RELEVANCE REVISITED : A REPLY TO HOCKNEY AND WILSON

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(1) In their welcome discussion of my paper, «Relevance» (Woods [4], pp. 130-7), Professors Hockney and Wilson have challenged my claim that there appears to be no satisfactory relevance condition which rules out Lewis' paradoxes of strict implication. (Hockney and Wilson [2], 211-20). Their own counter-claim ([2], p. 211) is that there is in fact just such a condition which, happily, is immune to any criticism advanced by me. Let us see.

(2) But first let me say something in response to my critics' treatment of a «preliminary matter». I had advanced ([4], p. 131) three conditions of a conception of relevance for which it can non-question beggingly be said that the *reason* the paradoxes fail is that they fail to exemplify relevance so defined. Those conditions are :

first : the paradoxes are clearly shown to exemplify irrelevance so defined;

second : the decision whether A is irrelevant to B is possible without first having to determine whether A entails B; and

third : the information yielded by the definition is sufficient to show that from the fact A is irrelevant to B it follows that A does not entail B.

Happily enough, the first two conditions Hockney and Wilson are disposed to accept. But what they cannot agree to is the third condition which they consider to be too strong. They ask, «why should it be required that... 'A is irrelevant to B' *entail* 'A does not entail B'?» ([2], p. 212. My italics). And they wish to know why it would not do to say that the information given by 'A is irrelevant to B' makes it *reasonable to suppose that* A does not entail B ([2], p. 212).

I think this will *not* do and for the following reasons : If 'A is

irrelevant to B' does *not* entail 'A does not entail B', then on the principle,
(P) If $\sim(p \mathrm{N} q)$ then $(p \circ \sim q)$; that is, if it is not the case that p entails q, then *p is consistent with* the denial of q it follows that 'A is irrelevant to B' is consistent with 'A *does* entail B'. Now on any of three conceptions of consistency this creates difficulties.

First, if we accept Lewis' notion of consistency, according to which $p \circ q$ if and only if $\mathrm{M}(p \,.\, q)$, if and only if, that is, the conjunction of p and q is logically possible, then to say that 'A is irrelevant to B' is consistent with 'A entails B' is to say that the conjunction of these is logically possible, and hence that each conjunct is logically possible. But if 'A entails B' is logically possible, it is clearly not logically impossible; *nor is it contingent*. Entailment-statements are insusceptible of a change of truth-value (without, i.e., undergoing an Aristotelian substantial change); from which it follows that the sense in which it is possible that A entails B cannot be the sense in which it *happens* to be false, but might *become* true. The only condition, therefore, under which 'A entails B' is possible is its being *logically necessary*. Thus the claim that ' A is irrelevant to B' is, on Lewis' interpretation, *consistent* with 'A entails B' actually entails the necessity of 'A entails B'. And the irrelevance of A and B, far from being a reason for supposing that 'A does not entail B' is a conclusive reason for supposing that 'A *does* entail B'.

It is possible that Hockney and Wilson would not accept Lewis' account of consistency, and I for one would agree with them. What, then, of a second sense of consistency according to which if $p \circ q$ then if $\mathrm{M}(p \vee q)$ then $\mathrm{M}(p \,.\, q)$; that is, if p is consistent with q then if either p or q is logically possible then so too are they both? According to P, if 'A is irrelevant to B' does not entail 'A does not entail B' then the former is consistent with 'A does entail B', which, on this second conception of consistency gives : if M((A is irrelevant to B) v (A entails B)) then M((A is irrelevant to B). (A entails B)). Obviously enough, the antecedent of this conditional is true—it is true at least because, in this context, it is true *ex hypothesi*. But what of the consequent? It is true only if both its conjuncts are possible and hence only if it is possible that A entail B, a necessary condition for which being that 'A entails B' is necessary. Either 'A is irrelevant to B' is *not* consistent with 'A entails B' (which is just to say that 'A is irrelevant to B' entails 'A does not entail B') or it is consistent with 'A entails B' (which is just to say that if the irrelevance-statement is true so also is the entailment-statement, and that the former, once again, not only does not give reason to suppose the latter to be false, but actually guarantees the latter to be true.)

Finally, on this point, there is a somewhat weaker conception of consistency which might better serve the point which Hockney and Wilson are here making. Let us say that $p \circ q$ only if $M(p \equiv q)$; that is, p is consistent with q only if it is logically possible that they share the same truth-values. On this third conception of consistency, to say that 'A is irrelevant to B' is consistent with 'A entails B' is to say that if the former is true it is logically possible both for it and 'A entails B' to be materially equivalent. Just so. *If* it is *true* that A is irrelevant to B then M((A is irrelevant to B) ≡ (A entails B)), and hence, M(A entails B), and thus 'A entails B' is necessary. Irrelevance, then, strictly confirms 'A entails B'; and if, perchance, A is *not* irrelevant to B, then one cannot conclude from this that A entails B, but neither can one use the *irrelevance* of A and B as reason for supposing that A does not entail B, simply because A and B are, in this case, not irrelevant.

The upshot is this: if 'A is irrelevant to B', does not entail 'A does not entail B', then, on each of our three conceptions of consistency, the irrelevance of A and B, if it does not *entail* that A does not entail B, interestingly enough *does* entail that A does entail B, in which case the former irrelevance can never be a non-deductive reason for A's not entailing B. This, then, is my justification of condition three of the required analysis of relevance. (This is not, by the way, to ignore the difficulty of understanding the force of 'because A is irrelevant to B, we have some (non-entailment-like) reason for supposing that A does not entail B'. On some interpretations I accept this, and so does Lewis. What is needed, therefore, is a much clearer exhibition of the relevance, in this context, of 'having a (non-deductive) reason for'.)

(3) I stand corrected (Hockney and Wilson [2], p. 213) on my sloppy use of 'definition'. It is feeble, I realize to point out that often a sufficient condition of x is sometimes *called* a definition

of x, even where there are conditions, distinct from the sufficient condition, which are necessary unto x. So, then, I quite agree that Hockney's and Wilson's IC («If A and B fail to share a variable then A is *irrelevant* to B», [2], p. 132) and Belnap's condition which I have dubbed 'CIII' ([4], p. 213 — «*The principle of relevance*. If A and B have no propositional variables in common then 'A entails B' is rejected as a theorem of the system») are to be construed as allegedly necessary conditions, the former of relevance and the latter of the adequacy of any analysis of entailment. My point, however, was that it does not follow from IC, even in conjunction with other acceptable logical laws, that if A is irrelevant to B then A does not entail B, in violation of my own condition three. Only if we accept Belnap's CIII can we move from the irrelevance of A to B to the fact that A does not entail B. And I think we cannot accept CIII without running afoul of CII (my designation [4], p. 132, for Belnap's principle that a theory of entailment should not rule out any non-controversial entailment-statement) — as I hoped my counterexamples would show.

Yet Hockney and Wilson take my counterexamples to be singularly misguided ([2], p. 215). I had said that CIII ruled out of court the following, true, entailments, in violation of CII :

1. 'x is blue' entails 'x is coloured'

2. 'A is true' entails 'A is truth-valued'

3. 'x is square' entails 'x is rectangular'

My critics reply ([2], pp. 215-16) that, properly understood, each consequent of *1* to *3* analyses into a disjunction or conjunction each containing at least one occurrence of its respective antecedent. Thus, for example, 'x is coloured' analyzes into the (finite but possibly indeterminate) disjunction 'x is blue or x is red or x is yellow or ...' which disjunction shares a term ('x is blue', namely) with the antecedent of *1*. Similar reductionist techniques are alleged to save *2* and *3*.

I must say that I find this line of reasoning rather perplexing. In each case the consequent is thought to analyze into some disjunction or conjunction containing an occurrence of the antecedent of the suspect entailment-statement. But just what is it for a term **F** to *analyze* into, say, the disjunction '$f_1 \vee f_2 \vee f_3 \vee \ldots$'? It is surely a necessary condition of the success of such an analysis that '**F**' *entail* '$f_1 \vee f_2 \vee f_3 \vee \ldots$'. But if '*x* is coloured' really does analyze into '*x* is blue or *x* is red or *x* is yellow, or ...' then the former must *entail* the latter, in out and out violation of CIII! Commonality of terms is nowhere in the offing. I don't mean to revert to the old Paradox of Analysis, and in any case I don't mean to rule out the possibility that an analysis may be something like a Carnapian *explication*. Just so, if G explicates **F** then we must say that **F** ought to entail G, and because it ought to, we shall say it does — our saying so making it so. Manifestly, 'explication' founders on the same shoal as does 'analysis'.

What then are we to say of '*x* is coloured'? If we say that it is atomic in Carnap's sense, and moreover, that being thus atomic rules out the possibility of its being entailed by any other atomic term, then as my critics point out, ([2], p. 216) *1*, *2* and 3 are false and do not embarrass CII. But clearly, *1*, *2* and *3* are true; and either their consequents are not atomic in Carnap's sense or the Carnapian atomicity of a term does not preclude its being entailed by another atomic term.

(4) I quite agree however that if, as I think is the case, my counterexamples are genuine, they may well be counterexamples to *all* systems of implication formulated to date, Lewis' included (Hockney and Wilson [2], p. 216). So be it. I do not, nor have I espoused Lewis' systems *in toto*. In fact for present purposes I need not even accept the paradoxes. All the same, we seem to have come upon an exciting, if not alarming, fact, to wit: entailments generated by the determinate-determinable and genus-species relations *are not axiomatizable*, save in the ersatz sense involving ludicrously many special postulates, one for every particular case of such entailments. This is just to say that the entailment- relation cannot be fully formalized. But am I not hoist upon my own petard? I have argued that, in violation of CIII, IC falsifies all statements of the form «**F***x* entails G*x*», some of which, surely, are true. Yet such of them as are true involve an entailment relation which systematically resists axiomatization; from which it follows, does it not, that such entailments cannot be counterexamples to Anderson's and Belnap's system which is designed to capture entailments only of the formalizable breed? That is, or so it might be argued, we have

at least two *senses* of entailment, the one susceptible to axiomatization and the other not. So it won't do to give examples of the latter as counters to theories of the former. On the contrary. The argument is just so much sleight of hand. Some entailments can be formalized and some cannot; some hold in virtue of form and some do not. There is a difference of criteria of the correct application of the word 'entails', but this difference does not guarantee a difference of meaning, does not show that instead of a single entailment relation, we have now two or more. There is a difference of criteria of the ascription of mental predicates as between oneself and another, but philosophers have, of recent years, recovered the good sense to resist concluding from this that we have at least two concepts of pain, the one being correctly ascribed only to oneself and the other only to another. Neither should philosophers of logic tolerate such conceptual pullulations in *their* domain.

(5) If I am right in supposing my counterexamples to stand up then, plainly, the only option open to Hockney and Wilson is actually to *refute* Lewis' independent arguments in behalf of the paradoxes. This, following Anderson and Belnap ([1], pp. 18-20), they attempt on pages 217 and 218. The Lewisean proof of the first paradox (that an impossible proposition entails any proposition) is dismissed as embodying the *fallacious* (!) inference-rule, disjunctive syllogism, (rule 4, Hockney and Wilson [2], p. 217). Since I have previously spoken in defense of disjunctive syllogism (Woods [5], pp. 312-20), I shall say here only that I cannot see how my critics' attack upon it upsets what I have said in its support — that attack consisting, as it does, in a mere paraphrase of Anderson and Belnap.

(6) As for the difficulties which Hockney and Wilson find ([2], p. 218) in Lewis' proof ([3] ,p. 251) of the second paradox (that a necessary proposition is entailed by any proposition), I must confess once again to a feeling of enormous puzzlement. The objection they make is that the second step of the proof $(A \rightarrow ((A . \sim B) \vee (A . B)))$ is inadmissible; and the *reason* for this is that if the consequent of this step, '$(A . \sim B) \vee (A . B)$', is put into conjunctive normal form, we obtain :

$$A . (A \vee B) . (A \vee \sim B) . (B \vee \sim B),$$

which latter does not follow from A because

$$A \rightarrow (A \,.\, (A \vee B) \,.\, (A \vee \sim B) \,.\, (B \vee \sim B))$$

is not a *tautological entailment*, since A *does not entail* (B v ~B). From which they conclude the invalidity of Lewis' step 2. Otherwise put, Lewis' proof of 'A entails (B v ~B)' fails at step 2 just because that step is equivalent to a statement in conjunctive normal form which is not a tautological entailment for the simple reason that it is not the case that A entails (B v ~B). Could the circularity of this reasoning be *more* blatant?

(7) Finally, one does not resolve the problem I raised ([4], p. 133) about simultaneous and piecemeal (uniform) substitution by the casual and unargued for observations (Hockney and Wilson [2], p. 219) that Anderson and Belnap do not in their system actually *use* substitution rules; that, of course, particular rules of inference are merely system-relative; and that logicians historically have had trouble with substitution rules. The crucial question nonetheless remains unexamined: *is* substitution, whether or not it occurs in the work of certain logicians, a valid mode of inference, and if not, why not? If in its customary employment, it is valid then my counterexample is left undisturbed. If it is not valid, then Hockney and Wilson might have shown this. And if it is neither valid nor invalid, save with respect to a given logistic system, this too might better have been shown. Actually though, on the assumption that substitution is *valid*, there just might be an objection which Hockney and Wilson could have raised to my argument. *If* the results of uniform substitutions into theorems of a system (into valid wffs of the system) are guaranteed to be theorems (to be valid wffs), and into atheorems (invalid wffs) are guaranteed to be atheorems (invalid wffs), this comes very close to saying, in the first case, that a necessary truth entails any necessary truth, and, in the second case, that a selfcontradiction entails any selfcontradiction — the former being Lewis' second paradox and the latter, with the aid of simplification, being the first paradox. So perhaps the claim that substitution is valid begs the question against those who reject the paradoxes.

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