NONCREATIVITY AND TRANSLATABILITY IN TERMS OF INTENSION (*)

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I intend to show in this paper that Carnap's notion of intension ([1]) may have interesting applications in logical inquiries. The notion of intension will be applied here to the problem of extending of theories. I will only consider the intensions of sentences (without free variables). The intensions of terms and of other formulas will not be taken into account here ([2]).

The symbols $\sim$, $\vee$, $\wedge$, $\equiv$ will be used for the sentential connectives (negation, alternative, conjunction and biconditional).

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Let T_1 and T_2 be two standard formalized theories ([3]) such that T_1 is a subtheory of T_2. It means that the language L_1 of T_1 is a sublanguage of the language L_2 of T_2 and T_1 is a subset of T_2. We may suppose that there exists a set A_1 of sentences in L_1 and a set A_2 of sentences in L_2 such that the following equivalence holds for each sentence p in L_K where $k = 1,2$:

the sentence p is in T_K i.e. p is a theorem of T_K if and only if p follows logically from A_K.

The assumption that T_1 is a subset of T_2 is equivalent to the following condition:

(*) English translation of a lecture given in Russian at the Faculty of Mathematics of the University of Sofia, June 6 1966, Sofia, Bulgaria.

(1) R. CARNAP, *Meaning and Necessity*, Chicago 1947. See also my paper An Essay in the Formal Theory of Extension and of Intension, *Studia Logica*, vol. 20, in print.

(2) A more extended paper concerning the applications of the notion of intension will be published elsewhere.

(3) I.e. the theories T_1 and T_2 are based on the engere Praedikaten-kalkül mit Identität, Funktionszeichen und Beschreibungssymbolen. See D. HILBERT und P. BERNAYS, *Grundlagen der Mathematik*, vol. I, Berlin 1943.

every sentence in A_1 follows logically from A_2.

The symbols T_1/T_2 means the extending transformation of T_1 to T_2. The transformation T_1/T_2 may be *noncreative* (St. Lesniewski) and/or *translatable* (K. Adjukiewicz). The noncreativity and translatability are defined as follows ([4]).

The transformation T_1/T_2 is noncreative if and only if every sentence in L_1 being a theorem of T_2 is also a theorem of T_1.

The transformation T_1/T_2 is translatable if and only if for every sentence p in L_2 there exists a sentence q in L_1 such that the biconditional $p \equiv q$ is a theorem of T_2.

We say that the transformation T_1/T_2 is a definitional one if and only if

1) $L_2 = L_1 +$ certain new extralogical constants and

2) $A_2 = A_1 +$ the standard definitions of all new extralogical constants.

We know very well that if the fransformation T_1/T_2 is definitional then it is both non creative and translatable.

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Let $k = 1,2$. If p is a sentence in L_K then $int_K(p) =$ the *intension* of p in T_K. The intensions of sentences in L_K may be introduced as follows:

$int_K(p) = int_K(q)$ if and only if the biconditional $p \equiv q$ is a theorem of T_K.

We know very well that the set I_K of all intensions of sentences in L_K is a Boolean algebra (called also Lindenbaum algebra of T_K in L_K) such that for all p, q in L_K:

1) $int_K(p \vee \sim p) =$ the unit-element in I_K,

2) $int_K(p \wedge \sim p) =$ the zero-element in I_K,

3) $-int_K(p) = int_K(\sim p)$,

4) $int_K(p) + int_K(q) = int_K(p \vee q)$,

5) $int_K(p) . int_K(q) = int_K(p \wedge q)$.

The signs -, +, . stand for the Boolean operations of complementation, of addition and of multiplication, respectively.

([4]) See K. Adjukiewicz, Die Definition, *Actes du Congrès International de Philosophie Scientifique* (Sorbonne 1935), vol. II, 1-7, Paris, 1936.

The algebras I_1 and I_2 are quite different, in general. Similarly, if p is a sentence in L_1 then $int_1(p)$ and $int_2(p)$ are different, in general. However, from the assumption that T_1 is a subtheory of T_2 it follows that there exists a special connection between I_1 and I_2.

The intensions of sentences in L_2 may be divide into *old* and *new*. If p is a sentence in L_2 then $int_2(p)$ is called old if and only if there exists a sentence q in L_1 such that $int_2(p) = int_2(q)$. Otherwise, $int_2(p)$ is called new.

It is clear that $int_2(p)$ is old for each sentence p in L_1. Of course, $int_2(p)$ may be new for some sentences p in L_2 only.

Let I^+_2 denote the set of all old intensions of sentences in L_2. Clearly, I^+_2 is a subset of I_2. Moreover, I^+_2 is a Boolean algebra and a subalgebra of I_2. It means that for all p, q in L_2 :

1) $int_2(p \vee \sim p)$ and $int_2(p \wedge \sim p)$ belong to I^+_2 and

2) if $int_2(p)$ and $int_2(q)$ belong to I^+_2 then $-int_2(p)$, $int_2(p) + int_2(q)$ and $int_2(p) . int_2(q)$ belong also to I^+_2.

Consider now the following functional correspondance H between the elements of I_1 and those of I^+_2 :

$$H(int_1(p)) = int_2(p)$$

for every sentence p in L_1. Clearly, I_1 = the set of all arguments of H and I^+_2 = the set of all values of H.

The function H is a homomorphism of I_1 onto I^+_2 and, consequently, a homomorphism of I_1 into I_2. This means that for all p, q in L_1 :

1) $H(-int_1(p)) = -H(int_1)p))$,

2) $H(int_1(p) + int_1(q)) = H(int_1(p)) + H(int_1(q))$,

3) $H(int_1(p) . int_1(q)) = H(int_1(p)) . H(int_1(q))$.

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An one-to-one homomorphism is called isomorphism. Therefore, the function H is an isomorphism between I_1 and I^+_2 if and only if the following condition holds for all p, q in L_1 :

$$(*) \text{ if } int_1(p) \neq int_1(q) \text{ then } int_2(p) \neq int_2(q).$$

Theorem 1. The extending transformation T_1/T_2 is noncreative if and only if the function H is an isomorphism between I_1 and I^+_2.

Proof. (1) Let the transformation T_1/T_2 be noncreative. Suppose that $int_2(p) = int_2(q)$ where p, q are in L_1. Consequently, the biconditional $p \equiv q$ is a theorem of T_2. It follows from the noncreativity that the biconditional $p \equiv q$ is also a theorem of T_1. This means that $int_1(p) = int_1/q/$. The condition (*) holds. (2) Let the transformation T_1/T_2 be creative. I.e. there exists a sentence p in L_1 such that p is a theorem of T_2 but it is not a theorem of T_1. It follows that the biconditional $p \equiv (p \vee \sim p)$ is in T_2 but it is not in T_1. Consequently, $int_1(p) \neq int_1(p \vee \sim p)$ and $int_2(p) = int_2(p \vee \sim p)$. The condition (*) does not hold.

It is easy to see that the transformation T_1/T_2 is translatable if and only if $int_2(p)$ is old for every sentence p in L_2, i.e. if the new intensions of sentences in L_2 do not exist ($I_2 = I^+_2$). Thus we have

Theorem 2. The extending transformation T_1/T_2 is translatable and noncreative if and only if the function H is an isomorphism between I_1 and I_2.

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