically true) sentence is one that holds in every state-description in which a specified set of meaning postulates holds. E.g., if 'All bachelors are unmarried men' is stipulated as a

(1) *Philosophical Studies*, 3 (1952), pp. 65-73. Repr. in *Meaning and Necessity*, Chicago, 2nd ed., 1956, pp. 222-229.

meaning postulate, then 'All bachelors are unmarried' will be A-true since it holds in every state-description in which the former holds. On the other hand, a sentence such as 'Horatio is a bachelor' will be F-true (factually true), for although it holds in *some* state-descriptions falling within the range of our meaning postulates, it does not hold in the *whole* range of those postulates.

One of the important features of Carnap's explication is that it brings out clearly the distinction between L-true and A-true sentences. Much of the controversy between defenders of the analytic-synthetic dichotomy, and those who oppose it, stems from confusion over the terms 'tautologous', 'analytic', and 'logically true'. On Carnap's analysis, 'tautologous' and 'L-true' are to be identified, but 'analytic' applies to something altogether different. For while all L-true sentences are analytic, the converse is not true.

The next question concerns the notion of informativeness. What complicates any discussion of this topic is that there are several non-logical ways in which the term is employed. But if we confine our attention to the factual or logical informativeness of a sentence, without regard for whether or not the information conveyed is new or surprising to the reader, then these non-logical senses may be ignored. Carnap defines the logical content of a sentence as a set of state-descriptions in which that sentence does not hold (2). This is reasonable since we tend to think of a sentence as being informative or having content when it rules out cases which seem to be otherwise possible. E.g., more information is conveyed by 'Horatio loves both Christine and Agatha' than by 'Horatio loves either Christine or Agatha', and the former holds in fewer state-descriptions than does the latter. The common view concerning the triviality of L-true sentence follows handily from this, for L-true sentences hold in every state-description, and, therefore, none are ruled out. For convenience, I shall henceforth refer to that portion of the total range of state-descriptions in which a given sentence holds as the L-range of that sentence, and the set of state-descriptions in which the sentence does not hold as its L-complement range. Carnap's explication of the notion of informativeness is intended to

(2) *Introduction to Symbolic Logic and Its Applications*, New York: Dover Publications, 1958, p. 21.

apply to the total set of state-descriptions. In other words, the informativeness of a sentence depends upon its L-complement range. A question arises about how we are to assess the logical content of A-true sentences. If we consider the total range, then any A-true sentence that is not L-true will have logical content. Thus analytic sentences will be informative. On the other hand, where the set of meaning postulates, P, is non-null, we may prefer to redefine 'logical content' in such a way as to consider only those state-descriptions in which P holds, i.e. the P-total range. Since every A-true sentence will hold in every state-description in the P-total range, then no A-true sentence will have logical content within that range. Also, since L-true sentences hold in the P-total range, then they too will have null logical content. F-true sentences may hold in some state-descriptions within the P-total range, but fail to hold in other state-descriptions within that range. In conformity with terminology used earlier, let us refer to the set of state-descriptions in which a given sentence holds within the P-total range as the P-range of that sentence, and the set of state-descriptions within the P-total range in which the sentence fails to hold as its P-complement range. The two foregoing notions of logical content may be stated as follows :

(A) A sentence S is informative (i.e., has L-content) $=_{df}$ the L-complement range of S is non-null.

(B) A sentence S is informative (i.e., has P-content) $=_{df}$ the P-complement range of S is non-null.

A peculiar consequence of these explications is worth noting. An L-false (i.e., self-contradictory) sentence fails to hold in any state-description, and thus would appear to be more informative than any consistent sentence (3). And yet it seems strange to maintain this. Rather, we would be inclined to say that L-false sentences are uninformative, though in a quite different sense from that in which we hold that L-true sentences are uninformative. The latter are uninformative because sentences which are true of every possible state say nothing, and L-false sentences are uninformative because

(3) A similar point is made by L. P. C. CUNNINGHAM in "Contradictory Premises Convey Infinite Information", *Analysis*, 23 (1963), p. 72.

their truth cannot be asserted of anything. The two senses can be reconciled by stipulating that the range of S must be non-null. Thus,

(C) A sentence S is informative (i.e., has L-content) $=_{df}$ both the L- and L-complement ranges of S are non-null.
(D) A sentence S is informative (i.e., has P-content) $=_{df}$ both the P- and P-complement ranges of S are non-null.

(C) provides us with an *absolute* notion of informativeness in the sense that the logical content of a sentence is independent of any other sentences except those occurring in state-descriptions. (D), on the other hand, would make the informativeness of a sentence dependent upon other sentences, viz., those appearing as meaning postulates. Thus, (D) provides an explication of a *relative* notion of informativeness. Yet neither of these explicata preserve certain features of our intuitive notion of informativeness.

Consider:

(i) A relict is a relict.

This sentence is L-true, and in any reasonable sense, uninformative.

(ii) A relict is a woman.

With respect to (i), (ii) is certainly informative. And yet, given the following definition of 'relict', (ii) is A-true.

(iii) A relict is a woman whose mate is dead.

This is the usual lexical definition, and would surely be included among the meaning postulates of any language in which 'relict' appears as a predicator. Thus (iii) is A-true, but it appears to say more than either (i) or (ii). Further,

(iv) A relict is a person whose mate is dead.

This is A-true since it holds in every state-description in which (iii) holds. Intuitively, (iv) is informative with respect to (i), but uninformative with respect to (iii). Moreover, although it is difficult to say whether (iv) is more (or less) informative than (ii), it does seem clear that the information conveyed by each of these sentences is in some sense different from the other.

Neither (C) nor (D) conforms to these intuitive considerations concerning the informativeness of sentences. If we were to adopt

(C), we would have to assert that (ii), (iii), and (iv) are always informative. If we adopt (D), then (i)-(iv) are each uninformative. What is called for is a *comparative* notion of informativeness.

(E) A sentence S is informative (i.e., has C-content) with respect to sentence $S' =_{df}$ (i) the range of S is non-null, and (ii) the set of state-descriptions in which S' holds, and S does not, is non-null.

According to (E), L-true and L-false sentences are uninformative with respect to each other and all other sentences. Moreover, any sentence is uninformative with respect to any sentence which entails it. All other intuitive characteristics concerning the informativeness of (i)-(iv) are likewise preserved.

Taking (E) as an explicatum for 'informativeness', the question whether analytic sentences are informative does not admit of a simple answer. Negative answers can be given for the special cases of L-true and L-false sentences. But the set of L-true sentences is only a proper subset of the set of analytic sentences. Of the remaining analytic sentences, one must ask, 'Informative with respect to *what*?)

A difficulty that confronts (E) as an explication is the following. Suppose that a factual sentence, S_1, holds in a set of state-descriptions, none of which describes an actual state of the universe. S_1, then, is F-false, but according to (E), it would be informative with respect to many other sentences, including all L-true sentences. I think that we would want to say that S_1 would be informative with respect to these other sentences *if it were true*, but not if it is false. This suggests the following modification of (E).

(F) A sentence S is informative (i.e., has C-content) with respect to sentence $S' =_{df}$ (i) S is true, and (ii) the set of state-descriptions in which S' holds, and S does not, is non-null.

In my subsequent remarks, (F) rather than (E) will be taken as an explicatum for the comparative notion of informativeness.

I want now to show how the comparative notion of informative-

ness bears on the solution of a particular philosophic problem, viz., that of the incompatibility of colors. Consider,

(v) Nothing is both red and green all over.

A problem arises because we want to hold that (v) is both informative and necessarily true. Some philosophers have argued that (1) since (v) is informative, then it must be synthetic, and (2) since (v) is necessarily true, it must be *a priori*. Therefore, there is at least one sentence which is *a priori* synthetic, and similar reasons can be given for thinking that there are also many others.

Both (1) and (2) have been defended and rejected by various authors. If we adopt (C) as an explicatum for 'informativeness', and, further, grant that (v) is informative, then the consequent of (1) does indeed follow. But it also follows that there are no A-true sentences beyond those which are L-true. For whatever reasons may be given for holding that (v) is informative in sense (C) can likewise be mustered in behalf of the view that (ii)-(iv) and other sentences of the same sort are informative. But restriction of the term 'analytic' to tautologies is clearly unacceptable. On the other hand, if we adopt (D) as our explicatum and (v) as a meaning postulate, then (v) will be A-true and thus necessarily true, but it follows that it will be uninformative. Thus the problem of color incompatibility will be solved by rejecting one of the premises that gives rise to it. This is often a legitimate procedure in philosophy, but it is surely counter-intuitive to contend that (v) is really uninformative. Still another alternative would be to concede that (v) is informative, but to deny that it is necessarily true (4). Again, this solves the problem by denying one of its assumptions. But it is simply not the case that we would say of a particular hue both that it is a shade of red and that it is a shade of green.

Explicatum (F) readily provides a way out of these difficulties. (v) is informative with respect to any L-true sentence because it is true and there are state-descriptions in which L-true sentences hold and (v) does not. But this is only half of the solution. What remains to be accounted for is the fact that (v) is necessary, i.e., that (v) is not only true, but analytic or A-true. A sentence will be A-true if

(4) This view is taken by John HILTON in "Red and Green All over Again", *Analysis*, 22 (1961), pp. 47-48.

it must be the case that one of the state-descriptions within its range is true. This is guaranteed for L-true sentences by the rules of truth for truth-functional connectives and quantifiers, and for A-true sentences by semantic rules for designators. To stipulate that a given sentence is a meaning postulate is not to completely characterize the semantic rules for designators occurring in it, but rather to restrict the class of admissible rules of designation. E.g., if 'M' and 'W' are predicators in L_i, and '(x) (Mx $\supset$ $\sim$Wx)' is included among the meaning postulates of L_i, then the rules of designation for L_i cannot include both " 'Mx' — 'x is a man' " and " 'Wx' — 'x is a man' ". Likewise, if both " 'Rx' — 'x is red' " and " 'Gx' — 'x is red' " are not admissible, then one of the state-descriptions in which '(x) (Rx $\supset$ $\sim$Gx)' holds must be descriptive of the real world, and, therefore, '(x) (Rx $\supset$ $\sim$Gx)' is necessarily true.

The contention that A-true sentences must be either wholly uninformative or somehow descriptive of linguistic usage has been responsible for a great deal of mischief in philosophy. In particular, the adoption of either alternative has led to the host of epistemological difficulties entailed by the doctrine of synthetic *a priori* sentences. It appears that much of this might have been avoided by paying closer attention to the senses in which a sentence may be asserted to be informative.

Florida State University Robert W. BEARD