A LOGIC OF COMMANDS (*)

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1. *Informal Account*

This paper presents a formalized language in which commands can be expressed. In particular, we shall add to the usual machinery of the predicate calculus of first order a connective that can reasonably be interpreted as 'It is commanded by *a* that *p*' ,where '*a*' is the name of an agent, and '*p*' is a declarative sentence. We shall also introduce formal techniques for specifying a hierarchy of agents such that some agents may be thought of as the superiors or supervisors of others. Hence we shall have a formal model of what we call a *command hierarchy*.

The existing literature on the logical analysis of commands seems to be divided into two groups. The first group centers around the approach taken by Hofstadter and McKinsey [5]. In [5] imperatives are assigned the values *obeyed* and *disobeyed* in much the same way that declaratives are assigned truth-values in classical systems. But it turns out that the system of [5] is trivial in the sense that given any formula of the system there is an equivalent formula that contains no imperative-connectives other than a single initial '!' ('!' being the singulary connective that forms imperatives out of declaratives). Other papers that seem to be more or less in the tradition of Hofstadter and McKinsey are Ross [10] and Bohnert [1].

The second (and more recent) approach to the logical analysis of commands is oriented toward deontic logic. In fact Kanger [6] and Fisher [3] take deontic logic and the logic of commands to be essentially the same in their formal structures. For example, Fisher's approach, roughly, is to use declaratives of the form 'It is commanded by *a* that *p*' in place of the actual imperative '*p* !' uttered by *a*. This

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approach, which is also adopted in this paper, turns out to be more fruitful than that of Hofstadter and McKinsey. It also seems quite reasonable. For the declarative sentence in question, when uttered by *a* himself, has all the force of the corresponding imperative. And the declarative has the additional advantage of describing the situation that obtains when *a* commands that *p* shall be brought about. Hence the declarative can be used either by *a* to issue a command or by someone else to describe a situation in which a command is issued.

We now give informal analyses of commands and command hierarchies that will provide motivation for the formal material to be given in subsequent sections. When an agent *a* issues a command, we might roughly characterize the situation by saying that *a* envisages some state of affairs *s* that he considers obtainable, and that *a* would like to see *s* brought about. We might further characterize the state of affairs *s*, which *a*'s command says shall be brought about, as a common element of all those states of affairs s_1, s_2, ..., that *a* considers allowable or unobjectionable, relative to his present point of view. More precisely, if s_1, s_2, ..., are all those states of affairs to which *a* has no objection from the point of view of his present state, and *S* is a description of the state of affairs *s*, then *a* commands that *s* shall be brought about if and only if *S* is true in each of s_1, s_2, ... Hence we analyze the commands of an agent in terms of what is common to all those «worlds» (i.e., state of affairs) that the agent considers unobjectionable from the point of view of his present «world».

Of course this analysis merely leaves us with another unanalyzed term (i.e., 'allowable' or 'unobjectionable') that may seem no more perspicuous that the one with which we started. But we shall see that by taking the notion of an allowable world as primitive we can provide a precise and plausible formalization of commanding. The allowable worlds that are associated with each agent will be nothing more than interpretations of the symbols of a formalized language, in the usual sense of 'interpretation' employed in mathematical logic. We shall then say that 'It is commanded by *a* that *p*' is true in a given world *w* if and only if '*p*' is true under all those interpretations that constitute the worlds that *a* considers allowable with respect to *w*.

Given this analysis of commanding for individual agents, the notion of a command hierarchy is then fairly easy to formulate. First, we postulare some finite set of agents and define a relation of superiority (i.e., supervisorhood) among them (1). If *a* is a superior or supervisor of *b*, then the commands issued by *a* ought to place a limit on the commands that can be issued by *b*—that is, *b* is not supposed to issue any command that would conflict with a command already issued by *a*. Of course subordinates are sometimes guilty of insubordination, but in the ideal case, at least, *b* would adopt all of *a*'s commands as his own, and such further commands as he issued would be concerned primarily with implementing in detail *a*'s more general directives. This situation will be reflected in our formalism as follows. Consider the set *W* of all those worlds that are allowable with respect to some other world according to any commander, and suppose that w_1 are w_2 and members of *W*. If *a* is a superior of *b* and if w_1 is allowable with respect to w_2 according to *b*, then we stipulate that w_1 is also allowable with respect to w_2 according to *a*. In other words, within the set *W*, *a*'s scope of responsibility and concern includes that of each of his subordinates *b*. Hence it follows that whatever is commanded by *a* in any member of *W* is also commanded by *b* in that member of *W*. And since *W* is the set of all the allowable worlds of all commanders, our interpretation incorporates the assumption that every commander commands that the commands of a superior shall also be the commands of his subordinates. This and other consequences of our interpretation of command hierarchies will be explained in more detail after our formalized language has been presented in the next section.

2. *A Formalized Language*

We present a formalized language L, that is a modification of a two-sorted predicate calculus of first order (2). The improper

(1) I am not aware of any work in the literature that takes this step. Fisher [3] comes the closest, since his system allows that there may be more than one commander. But he is not concerned with arranging these commanders in any kind of hierarchical structure.

(2) See church [2], pp. 339-341.

symbols of L are parentheses, comma, horseshoe ($\supset$), tilde ($\sim$), and the letter '*C*'. The proper symbols of L include a nonempty, finite or enumerable set of individual constants of the first sort

$$a_1, a_2, \ldots;$$

a nonempty, finite or enumerable set of individual constants of the second sort

$$c_1, c_2, \ldots;$$

an enumerable set of individual variables of both first and second sort

$$x_1, x_2, \ldots;$$

an enumerable set of individual variables of the second sort

$$y_1, y_2, \ldots.$$

and, for each k, a finite or enumerable, possibly empty, set of predicates

$$A^K{}_1, A^K{}_2, \ldots.$$

We interpret the individual constants of the first (second) sort as names of the members of a nonempty domain of individuals D (E). The individual variables x_i range over D $\cup$ E, while the individual variables y_i range over E. Each *k*-ary predicate names a function from $(D \cup E)^K$ to {T, F}. Intuitively, we think of E as a set of commanders (i.e., agents who issue commands) and D as a set of all individuals other than the commanders themselves (including perhaps both men and machines) that are within the jurisdiction of some member of E.

The well-formed formulas (wffs) of L are defined as follows:

1. The result of applying a *k*-ary predicate to *k* individual symbols (either constants or variables of either sort) is a wff (3).

(3) Notice that L could be refined by dividing the predicate letters into different classes on the basis of the sorts of individual symbols they accept as arguments. We do not undertake this refinement here.

2. If α and β are wffs, then $(\sim\alpha)$, $(\alpha \supset \beta)$, $((x_i)\alpha)$, $((y_i)\alpha)$, $((Cc_i)\alpha)$, and $((Cy_i)\alpha)$ are wffs, for all appropriate i.
3. No expression is a wff unless it is so by 1-2.

The tilde, horseshoe, and an individual variable enclosed in parentheses play their usual roles as negation, material implication, and universal quantification, respectively. In L, however, we have the option of quantifying over D $\cup$ E or only over E (i.e., of quantifying only over the commanders). A wff of the form $((Cc_i)\alpha)$ is read «It is commanded by c_i that α». Analogously, a wff of the form $((y_i)(((Cy_i)\alpha))$ is read «It is commanded by all commanders that α». Notice that the individual symbols a_i and x_i may not be used with the connective C to form wffs.

The procedure for interpreting L will now be precisely specified. We wish to make any interpretation relative to definite sets of individuals (both commanders and noncommanders) and relative to some definite hierarchical ordering of the commanders. The prefix 'DEH' on the terms defined below indicates this relativization. Specifically D is the set of noncommanders, E is the set of commanders, and H is certain ordering relation defined on the constants c_i and a concomitant condition on certain other relations which together give the hierarchical ordering of the commanders. In any practical application the set of commanders, E, will always be finite. Hence in what follows we always take E to contain a finite number of members, n, and we limit the individual constants of the second sort to $c_1, c_2, \ldots, c_n$. It should be pointed out, however, that this restriction is not in any way a necessary part of our formalism.

We define a *DEH model structure* (4) for L as an ordered $(n+2)$-tuple $(G, K, R_1, \ldots, R_n)$, where K is a nonempty set, $G \varepsilon K$, and each $R_i (1 \leq i \leq n)$ is a relation on K with the following properties: (1) For each $W_1 \varepsilon K$ there is at least one $W_2 \varepsilon K$ such that $W_1 R_i W_2$; (2) R_i is reflexive over K-{G} (i.e., over the set that consists of all members of K except G). Furthermore, each DEH model structure has associated with it the following features, which it shares

(4) The definitions of model structures and models given here are extensions of corresponding definitions for the writer's system DM [4]. The basic ideas involved derive ultimately from Kripke [7], [8].

with every other DEH model structure: (a) a fixed nonempty set D; (b) a fixed nonempty set E with n members; (c) a fixed assignment of a unique member of D(E) to each constant $a_i(c_i)$; (d) the stipulation H, consisting of a fixed ordering $>$ of the constants c_i and the following condition that depends on $>$: For any W_1, $W_2 \varepsilon$ K-{G}, if $W_1 R_i W_2$ and $c_j > c_i$, then $W_1 R_j W_2$.

We now define a *DEH model* for a wff α of L as a binary function $\Phi(P, W)$ associated with a given DEH model structure $(G, K, R_1, \ldots, R_n)$. P ranges over variables, predicates, and well-formed subformulas of α, while W ranges over members of K. $\Phi(x_i, W)$ is a member of $D \cup E$, and $\Phi(y_i, W)$ is a member of E, for each x_i and y_i, respectively. $\Phi(A^K{}_i, W)$ is a subset of $(D \cup E)^K$, for each k-ary predicate $A^K{}_i$. Suppose that the k individual symbols $u_1, \ldots, u_k$ are assigned $b_1, \ldots, b_k$ (each $b_j \varepsilon D \cup E$), respectively, by $\Phi(u_j, W)$, if u_j is a variable, or by the assignment that goes with DEH model structures, if u_j is a constant. Then $\Phi(A^K{}_i(u_1, \ldots, u_k), W) = T$ if the k-tuple $(b_1, \ldots, b_k)$ is a member of $\Phi(A^K{}_i, W)$; otherwise $\Phi(A^K{}_i(u_1, \ldots, u_k), W) = F$. The value of $\Phi(P, W)$ for all well-formed subformulas P of α (including α itself) can now be defined by induction as follows.

If $\Phi(\beta, W) = T$, then $\Phi((\sim\beta), W) = F$; otherwise $\Phi((\sim\beta), W) = T$. If $\Phi(\beta, W) = T$ and $\Phi(\gamma, W) = F$, then $\Phi((\beta \supset \gamma), W) = F$; otherwise $\Phi((\beta \supset \gamma), W) = T$. If $\Phi'(\beta(x_i), W) = T$, for every Φ' that differs from Φ at most in its assignment of a member of $D \cup E$ to x_i, then $\Phi(((x_i)\, \beta\, (x_i)), W) = T$; otherwise $\Phi(((x_i)\beta(x_i)), W) = F$. Similarly, if $\Phi'(\beta(y_i), W) = T$, for every Φ' that differs from Φ at most in its assignment of a member of E to y_i, then $\Phi(((y_i)\beta(y_i)), W) = T$; otherwise $\Phi(((y_i)\beta(y_i)), W) = F$. If $\Phi(\beta, W') = T$ for every $W' \varepsilon K$ such that $W R_i W'$, then $\Phi(((Cc_i)\beta), W) = T$; otherwise $\Phi(((Cc_i)\beta), W) = F$. Finally, if $\Phi(\beta, W') = T$ for every $W' \varepsilon K$ such that $W R_j W'$, where $\Phi(y_i, W)$ is the member of E assigned to c_j by all DEH model structures, then $\Phi(((Cy_i)\beta), W) = T$; otherwise $\Phi(((Cy_i)\beta), W) = F$.

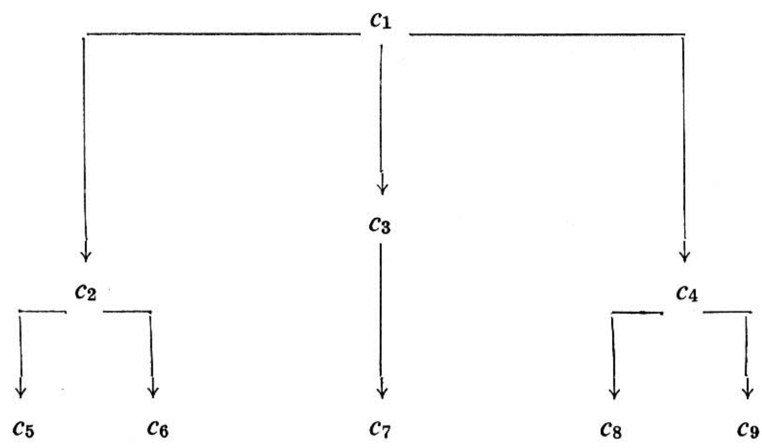
We now say that a wff α is *true in a DEH model* Φ associated with a DEH model structure $(G, K, R_1, \ldots, R_n)$ if and only if $\Phi(\alpha, G) = T$. We also say that a wff α is *DEH valid* if and only if it is true in all its DEH models.

For a given model structure we think of K as the set whose members are the real world (G) and all those other worlds (i.e., all the members of K-{G}) that are allowable with respect to some world according to at least one member of the set E of commanders. For any two worlds W_1, $W_2 \varepsilon K$ we read '$W_1 R_i W_2$' as 'W_2 is allowable with respect to W_1, according to commander c_i'. In view of the formal definition given above, it follows that we think of a sentence $(Cc_i)\beta$ as being true in a world W if and only if β is true in every world that is allowable with respect to W in the eyes of commander c_i. The restrictions that the definition of a DEH model structure placed on each relation R_i can be understood as follows. Restriction (1) says that for every world W_1 there is at least one world W_2 such that W_2 is allowable with respect to W_1, according to commander c_i. Restriction (2) says that every world that is allowable with respect to some other world according to any commander (i.e., every member of K-{G}) is allowable with respect to itself, according to commander c_i. Finally, the stipulation H determines a command hierarchy in the following way.

We read '$c_j > c_i$' as 'c_j is a superior of c_i' or 'c_i is a subordinate of c_j'. Whenever '$c_j > c_i$' holds it is reflected in the interpretation of L by a condition on the relations R_j and R_i. Specifically, given any two members of K-{G}, say W_1 and W_2, if $c_j > c_i$ and $W_1 R_i W_2$, then $W_1 R_j W_2$. In other words, within the set of all those worlds that are allowable with respect to some other world in the eyes of some commander (i.e., within the set K-{G}) the relation R_i is included in the relation R_j, whenever $c_j > c_i$. Hence, for any wff α, if $(Cc_j)\alpha$ is true in any of these worlds then $(Cc_i)\alpha$ is true in the same worlds — this is the way that '$c_j > c_i$' is reflected in the interpretation of L. Our reason for restricting this condition to K-{G} is that in the real world subordinates do not always obey their superiors, and hence $(Cc_j)\alpha \supset (Cc_i)\alpha$ is not in general true, even though $c_j > c_i$.

3. *Testing wffs for DEH validity*

We shall now consider, as an example, the result of applying L to a particular command hierarchy. The hierarchy we have in mind contains nine commanders, $c_1, \ldots, c_9$, arranged in the following way:



This diagram expresses the superior-subordinate relation among the commanders $c_1, \ldots, c_9$. If it is possible to pass from c_i to c_j by following arrows on the diagram we will say that c_i is a superior of c_j(i.e., $c_i > c_j$).

We wish to reflect the structure of this hierarchy in DEH model structures. Since there are just nine commanders in the present application, the set E of any DEH model structure will have just nine members, one of which is named by each of the individual constants $c_1, \ldots, c_9$. Each commander c_i will also have a relation R_i associated with it in any DEH model structure. The superior-subordinate relation among commanders can therefore be expressed in a DEH model structure as follows: If $c_j > c_i$ (for any i and j such that $1 \leq i \leq 9$ and $1 \leq j \leq 9$), then, for any W_1, $W_2 \varepsilon$ K-{G}, if $W_1 R_i W_2$, then $W_1 R_j W_2$.

The set E and the stipulation H have now been specified. If we wish to have a definite set of DEH concepts as defined in the previous section we must specify the set D (the set of non-commanders). But rather than do this we shall be concerned instead with exhibiting schemes of wffs that are DEH valid for any choice of D, given the above choices of E and H. Although it is often difficult to use model theory itself to determine if a given wff is DEH valid, it is known from the work of Kripke [7], [8] that the much simpler method of semantic tableaux can often be made equivalent to a given model theory. We shall specify informally a method of semantic tableaux that is equivalent to our model theory and then carry out our discussion of examples in terms of semantic tableaux.

We presuppose familiarity with the method of semantic tableaux as given in [7] and [8]. Tableau rules for $\sim$ and $\supset$ follow the standard truth-table interpretations of these connectives. Similarly, *mutatis mutandis*, for any other connectives that can be defined in terms of these primitives. Quantifier rules for the variables y_i (i.e., rules for quantification over the finite domain E) can be easily specified by treating universal quantification on these variables like conjucntion. If D is finite, quantification on the variables x_i can be treated similarly; for infinite D the method of [7] can be used. We further stipulate that each commander c_i has associated with it a relation R_i over each set of tableaux such that R_i is reflexive over all the auxiliary tableaux in the set. We also give the stipulation H for tableaux as follows: For any auxiliary tableaux t_1 and t_2, if $t_1R_it_2$ and $c_j > c_i$, then $t_1R_jt_2$. The tableaux rules for the connective C are then as follows:

> Cl. If $((Cc_i)\beta)$ (5) appears on the left of a tableau t, put β on the left of each tableau t′ such that t R_it′. If there is no such t′ (i.e., if t is a main tableau with no auxiliary stemming from it), then start a new tableau t′ with β on the left and stipulate that t R_i t′ (5).
>
> Cr. If $((Cc_i)\beta)$ (5) appears on the right of a tableau t, then start a new tableau t′ with β on the right and stipulate that t R_i t′.

It would not be difficult, using methods analogous to those of [7] and [8], to establish that a wff is DEH valid if and only if its semantic-tableau construction (as sketched above) is closed. And if D as well as E is a finite set, this method of semantic tableaux

(5) A wff of the form $((Cy_i)\beta)$ can be treated like its universal closure. In view of our intention of treating universal quantification over the finite domain E as conjunction, this would result in $((Cy_i)\beta)$ being construed as $((Cc_1)\beta) \wedge \dots \wedge ((Cc_9)\beta)$.

(6) The second sentence of this rule gives the effect, for semantic tableaux, of the stipulation in the model theory that every world bears the relation R_i to some world.

constitutes a decision procedure for DEH validity. In the remainder of this section we shall give some examples of the use of semantic tableaux for determining DEH validity.

Consider first the wff scheme

$$(Cc_1)((Cc_2)\,\alpha \supset (Cc_5)\alpha) \qquad (1).$$

Its tableau construction is as follows:

t_1

	$(Cc_1)((Cc_2)\,\alpha \supset (Cc_5)\alpha)$

$\downarrow$ 1

t_2

	$(Cc_2)\,\alpha \supset (Cc_5)\alpha$
$(Cc_2)\alpha$	$(Cc_5)\alpha$
α	

$\downarrow$ 5,2

t_3

α	α

For convenience we label the main tableau in this construction t_1 and the auxiliary tableaux t_2 and t_3, although such labeling is not necessary. The construction begins with (1) on the right (i.e., the false side) of t_1. Applying Cr we introduce t_2 with $(Cc_2)\alpha \supset (Cc_5)\alpha$ on the right. The arrow with the '1' along side indicates that $t_1\ R_1\ t_2$. The rule for implication on the right then puts $(Cc_2)\alpha$ on the left and $(Cc_5)\alpha$ on the right of t_2. Next we use Cr again to introduce t_3 with α on the right. The arrow and '5' indicate that $t_2\ R_5\ t_3$. But since t_2 and t_3 are both auxiliary tableaux and $c_2 > c_5$, it follows by stipulation H that $t_2\ R_2\ t_3$. This is indicated by the '2' that also appears along side the second arrow. We also have $t_2\ R_2\ t_2$ since t_2 is an auxiliary tableau and each R_i is reflexive over the entire set of auxiliary tableaux. Application of Cl to $(Cc_2)\alpha$ therefore puts α on the left of t_2 and on the left of t_3. Hence the construction for (1) is closed, and (1) is DEH valid for any D, given our choice of E and H.

From this example it is easy to see that for any i and for any j and k such that $c_j > c_k$, $(Cc_i)((Cc_j)\alpha \supset (Cc_k)\alpha)$ is DEH valid. Hence our interpretation of L incorporates the assumption that every commander commands that subordinates be bound by the commands of their superiors. Notice, however, that wffs of the form $(Cc_j)\alpha \supset (Cc_k)\alpha$ are not in general DEH valid, even if $j > k$. This is as it should be since it is not even factually true, much less logically true, that subordinates always obey their superiors.

As a second example consider the wff scheme

$$(Cc_6)((Cc_9)\alpha \supset \alpha) \qquad (2).$$

Its construction is:

t_1

| $(Cc_6)((Cc_9)\alpha \supset \alpha)$

$\downarrow$ 6

t_2

| $(Cc_9)\alpha \supset \alpha$

$Cc_9\alpha$ | α

α |

Notice that closure in this case (and hence DEH validity) results from the fact that R*9* is reflexive over all auxiliary tableaux and hence that α can be obtained on the left of t_2 by Cl. It is obvious from this example that any wff of the form $(Cc_i)((Cc_j)\alpha \supset \alpha)$ is DEH valid, for any i and j. (Indeed our motivation in adopting the restriction that each R_i must be reflexive over K-{G} — or, equivalently, over the set of auxiliary tableaux — was to insure that wffs of the form of (2) would be DEH valid.) Hence our interpretation of L incorporates the assumption that every commander commands that the commands of each commander actually be carried out. Of course wffs of the form $(Cc_i)\alpha \supset \alpha$ are not in general DEH valid.

We consider now two wff schemes that are most conveniently discussed in terms of a defined connective, *P*, that expresses permission. Specifically, we define $(Pc_i)\alpha$ as $\sim(Cc_i)\sim\alpha$ and $(Py_i)\alpha$ as $\sim(c_i)\sim\alpha$. Hence we interpret sentences of the form 'α is permitted by commander c_i' as 'It is not the case that c_i commands that $\sim\alpha$'. This is in accordance with the standard definition used in deontic logic.

First we notice that for any *i* and for any *j* and *k* such that $c_k > c_j$ all wffs of the form

$$(Cc_i)((Pc_j)\alpha \supset (Pc_k)\alpha) \tag{3}$$

are DEH valid. The reader can easily verify this by means of semantic tableaux. Hence just as example (1) showed that commands are intended to flow «down» a command hierarchy, so (3) shows that permissions are intended to flow «up» a command hierarchy. Although this result may not seem quite as intuitive as (1), it is nevertheless a consequence of the stipulation H (which is also crucial for (1)) and our definition of *P*.

The second wff scheme involving *P* that we wish to consider is

$$(Pc_i)(Ex_j)\alpha(x_j) \supset (Ex_j)(Pc_i)\alpha(x_j) \qquad (4).$$

(Here we assume that the existential quantifier (*E*) is defined in the usual way.) The scheme (4) is the command-logic analogue of the well-known Barcan formula (7), which has been widely discussed in the literature on modal logic. The controversy over expressions like (4) centers around whether or not we should be allowed to conclude that something exists from the mere hypothesis that (in

(7) So named by Prior [9], p. 26.

this case) it is permitted that something should exist. Wffs of the form (4) are DEH valid, as the reader can verify by constructing the appropriate semantic tableaux. And this is to be expected, since our interpretation of L specifies that the set of individuals D ∪ E is the same in all worlds, both actual and allowable. If we were to allow different sets of individuals to be associated with each world of a model structure, then of course there would be countermodels to (4).

We have now seen several examples of the way that semantic tableaux can be used to determine whether or not wffs of L are DEH valid for given choices of D, E, and H. In general, we can see that L is a language that could be used both within a command hierarchy for issuing commands and outside a command hierarchy for describing the activity that takes place within it. And such a language, I would argue, ought to be a useful tool in understanding the complex workings of real command hierarchies. Of course the artificial language L and the analysis that we have given of command hierarchies are simple by comparison with their analogues in the real world. But I nevertheless hope that this simple beginning will lead to more realistic analyses.

4. *Application*

I shall conclude by sketching briefly a problem that might arise within a command hierarchy and that is solvable by means of the formal machinery we have developed for L. In order for the solution given to be generally effective, the particular interpretation of L that is being used must be one for which DEH validity is effectively decidable. Hence any interpretation in which D and E are both finite will do. But even for interpretations in which D, say, is infinite, there will be classes of wffs for which DEH validity is effectively decidable. The solution given will therefore also be effective in cases that involve only wffs of this kind.

The problem is that of locating an inconsistency among the commands of the various members of a command hierarchy. Suppose that each member of the hierarchy issues a statement (perhaps a very complex statement including commands, permissions, commands that are conditional on the statement of some other commander or on the state or the world, etc.) within a given period of time. Let each of these statements be expressed in the language L, and let the statement of commander c_i be called A_i. Suppose that $A_1 \wedge \ldots \wedge A_n$, the conjunction of all these statements, is inconsistent in the sense that a subordinate has issued a statement that conflicts with the statement of one of his superiors. As an example, consider the very simple case in which $n = 2$, $c_1 > c_2$, $A_1 = (Cc_1)\alpha$, and $A_2 = (Pc_2) \sim \alpha$. Here there is an obvious conflict between A_1 and A_2, but $A_1 \wedge A_2$ is not a contradiction (i.e., $\sim(A_1 \wedge A_2)$ is not DEH valid). Notice, however, that $(Cc_i) \sim (A_1 \wedge A_2)$ is DEH valid (both for i = 1 and i = 2). Hence, in the general case, in order to identify the kind of conflict that we are interested in it seems appropriate to test

$$(Cc_i) \sim (A_1 \wedge \ldots \wedge A_n) \qquad (5).$$

for DEH validity (for any i).

In general if a wff of the type (5) is DEH valid it may be because (a) one or more commanders has issued an inconsistent statement; or (b) the statements of some subset of members of the hierarchy are jointly inconsistent. The source of the inconsistency can be pinpointed by first testing $(Cc_i) \sim A_j$ for DEH validity (for each j), then testing $(Cc_i) \sim (A_j \wedge A_k)$ for DEH validity (for each j and k), and so on. If (a) an inconsistency is found by this procedure in a set of two or more statements, and (b) the procedure does not disclose an inconsistency in any subset of this set, and (c) one of the commanders involved is a superior of one or more of the other commanders involved, then we will have discovered, by purely formal methods, a case of insubordination.

It does not seem unreasonable to hope that there may be other problems, similar to the one discussed above, that are solvable by means of formal techniques related to L. But independently of L's ability to solve specific problems, the existence of a precisely defined and interpreted language that incorporates at least some of the important features of discourse concerning commands and command hierarchies may be of value and interest.

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