THE EXTENSIONAL CLASSIFICATION OF CATEGORICAL PROPOSITIONS AND TRIVALENT LOGIC

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Recently, the 16 elementary propositions on two properties and 256 elementary propositions on three properties in divalent logic were classified on a purely extensional basis ([1]) clarifying and solving many problems in syllogistic theory. The expansion of this description to many-valued (polyvalent or, perhaps rather, multivalent) logic was shortly mentioned. Before reporting below further results of such research, it may be useful to consider certain aspects of the divalent case. Four elementary propositions on one property are conceivable:

αa : a exists and defines the universe of discourse U (this can be written as the conjunction $(Ea)(\overline{E}\bar{a})$).

βa : a and $\bar{a}$ exist, $(Ea)(E\bar{a})$.

γa : $\bar{a}$ exists but not a, $(\overline{E}a)(E\bar{a})$.

τa : nothing exists in U, $(\overline{E}a)(\overline{E}\bar{a})$.

The sixteen elementary propositions on two properties ωab are given in Table 1 in a somewhat different order from that of ref. [1], showing a close analogy to Table 1 of ref. [1]. The interpretation $\omega_1 a$, $\omega_2 b$ of each of the sixteen ωab is given, but has no bi-unique relation to the sixteen *signs of quantity* ωab. On the other hand, the conjunctions $(\omega' a[b])(\omega'' a[\bar{b}])$ taking in the case of ω' the universe of discourse consisting of b and in the case of ω'' U consisting of $\bar{b}$ form sixteen alternatives, each of which represents exactly one of the ωab. The same is true for the conjunctions $(\omega' b[a])(\omega'' b[\bar{a}])$ or in general, for any conjunction of the type $(\omega' \bar{a}[b])(\omega'' \bar{a}[\bar{b}])$ or $(\omega' \bar{b}[a])(\omega'' \bar{b}[\bar{a}])$. This rather surprising property comes from the fact that "immediate inferences" always are biunique for elementary propositions, e.g.

$$\alpha a \equiv \gamma\bar{a} \qquad \beta a \equiv \beta\bar{a} \qquad \gamma a \equiv \alpha\bar{a} \qquad \tau a \equiv \tau\bar{a}$$

The propositions nos. 7, 8, 11, 12, 13, 14 and 15 of our Table 1 were called *heterogeneous* in ref. [1] because they represent situations where both a and $\bar{a}$, as well as both b and $\bar{b}$ are non-empty. We are here going to refine the classification of the other, nine, homogeneous propositions. In the column $\omega_1 a$, $\omega_2 b$, the heterogeneous propositions are all marked $\beta\beta$. We call the four propositions nos. 5, 6, 9 and 10 homogeneous in one property, because one β is combined with one α or γ. We call the four propositions nos. 1, 2, 3 and 4 *homogeneous in two properties*, because both ω_1 and ω_2 are α or γ. Finally, we call the proposition no. 0 *hyperhomogeneous*, because $\omega_1 = \omega_2 = \tau$. In Appendix I, we are going to extend these remarks to the 256 elementary propositions on three properties in divalent logic.

We write composite propositions on n properties

$$\omega_1 + \omega_2 + \ldots + \omega_m \qquad a_1 a_2 \ldots a_n$$

as an exclusive disjunction of m different elementary propositions (either "$\omega_1 a_1 a_2 \ldots a_n$" or "$\omega_2 a_1 a_2 \ldots a_n$" or... or "$\omega_m a_1 a_2 \ldots a_n$" and only one of these propositions) and we introduced in ref. [1] special signs of quantity for four of the composite propositions on two properties:

$$\alpha ab \equiv \delta + \varepsilon + \lambda + \pi \; ab$$
$$\beta ab \equiv \zeta + \eta + \theta + o \; ab$$
$$\gamma ab \equiv \iota + \kappa + \mu + \rho \; ab$$
$$\tau ab \equiv \nu + \xi + \sigma + \upsilon \; ab$$

(The similarity of notation is suggested by the fact that αab, βab, ... are αb, βb, ... using [a] as universe of discourse). These composite propositions and four of their combinations are particularly frequent:

αab : all A are B, and A exist.
α + βab : some (or all) A are B.
α + τab : all A are B (without existential implication).
βab : some, but not all, A are B.
β + γab : some A are not B.
γab : no A are B, and A exist.
γ + τab : no A are B.
τab : A do not exist.

In ref. [1], it was emphasized that in many-valued logic, the number of elementary classes b^n produced by n properties may not necessarily have the same value for b as the number a of states of a given class. The number of elementary propositions is $a^{(b^n)}$; in divalent logic, where the classes are empty or not-empty, $a = 2$ and equal to b. Rosser and Turquette ([2]) describe many-valued logics by two numbers, M truth-values corresponding to our b and S which is so constituted that for S "designated" truth-values, propositions can be asserted, whereas for (M — S) "undesignated" truth-values, the propositions cannot be asserted. S is positive, but smaller than M (for divalent logic, M = 2 and S = 1). The writer hopes that he is excused to have a Peripatetic rather than a Stoic mind; he would not consider sentential calculus and deductive theory as much as the extensional classification discussed above. This classification presents a different existence-values. Hence, we have two mixed forms and one pure form of trivalent logic :

I. *Di-trivalent logic* $a = 3$ and $b = 2$. Each of the classes a and $\bar{a}$ have the existence-value E, $\bar{E}$ or $\tilde{\bar{E}}$. Hence, there occurs $3^2 = 9$ alternative ωa and $3^4 = 81$ ωab. They are the combinations of nine ωa[b] and nine ωa[$\bar{b}$]. The syllogistic theory is fairly complicated, since $3^8 = 6561$ ωabc are possible.

II. *Tri-divalent logic* $a = 2$ and $b = 3$. Each of the classes a, $\bar{a}$ and $\tilde{\bar{a}}$ have the existence-value E or $\bar{E}$. Hence, we have $2^3 = 8$ different ωa, $2^9 = 512$ different ωab (each being the conjunctions (ω'a[b]) (ω''a[$\bar{b}$]) (ω'''a[$\tilde{\bar{b}}$]) of three different universes of discourse consisting of b, $\bar{b}$ and $\tilde{\bar{b}}$) and 2^{27} different ωabc.

III. *Pure trivalent logic* $a = 3$ and $b = 3$. Each of the classes a, ā and ã have the existence-value E, $\bar{E}$ or $\tilde{\bar{E}}$. Hence, we have $3^3 = 27$ different ωa, $3^9 = 19683$ different ωab and 3^{27} different ωabc (as compared to 256 in divalent logic).

In case II, one can introduce the notation for the eight propositions on one property:

$$\alpha_1 a \equiv (Ea)(\overline{E\bar{a}})(\overline{E\tilde{a}})$$

$$\alpha_2 a \equiv (Ea)(E\tilde{a})(\overline{E\bar{a}})$$

$$\beta_1 a \equiv (Ea)(E\bar{a})(\overline{E\tilde{a}})$$

$$\beta_2 a \equiv (Ea)(E\bar{a})(E\tilde{a})$$

$$\gamma_1 a \equiv (E\bar{a})(\overline{Ea})(\overline{E\tilde{a}})$$

$$\gamma_2 a \equiv (E\bar{a})(E\tilde{a})(\overline{Ea})$$

$$\tau_1 a \equiv (\overline{Ea})(\overline{E\bar{a}})(\overline{E\tilde{a}})$$

$$\tau_2 a \equiv (E\tilde{a})(\overline{Ea})(\overline{E\bar{a}})$$

If one neglects the class ã, one obtains the four corresponding αa, βa, γa and τa.

Table 2 compares the 27 propositions on one property in the pure trivalent case III with the cases I and II. Thus, the nine examples involving the logical factor $(\bar{E}\tilde{a})$ in case III can be directly compared with the nine propositions in case I.

We have introduced a *"realistic" interpretation* where $\tilde{\bar{E}}$a indicates the possible, but not certain, existence of the class a; where $\tilde{\bar{E}}\bar{a}$ indicates the possible, but not certain, existence of the class ā; and where $\tilde{\bar{E}}\tilde{a}$ and Eã indicate the possible, but not certain, existence of both the classes a and ā (but Eã being incompatible with a and ā both being empty). As seen from Table 2, ten among the fifteen possible composite propositions formed from one, two, three or all four ω a in divalent logic are obtained by the "realistic" interpretation. The tautology $\alpha + \beta + \gamma + \tau$ a is obtained for the five last of the 27 propositions in Table 2.

Naïvely, one might expect that Eã and $\tilde{\bar{E}}$a give the same results in the "realistic" interpretation. However, there is one profound difference: $\tilde{E}a$ is compatible with τa whereas $E\tilde{a}$ prevents τa from subsisting. In other words, the doubtful existence of well-defined classes is not exactly equivalent with the certain existence of indefinite classes. This shows how important arguments based on the theory of types (the properties of classes being of a higher type than the properties of their members) can be in many-valued logics.

As expected, Table 2 is symmetric with respect to an exchange of a and $\bar{a}$ and a concomitant exchange of α and γ. A source of asymmetry is the atomic proposition $\tilde{E}\tilde{a}$ which is "realistically' compatible with the existence or not of both a and $\bar{a}$. However, it cannot nullify terms Ea or $E\bar{a}$ already present, and hence, five of the nine propositions containing $\tilde{E}\tilde{a}$ escape the "realistic" interpretation as tautologies.

It is indeed pertinent to ask with Rosser and Turquette ([2]) whether trivalent logic has any application to the real world. In the writer's opinion, the obvious candidate is scientific observations, where the existence of A can be confirmed, but where the absence of A frequently leaves the possibility open of a later observation of A. This line of thought may end with the di-trivalent case I showing the least numerical expansion of ωab and ωabc. It is remarkable that nine of the ten composite "realistic" propositions in Table 1 are represented once, and that $\alpha + \beta + \gamma$ a is the only one absent in case I. Colloquially speaking, the reason is the absence of $E\tilde{a}$ (as contrasted to $\tilde{E}a$ and $\tilde{E}\bar{a}$) in case I.

On the other hand, quantum-mechanical measurements might sometimes constitute an argument for case II, the certain existence of something about which it cannot be decided whether it is a or $\bar{a}$.

It is tempting to argue according to the severe of the two linear combinations of the authors in the dialogue (ref. [2], p. 3) that all many-valued logics give the impression of being combinations of divalent propositions. For instance, our argument above in case I would make (Ea) ($\tilde{E}\bar{a}$) more or less equivalent to a *non-exclusive* disjunction $\alpha + \beta$a. However, this distinction is slightly metaphysical. Somebody asserting "$\alpha + \beta$a" in the sense of ref. [1] is convinced that either "αa" or "βa", but he does not know which; somebody asserting "(Ea) ($\tilde{E}\bar{a}$)" in the "realistic "interpretation implicitly means "$\alpha + \beta$a" without stressing the point that one and only one, must be true. It is even more striking that the four composite propositions $(\alpha + \beta a)$, $(\alpha + \tau a,)$ $(\beta + \gamma a)$ and $(\gamma + \tau a)$ are exactly those commonly used by logicians in contrast to the two other involving two signs of quantity $(\alpha + \gamma a)$ and $(\beta + \tau a)$ and the four composite propositions involving three signs of quantity. Said in other words, the information supplied by factors such as (Ea), $(\bar{E}a)$ and $(\tilde{\bar{E}}a)$ produces the nine types of propositions found for case I. In the cases where only the universes of discourse [b] and $[\bar{b}]$ need to be considered, the combined propositions on two properties are the products $(\omega_1 + \omega_2 + \ldots a[b])$ $(\omega'_1 + \omega'_2 + \ldots a[\bar{b}])$ as can easily be constructed from Table 1. For instance

$$(\alpha + \beta a[b])\,(\gamma + \tau a[\bar{b}]) \equiv \delta + \lambda + \varepsilon + \pi ab$$

which, incidentally, is another way of writing αab.

Three similar cases occur:

$$(\alpha + \beta a[b])\,(\alpha + \beta a[\bar{b}]) \equiv \beta ab$$

$$(\gamma + \tau a[b])\,(\alpha + \beta a[\bar{b}]) \equiv \gamma ab$$

$$(\gamma + \tau a[b])\,(\gamma + \tau a[\bar{b}]) \equiv \tau ab$$

In a certain sense, αa and γa have equal levels of complication, whereas βa is distinctly more complicated, and τa is an outsider, quite different, saying something about the universe of discourse rather than about a. It may very well be that we have gone the long way over di-trivalent logic of case I to find propositions such as $(\alpha + \beta a)$ (but not $\beta + \tau a$) which have a higher and comparable level of complication in a purely divalent description.

Another plausible candidate for application of trivalent logic is Aristotle's *modal syllogisms*. Łukasiewicz (3) and McCall (4) have constructed formal models which to a smaller or larger extent reproduce Aristotle's results. It is, of course, very interesting to analyze these, highly complex, structures, but one may be interested in differing approaches as well. Thus, the extensional classification (1) concords with Aristotle in neglecting singular terms which would be represented by their corresponding one-member classes. On the other hand, the extensional classification accepts negative terms.

Łukasiewicz (³) emphasized that Aristotle formulated his syllogisms as implications rather than as inferences. However, it may be a dangerous dogma that, everything done by Medieval logicians was a regression relative to Aristotle (cf. also Bocheński's fascinating book (⁵)). In other cases, one may not have understood Aristotle at a certain time; when neglecting empty terms A, writing „All A are B" as our "αab" rather than our "$\alpha + \tau ab$", he has presumably made a deliberate decision and not a confusion between the two forms.

In the modal syllogisms, one uses a stronger property of A being *necessarily* B and a weaker property of A being *possibly* B. In the trivalent description, one might try to compare necessity with b, possibility with $b + \tilde{b}$, contingence (excluding necessity) with $\tilde{b}$ and impossibility with $\bar{b}$. However, these distinctions are of intensional character and not easy to make compatible with an extensional point of view, though it was discussed in ref. [1] why the intensional properties to some extent can be represented by classes having fewer or more properties defined. This is the reason why the singular terms do not occur in ref. [1]; it is sometimes acceptable to consider classes consisting only of other classes and not "modern" individuals as members.

There is one sense where our di-trivalent logic case I corresponds closely to one aspect of modal propositions. Łukasiewicz (³) discussed future events not yet known. If there occurs properties which cannot be verified at the moment, but which under other (temporal and spatial) conditions can be verified with certainty, the description $\tilde{E}a$ and $\tilde{E}\bar{a}$ seems satisfactory. It is worth emphasizing that these two atomic propositions are different; $E\bar{a}$ is the existence of something not having the property a and hence (in divalent logic) having the property $\bar{a}$, whereas $\bar{E}a$ corresponds to a not existing at all in the universe of discourse, and $\tilde{E}$ is the corresponding possibility of both E and $\bar{E}$.

It is relatively easy to perform syllogistic calculations in case I. If we denote the "realistic" interpretation by "$=$" (and not the equivalence "$\equiv$"), it is for instance a valid rule of inference (the specific universe of discourse each time being indicated in sharp brackets):

$$(Ea[m])(\tilde{E}\bar{a}[m])(\tilde{E}a[\bar{m}])(E\bar{a}[\bar{m}]) = (\alpha + \beta a[m])(\beta + \gamma a[\bar{m}]) \equiv \delta + \varepsilon + \zeta + \theta am$$

$$(\tilde{E}m[b])(E^{m}[b])(Em[\bar{b}])(\tilde{E}\bar{m}[\bar{b}]) = (\beta + \gamma m[b])(\alpha + \beta m[\bar{b}]) \equiv \eta + \theta + \iota + \kappa mb$$

$$\overline{\delta + \varepsilon + \zeta + \eta + \theta + \iota + \kappa ab}$$

and the total conclusion (1), consisting of all seven heterogeneous ωab, is fairly typical for such cases. (cf. also Appendix II and Table 3, propositions no. 69 and no. 68). Technically, the example is a syllogism of the fourth figure.

If we concentrate our attention on this di-trivalent case, one of the three states of a class, Ẽ, has very much the same relation to E and $\bar{E}$ as βa has to αa and γa. This is one extreme of making trivalent logic as similar to divalent logic as possible. By the same token, quadrivalent logic has a nearly unsurmountable tendency to disintegrate into the combination of two divalent logics, four being two times two. The opposite extreme is to consider all 27 propositions of Table 2 on equal footing and to make no divalent-like distinction between a, ā, ã or E, $\bar{E}$, Ẽ. The latter purely trivalent description may indeed look much more like a formal game. The writer does not intend to make appeal to great electronic computers, and hence, he does not intend to make syllogistic theory for this, very voluminous, case III. However, it is quite conceivable that this case may be of similar importance to formal logic as complex numbers to mathematics.

Appendix I. If the elementary propositions on more than one property in divalent logic are interpreted as propositions of the type αa_m, βa_m, γa_m or τa_m on each individual property, we may distinguish various possibilities:

Hyperhomogeneous proposition: if the universe of discourse is empty, one (no. zero) of the $2^{(2^n)}$ elementary propositions on n properties corresponds to τa_m for all a_m ($m = 1, 2, \ldots, n$).

Homogeneous proposition in one property: In (n — 1) cases, the interpretation is βa_m, and in one case, it is αa_m or γa_m.

Homogeneous proposition in two properties: In (n — 2) cases, the interpretation is βa_m, and in two cases it is αa_m or γa_m.

Homogeneous proposition in k properties: In (n — k) cases (non-negative integer) the interpretation is βa_m, and for k values of m, the interpretation is αa_m or γa_m.

Heterogeneous proposition: In all n cases, the interpretation is βa_m.

If the 256 elementary propositions on three properties ωabc in Table 1 of ref. [1] are considered,

no. 0 is hyperhomogeneous,

nos. 1-8 are homogeneous in three properties,

among nos. 9-36, twelve (nos. 9, 10, 12, 17, 19, 22, 25, 30, 31, 32, 35 and 36) are homogeneous in two properties; twelve (nos. 11, 13, 14, 16, 18, 21, 23, 26, 28, 29, 33 and 34) are homogeneous in one property; and four (nos. 15, 20, 24 and 27) are heterogeneous.

among nos. 37-92, twenty-four (nos. 37, 38, 39, 40, 43, 44, 46, 52, 53, 58, 64, 66, 67, 71, 75, 76, 78, 82, 87, 88, 89, 90, 91, 92) are homogeneous in one property and the remaining thirty-two heterogeneous.

among nos. 93-162, six (nos. 93, 102, 113, 142, 153 and 162) are homogeneous in one property and the sixty-four others are heterogeneous.

the nos. 163-255 are all heterogeneous.

If the interpretation of ωamb as $\omega_1$am, $\omega_2$mb, $\omega_3$ab is considered, the cases homogeneous in two or three properties have all three ω_1, ω_2, ω_3 homogeneous, whereas the cases homogeneous in one property have two of the three ω_k homogeneous.

A comparison of n = 1, 2, 3 does not reveal very conspicuous regularities:

	n = 1	n = 2	n = 3
heterogeneous	1	7	193
homogeneous in 1 property	2	4	42
homogeneous in 2 properties	—	4	12
homogeneous in 3 properties	—	—	8
hyperhomogeneous	1	1	1

The only obvious general rules are that 2^n of the elementary propositions on n properties are homogeneous in n properties; and that $2^n(n/2)$ propositions are homogeneous in (n — 1) properties.

Appendix II. Table 3 gives the 81 elementary propositions ωab on two properties in the di-trivalent case I with "realistic" interpretation. The arrangement is not exactly the same as in Table 1; the propositions nos. 0-15 are the ωab in divalent logic; then, the existing elementary classes (+) in the propositions nos. 1-15 are systematically replaced first by one, then by two, then by three, and finally by four (~).

It may be noted that the interpretation is symmetric in the sense that each of the sixteen ωab in divalent logic is represented an equal number of times, i.e. one among the propositions nos. 0-15, four among nos. 16-47, six among nos. 48-71, four among nos. 72-79 and once for proposition no. 80. This binomial coefficient structure (1, 4, 6, 4, 1) is similar to several other results discussed in ref. [1]. Hence, the 81 propositions are interpreted by disjunctions of altogether 256 ωab from divalent logic.

It is remarkable that proposition no. 62 represents αab, no. 71 βab, no. 65 γab, no. 53 τab, no. 79 α + βab, no. 74 α + τab, no. 78 β + γab and no. 75 γ + τab. This illustrates once more the relation between these particular composite propositions in divalent logic and a few of the elementary propositions in di-trivalent logic. One can form (16.15.14.13)/(2.3.4) = 1820 composite propositions from four elementary ωab in divalent logic; the four special propositions αab, ... are hence unexpectedly frequent in Table 3. This "unstatistical" distribution can be connected with another fact: Table 4 gives the sixteen interpretations of the conjunctions $(\omega_1 + \omega_2 a[b])\ (\omega_3 + \omega_4 a[\bar{b}])$ which are abbreviated $[\omega_1 + \omega_2]\ [\omega_3 + \omega_4]$, and the eight interpretations of the conjunctions $(\omega_1 a[b])\ (\alpha + \beta + \gamma + \tau a[\bar{b}])$ and $(\alpha + \beta + \gamma + \tau a[b])\ (\omega_2 a[\bar{b}])$ which are abbreviated $[\omega_1]\ [\psi]$ and $[\psi]\ [\omega_2]$, respectively, introducing the notation for the tautology

$$\psi a \equiv \alpha + \beta + \gamma + \tau a$$

These twenty-four propositions are represented each once among the propositions nos. 48-71 in Table 3. By the same token, the propositions nos. 16-47 correspond each either to one of the conjunctions $[\omega_1 + \omega_2]\,[\omega_3]$ or to one of the $[\omega_4]\,[\omega_5 + \omega_6]$, taken from the four composite propositions having the signs of quantity $\alpha + \beta$, $\beta + \gamma$, $\gamma + \tau$ and $\alpha + \tau$. Finally, the propositions nos. 72-79 in Table 3 correspond to the eight possible conjunctions of this kind $[\omega_1 + \omega_2]\,[\psi]$ and $[\psi]\,[\omega_3 + \omega_4]$. They are also included in Table 4. In an extensional treatment, one is asserting nothing new by forming a conjunction with the tautology ψ and one might as well write $[\psi]\,[\omega_3 + \omega_4]$ as $(\omega_3 + \omega_4 a[\bar{b}])$.

Hence, a system consisting of the nine signs of quantity

$$\omega_x : \alpha, \beta, \gamma, \tau, \alpha + \beta, \beta + \gamma, \gamma + \tau, \alpha + \tau, \psi$$

is complete in the sense that each of the 81 propositions in Table 3 has the "realistic" interpretation $(\omega'_x a[b])\,(\omega_x'' a[\bar{b}])$ written $[\omega_x']\,[\omega_x'']$ in Table 4.

Appendix III. Errata to ref. [1].

The writer did not have the opportunity to correct the proofs of the otherwise very carefully printed article. It may be useful here to correct a few distinct errors:

p. 235. The paragraph below the first two columns of equivalencies should read:

and all the other "immediate inferences" e.g. of the form $\omega_1 ab \equiv \omega_2 \bar{a}b$ or $\omega_1 ab \equiv \omega_3 \overline{\bar{a}b}$, are also biunique. These operations transform the signs of quantity within six closed groups:

$$(\delta\ \iota)\ (\varepsilon\ \zeta\ \eta\ \kappa)\ (\theta)\ (\lambda\ \mu\ \nu\ \xi)\ (o\ \pi\ \rho\ \sigma)\ (\upsilon).$$

p. 250. Proposition no. 126 should read:

$+++$ $\quad -++$ $\quad --+$ $\quad ---$.

TABLE 1

Systematic classification of elementary propositions ωab on two properties in divalent logic arranged according to the existence of non-empty classes, ++ denoting ab; +— $a\bar{b}$; —+ $\bar{a}b$; and —— $\overline{ab}$. The interpretation $\omega_1 a$, $\omega_2 b$, and the biunique representation (ω′a[b]) (ω″a[$\bar{b}$]) are discussed in the text. The converted propositions ω*ba are given for convenient comparison.

Elementary Proposition No.	Non-empty Classes	ωab	ω*ba	$\omega_1 a$	$\omega_2 b$	ω′a[b] (= ω′ba)	ω″a[b] (= ω″b
0	none	υ	υ	τ	τ	τ	τ
1	++	λ	λ	α	α	α	τ
2	+—	μ	ν	α	γ	τ	α
3	—+	ν	μ	γ	α	γ	τ
4	——	ξ	ξ	γ	γ	τ	γ
5	++ +—	ο	π	α	β	α	α
6	++ —+	π	ο	β	α	β	τ
7	++ ——	δ	δ	β	β	α	γ
8	+— —+	ι	ι	β	β	γ	α
9	+— ——	ρ	σ	β	γ	τ	β
10	—+ ——	σ	ρ	γ	β	γ	γ
11	++ +— —+	η	η	β	β	β	α
12	++ +— ——	ζ	ε	β	β	α	β
13	++ —+ ——	ε	ζ	β	β	β	γ
14	+— —+ ——	κ	κ	β	β	γ	β
15	++ +— —+ ——	θ	θ	β	β	β	β

TABLE 2

Elementary propositions ωa on one property in trivalent logic in the di-trivalent case I, tri-divalent case II, and purely trivalent case III. The "realistic" interpretation in terms of composite propositions on one property in divalent logic is discussed in the text.

"Realistic" interpretation	I	II	III
αa	(Ēa) (Ēā)	α_1a	(Ea) (Ēā) (Ēã)
βa	(Ea) (Eā) — —	β_1a β_2a —	(Ea) (Eā) (Ēã) (Ea) (Eā) (E̲ã) (Ea) (Eā) (Ẽã)
γa	(Ēa) (Eā)	γ_1a	(Ēa) (Eā) (Ēã)
τa	(Ēa) (Ēā)	τ_1a	(Ēa) (Ē)ā (Ēã)
α + βa	— — — (Ea) (Ẽā) —	α_2a — — — —	(Ea) (Ē̲ā) (E̲ã) (Ea) (Ē̲ā) (Ẽã) (Ea) (Ẽā) (E̲ã) (Ea) (Ẽā) (Ēã) (Ea) (Ẽā) (Ẽã)
β + γa	— — — (Ẽa) (Eā) —	γ_2a — — — —	(Ē̲a) (Eā) (E̲ã) (Ē̲a) (Eā) (Ẽã) (Ẽa) (Eā) (E̲ã) (Ẽa) (Eā) (Ēã) (Ẽa) (Eā) (Ẽã)
γ + τa	(Ēa) (Ẽā)	—	(Ēa) (Eā) (Ēã)
α + τa	(Ẽa) (Ēā)	—	(Ẽa) (Ēā) (Ēã)
α + β + γa	— — — —	τ_2a — — —	(Ē̲a) (Ē̲ā) (Eã) (Ea) (Ẽā) (Eã) (Ẽa) (Ē̲ā) (Eã) (Ẽa) (Ẽā) (Eã)
α + β + γ + τa	(Ẽa) (Ẽā) — — — —	— — — — —	(Ẽa) (E̲ā) (Ēã) (E̲a) (E̲ā) (Ẽã) (Ēa) (Ẽā) (Ẽã) (Ẽa) (Ēā) (Ẽã) (Ẽa) (Ẽā) (Ẽã)

TABLE 3

Elementary propositions ωab on two properties in the di-trivalent case I interpreted "realistically" as discussed in the text. The four symbols +, —, ~ in the parentheses such as (++++) indicate the existence-values E, $\bar{E}$, $\tilde{E}$ of the four elementary classes ab, a$\bar{b}$, $\bar{a}$b, $\bar{a}\bar{b}$ respectively.

No.		No.	
0	(————) υ	41	(—~++) κ+σ
1	(+———) λ	42	(—+~+) κ+ρ
2	(—+——) μ	43	(—++~) ι+κ
3	(——+—) ν	44	(~+++) θ+κ
4	(———+) ξ	45	(+~++) ε+θ
5	(++——) ο	46	(++~+) ζ+θ
6	(+—+—) π	47	(+++~) η+θ
7	(+——+) δ	48	(~~——) λ+μ+ο+υ
8	(—++—) ι	49	(~—~—) λ+ν+π+υ
9	(—+—+) ρ	50	(~——~) δ+λ+ξ+υ
10	(——++) σ	51	(—~~—) ι+μ+ν+υ
11	(+++—) η	52	(—~—~) μ+ξ+ρ+υ
12	(++—+) ζ	53	(——~~) ν+ξ+σ+υ≡τ
13	(+—++) ε	54	(~~+—) η+ι+ν+π
14	(—+++) κ	55	(~+~—) η+ι+μ+ο
15	(++++) θ	56	(+~~—) η+λ+ο+π
16	(~———) λ+υ	57	(~~—+) δ+ζ+ξ+ρ
17	(—~——) μ+υ	58	(~+—~) ζ+μ+ο+ρ
18	(——~—) ν+υ	59	(+~—~) δ+ζ+λ+ο
19	(———~) ξ+υ	60	(~—~+) δ+ε+ξ+σ
20	(~+——) μ+ο	61	(~—+~) ε+ν+π+σ
21	(+~——) λ+ο	62	(+—~~) δ+ε+λ+π≡α
22	(~—+—) ν+π	63	(—~~+) κ+ξ+ρ+σ
23	(+—~—) λ+π	64	(—~+~) ι+κ+ν+σ
24	(~——+) δ+ξ	65	(—+~~) ι+κ+μ+ρ≡γ
25	(+——~) δ+λ	66	(~~++) ε+θ+κ+σ
26	(—~+—) ι+ν	67	(~+~+) ζ+θ+κ+ρ
27	(—+~—) ι+μ	68	(~++~) η+θ+ι+κ
28	(—~—+) ξ+ρ	69	(+~~+) δ+ε+ζ+θ
29	(—+—~) μ+ρ	70	(+~+~) ε+η+θ+π
30	(——~+) ξ+σ	71	(++~~) ζ+η+θ+ο≡β
31	(——+~) ν+σ	72	(~~~—) η+ι+λ+μ+ν+ο+π+υ
32	(~++—) η+ι	73	(~~—~) δ+ζ+λ+μ+ξ+ο+ρ+υ
33	(+~+—) η+π	74	(~—~~) δ+ε+λ+ν+ξ+π+σ+υ≡α+τ
34	(++~—) η+ο	75	(—~~~) ι+κ+μ+ν+ξ+ρ+σ+υ≡γ+τ
35	(~+—+) ζ+ρ	76	(~~~+) δ+ε+ζ+θ+κ+ξ+ρ+σ
36	(+~—+) δ+ζ	77	(~~+~) ε+η+θ+ι+κ+ν+π+σ
37	(++—~) ζ+ο	78	(~+~~) ζ+η+θ+ι+κ+μ+ο+ρ≡β+γ
38	(~—++) ε+σ	79	(+~~~) δ+ε+ζ+η+θ+λ+ο+π≡α+β
39	(+—~+) δ+ε	80	(~~~~) tautology α+β+γ+τ
40	(+—+~) ε+π		

TABLE 4

The conjunctions $(\omega_x{}'a[b])$ $(\omega_x{}''a[\bar{b}])$ discussed in Appendix II. ψa is the tautology $\alpha + \beta + \gamma + \tau a$. The universe of discourse considered in each atomic proposition is indicated by sharp brackets [b] or $[\bar{b}]$, and the resulting $\omega_1 + \omega_2 + \ldots ab$ are given. In some cases, other composite propositions are given in round parentheses.

$[\alpha+\beta]$	$[\alpha+\beta]$	$\zeta+\eta+\theta+o \equiv \beta$	$[\alpha]$	$[\psi]$	$\delta+\zeta+\lambda+o$ $(\equiv \alpha ba)$
$[\alpha+\beta]$	$[\beta+\gamma]$	$\delta+\varepsilon+\zeta+\theta$	$[\beta]$	$[\psi]$	$\varepsilon+\eta+\theta+\pi$ $(\equiv \beta ba)$
$[\alpha+\beta]$	$[\gamma+\tau]$	$\delta+\varepsilon+\lambda+\pi \equiv \alpha$	$[\gamma]$	$[\psi]$	$\iota+\kappa+\nu+\sigma$ $(\equiv \gamma ba)$
$[\alpha+\beta]$	$[\alpha+\tau]$	$\eta+\lambda+o+\pi$	$[\tau]$	$[\psi]$	$\mu+\xi+\rho+\upsilon$ $(\equiv \tau \underline{b}a)$
$[\beta+\gamma]$	$[\alpha+\beta]$	$\eta+\theta+\iota+\kappa$	$[\psi]$	$[\alpha]$	$\eta+\iota+\mu+o$ $(\equiv \alpha \underline{b}a)$
$[\beta+\gamma]$	$[\beta+\gamma]$	$\varepsilon+\theta+\kappa+\sigma$ $(\equiv \beta \bar{a}b)$	$[\psi]$	$[\beta]$	$\zeta+\theta+\kappa+\rho$ $(\equiv \beta \underline{b}a)$
$[\beta+\gamma]$	$[\gamma+\tau]$	$\varepsilon+\nu+\pi+\sigma$	$[\psi]$	$[\gamma]$	$\delta+\varepsilon+\xi+\sigma$ $(\equiv \gamma \underline{b}a)$
$[\beta+\gamma]$	$[\alpha+\tau]$	$\eta+\iota+\nu+\pi$ $(\equiv \alpha \bar{a}b)$	$[\psi]$	$[\tau]$	$\lambda+\nu+\pi+\upsilon$ $(\equiv \tau ba)$
$[\gamma+\tau]$	$[\alpha+\beta]$	$\iota+\kappa+\mu+\rho \equiv \gamma$	$[\alpha+\beta]$	$[\psi]$	$\delta+\varepsilon+\zeta+\eta+\theta+\lambda+o+\pi \equiv \alpha+\beta$
$[\gamma+\tau]$	$[\beta+\gamma]$	$\kappa+\xi+\rho+\sigma$	$[\beta+\gamma]$	$[\psi]$	$\varepsilon+\eta+\theta+\iota+\kappa+\nu+\pi+\sigma$
$[\gamma+\tau]$	$[\gamma+\tau]$	$\nu+\xi+\sigma+\upsilon \equiv \tau$	$[\gamma+\tau]$	$[\psi]$	$\iota+\kappa+\mu+\nu+\xi+\rho+\sigma+\upsilon \equiv \gamma+\tau$
$[\gamma+\tau]$	$[\alpha+\tau]$	$\iota+\mu+\nu+\upsilon$	$[\alpha+\tau]$	$[\psi]$	$\delta+\zeta+\lambda+\mu+\xi+o+\rho+\upsilon$
$[\alpha+\tau]$	$[\alpha+\beta]$	$\zeta+\mu+o+\rho$	$[\psi]$	$[\alpha+\beta]$	$\zeta+\eta+\theta+\iota+\kappa+\mu+o+\rho \equiv \beta+\gamma$
$[\alpha+\tau]$	$[\beta+\gamma]$	$\delta+\zeta+\xi+\rho$ $(\equiv \gamma \bar{a}b)$	$[\psi]$	$[\beta+\gamma]$	$\delta+\varepsilon+\zeta+\theta+\kappa+\xi+\rho+\sigma$
$[\alpha+\tau]$	$[\gamma+\tau]$	$\delta+\lambda+\xi+\upsilon$	$[\psi]$	$[\gamma+\tau]$	$\delta+\varepsilon+\lambda+\nu+\xi+\pi+\sigma+\upsilon \equiv \alpha+\tau$
$[\alpha+\tau]$	$[\alpha+\tau]$	$\lambda+\mu+o+\upsilon$ $(\equiv \tau \bar{a}b)$	$[\psi]$	$[\alpha+\tau]$	$\eta+\iota+\lambda+\mu+\nu+o+\pi+\upsilon$

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