ON THE CHURCH-FREGE ANALYSIS OF BELIEF SENTENCES

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The present status of attempts to formalize sentences of the form 'A believes ...' as well as other instances of what Russell used to call 'propositional attitudes', is still far from satisfactory. The most promising effort is Carnap's in *Meaning and Necessity* in which he tries to retain sentences as objects of belief. But Church's criticism of the position is quite formidable (1), and it appears that any attempt to employ sentences as objects of belief will be open to the same or very similar objections. The difficulty in formalizing belief-sentences — or for that matter, any other sorts of oblique reference — is that we want to employ a rule of substitution for identicals, and at the same time to prevent such obviously invalid inferences as the following.

(1) Sir Walter Scott = the author of *Waverley*.

(2) George IV believed that (Sir Walter Scott = Sir Walter Scott). Therefore,

(3) George IV believed that (Sir Walter Scott = the author of *Waverley*).

There may well have been a time at which (1) and (2) were true, and at that time George IV may have doubted that Scott was in fact the author of *Waverley*.

One of the most plausible alternatives to Carnap's formulation is that suggested by Frege. Although the general outline of Frege's analysis of oblique contexts is widely known, there has been surprisingly little effort directed toward a systematic development and evaluation of it, and particularly to its application as a possible solution for the problem of belief-sentences. While Carnap's discussion of Frege in *Meaning and Necessity* is quite helpful, it surely

(1) «On Carnap's Analysis of Statements of Assertion and Belief», *Analysis*, 10 (1950), pp. 97-99, and «A Formulation of the Logic of Sense and Denotation», in *Structure, Method and Meaning: Essays in Honor of Henry M. Sheffer*, Paul HENLE, et al. (eds.), New York, 1951, pp. 3-24 (see n. 5, pp. 5, 6). The latter paper will be referred to henceforth as 'FLSD'.

cannot be counted as, nor was it intended to be, a rigorous development of Frege's views. And neither can it be claimed that Frege himself dealt with the question at sufficiently great length. As Church suggests, the paper, «*Über Sinn und Bedeutung*», was probably intended to disentangle certain problems so that Frege could proceed with the main business at hand, viz., the rigorous formalization of an extensional logic.

The first major effort to formulate a general system of intensional logic in accord with Frege's suggestion was Church's FLSD. While this paper does not explicitly deal with the problem of formalizing oblique contexts — except, perhaps, in connection with modal operators — it does provide the logical devices, and especially notation, for doing so. Church himself has not written extensively on the topic of the analysis of belief-sentences in accord with Frege's suggestion, presumably because he is aware that the axiom system of his paper is not wholly satisfactory. Nevertheless, it may be profitable to explore the possibility of an analysis of belief-sentences along Frege's line as a relatively informal way, with the hope that there will emerge a clearer notation of the direction that an adequate formalization will take. In my subsequent remarks, I shall undertake this, and try to show how a defensible theory of belief-sentences can be formulated. No claim will be made that a rigorous formalization will not reveal new problems or obviate some of those that appear here.

Before getting down to the details of the solution, something should be said about the reluctance of many philosophers to treat the Frege approach sympathetically. It is difficult to conceal a certain skepticism about the bewildering ontology that it seems to involve. And as Myhill remarks: «... few would see anything to object to in the quest for an approach which is immune from all the criticisms which the Frege-Church theory is designed to answer, and at the same time escapes the formidable complexities of the latter (2)». Philosophers, especially empiricists, may be quite fond of Goodman's asceticism, but there are reasons to think that his frugality may be achieved at too great a cost. In contrast with Good-

(2) «Problems Arising in the Formalization of Intensional Logic», *Logique et Analyse*, 1 (1958), p. 74.

man's austere exhortation, «Let us, as philosophers, be utterly fastidious in choosing linguistic forms,» (3) Church adopts Carnap's Principle of Tolerance with a measure of vengence. But Church can plausibly argue that his system is justified to the extent that it can solve problems that no other existing system can solve.

One of the chief recommendations for the Church-Frege distinction between sense and denotation is the simplicity with which it lends itself to the solution of Frege's puzzle about the Morning and Evening Stars, and special cases of this puzzle such as Langford's formulation of the Paradox of Analysis (4).

(4) Sir Walter Scott = Sir Walter Scott.

According to the Church-Frege view, although (1) and (4) have the same denotation, they express quite different propositions. This is inferred from,

(5) The sense of the name 'Sir Walter Scott' ≠ the sense of the name 'the author of *Waverley*'.

(6) The sense of the name 'Sir Walter Scott' = the sense of the name 'Sir Walter Scott'.

(7) If a name-forming part of a longer name is replaced by another having a different sense, the sense of the whole is altered.

(8) The sense of a sentence is a proposition.

The fundamental notions of the Chruch-Frege view are those of 'name', 'sense', 'denotation', 'concept of', 'denotes', and 'expresses'. A name 'N' denotes its denotatum *a*, and expresses a sense that is denoted by 'N_1', while N_1 is said to be a concept of *a*. In turn, 'N_1' denotes N_1, expresses a sense that is denoted by 'N_2', and N_2 is a concept of N_1. And so on *ad infinitum*. In addition, there will be a name having 'N' as its denotation, and so on, each of these names expressing a sense and thus entailing a further hierarchy. In accordance with the device of subscripts (5), (5) and (6) may be rewritten as

(3) «A World of Individuals,» in I. M. BOCHENSKI, Alonzo CHURCH, and Nelson GOODMAN, *The Problem of Universals*, Notre Dame, Indiana : University of Notre Dame Press, 1965, p. 30.

(4) «The Notion of Analysis in Moore's Philosophy,» in P. A. SCHILPP (ed.), *The Philosophy of G. E. Moore*, Chicago, 1942, p. 323.

(5) This device is borrowed with modifications from Church's FLSD. Also, it will be stipulated that compound names of the form $(\alpha\beta\gamma)_1$ be understood as abbreviations for the form $(\alpha_1\beta_1\gamma_1)$. Parentheses will be omitted when the scope of the subscript is obvious.

(5′) Sir Walter $Scott_1 \neq$ the author of *$Waverley_1$*,

(6′) Sir Walter $Scott_1 =$ Sir Walter $Scott_1$.

Frege holds (i) that sentences are names, and (ii) that names occurring in oblique contexts denote those senses that they express when used in ordinary contexts. According to (ii), the proper formulations of (2) and (3) would be, respectively,

(9) George IV believed that (Sir Walter $Scott_1 =_1$ Sir Walter $Scott_1$),

(10) George IV believed that (Sir Walter $Scott_1 =_1$ the author of *$Waverley_1$*).

One of the prominent features of the Frege system is that sentences in ordinary contexts denote truth-values, i.e., the True of the False. Consider, however,

(11) Sir Walter $Scott_1 =_1$ the author of *$Waverley_1$*.

According to the Church-Frege view, (11) is the name of a proposition, not a truth-value. Thus, since *what* George IV believes in (10) is what is denoted by (11), he would believe a proposition, as we would expect. The difference between (5′) and (11) is easily overlooked. Both of these names will, of course, express a sense, but the sense expressed by (5′) is a proposition, while the sense expresseb by (11) is not.

Now, it is obvious that the Church-Frege translations of (2) and (3) would preclude our validly inferring (3) from (1) and (2). (1) and (9) simply do not contain common terms from which (10) might follow. But a translation that would prevent this sort of inference is not all that we want to accomplish. After all, the same result could have been achieved by restricting our rule of substitution to ordinary contexts. But the beauty of the Church -Frege view is that the same inferences that are permitted in ordinary contexts should also be permitted in oblique ones. Carnap's analysis, on the other hand, does not exhibit this feature. Substitution of identicals is permitted only in ordinary contexts, and the condition of intensional isomorphism is imposed for indirect contexts.

If we think back on Frege's original puzzle, viz., how it can be that (1) and (4) differ from one another, we note that the solution consists in maintaining that 'Sir Walter Scott' and 'the author of *Waverley*' express two different senses but have the same denotation. But there is no good reason why the same problem cannot also occur with two names that express different senses, and yet denote the same sense. Assume, e.g.,

(12) Othello believes that (Desdemona loves Cassio)$_1$,

(13) Iago believes that (Desdemona loves Cassio)$_1$.

Thus we may assert that,

(14) What Othello believes$_1$ = what Iago believes$_1$ [6].

And it is also true that,

(15) What Othello believes$_1$ = what Othello believes$_1$.

(14) and (15) are higher-level counterparts of (1) and (4), and thus the Frege puzzle can appear at any level of the hierarchy. Its solution depends on recognizing that there can be more than one concept of a given concept, just as there can be more than one concept of a given individual. In order to explain why (14) is informative in a way in which (15) is not, we require the following premise.

(16) What Othello believes$_2$ $\neq$ what Iago believes$_2$.

In a sentence of the form 'A believes $(\ldots)_1$', what A believes is what is *denoted*, not what is expressed, by the name '$(\ldots)_1$'. Any name with the same denotation that is substituted for '$(\ldots)_1$' will leave the truth-value of 'A believes $(\ldots)_1$' unaltered, and thus would be a valid inference. This is the principle to which any appropriate rule of substitution must conform, and it suggests a strikingly simple rule:

R1. From 'A believes N_n' and '$N_n = M_n$', where 'N_n' and 'M_n' are names of propositions, we may infer 'A believes M_n'.

One difficulty with this rule is that it seems obviously *wrong*, but I want to contend that, appearances notwithstanding, it is perfectly legitimate provided that certain criteria for identity of sense are not adopted.

It would be helpful to carefully distinguish two questions: (a) Does *R1* permit inferences from true premisses to a false conclusion? and (b) What grounds can be given for holding that particular instances of premisses of the form '$N_n = M_n$' are true? The usual justification for a negative answer to (a) is supplied by a counter-example: «A person may well hold inconsistent beliefs, i.e., he may believe a particular proposition of logic and yet at the

(6) This is, of course, overly simplified. Presumably both Othello and Iago hold many beliefs, some of which may differ. Also, there is an implicit time reference.

same time not believe another that can be proved to be logically equivalent to it» (7). The objection appears to be quite forceful, but it is important to make clear one of its assumptions. For as it stands, this objection is simply not an instance of *R1*. What *R1* requires is a premiss of the form '$N_n = M_n$', while what the counter-example asserts is that the propositions denoted by 'N_n' and 'M_n' are logically equivalent. It is not clear, however, that given the logical equivalence of two propositions, we are entitled to assert their identity. But in any case, the premiss required to render the objection valid would be a partial answer to question (b), and it is to this question that we shall next turn our attention.

Carnap recognizes in Frege's view two principles of interchangeability which I shall reformulate and refer to as *R2* and *R3*.

R2. If 'N_n' and 'M_n' are name-forming expressions whose denotations are propositions, and 'N_n' is exactly like 'M_n' except that 'N_n' contains the name-forming part 'A_n' in a place where 'M_n' contains the name-forming part 'B_n', then if $A_n = B_n$, then $N_n = M_n$.

Now, all that *R2* does is to give us a sufficient condition for the identity of denotation of compound names provided that we can recognize that two simpler constituent names have the same denotation. But it surely does not, as it stands, provide a logical criterion for identity of denotation.

R3. If (i) 'N_n' and 'M_n' are name-forming expressions whose denotations are propositions and whose senses are denoted by 'N_{n+1}' and 'M_{n+1}' respectively, (ii) 'N_n' is exactly like 'M_n' except that 'N_n' contains the name-forming part 'A_n' at a place where 'M_n' contains the name-forming part 'B_n', and (iii) 'A_{n+1}' and 'B_{n+1}' denote the senses of 'A_n' and 'B_n' respectively, then if $A_{n+1} = B_{n+1}$, then $N_{n+1} = M_{n+1}$.

Again, what *R3* does is to give a sufficient condition for the identity of the senses of two compound names provided that we can recognize the identity of the senses of their corresponding constituents. But the real importance of *R3* lies in the fact that it can be combined with still another principle, viz.,

(7) For lack of a better name, I shall henceforth refer to this as «the puzzle of the equivalent propositions».

R4. If 'N_n' and 'M_n' are name-forming expressions whose denotations are propositions, and 'N_{n+1}' and 'M_{n+1}' are names whose denotations are the senses of 'N_n' and 'M_n' respectively, then if $N_{n+1} = M_{n+1}$, then $N_n = M_n$.

The problem posed by *R3* and *R4* is that of formulating a criterion for identity of sense, or what comes to the same thing, a criterion for the truth of '$N_{n+1} = M_{n+1}$'. The most tempting solution to this problem is provided by one or more proposed logical criteria. Church suggests three such criteria in FLSD, and a variant of Carnap's intensional isomorphism would certainly be counted as a fourth. But actually a whole range of criteria may be advanced in addition to these. Any acceptable proposal must, of course, satisfy certain minimal formal conditions, it must be in reasonable accord with our expectations as to what sort of answer we are looking for, it must lend itself to the solution of Frege's puzzle, and it must lend itself to a formalization of indirect discourse. By conformity with our expectations, I mean something like this: We normally expect the sentence, «Scott is the author of *Waverley*», to convey information, and thus a criterion for identity of sense that entails that this sentence is logically true would be unaccceptable. The same sort of thing applies to talk about propositions, i.e., we want the sentence, «What Othello believes is the same as what Iago believes», to express an informative proposition. In fact, Frege's puzzle arises simply because two expressions can denote the same entity without being logically equivalent.

Church's Alternative (2) puts down as an axiom or theorem '$\Box(N_1 = M_1) \supset N_1 = M_1$', where '$N_1$' and '$M_1$' denote propositions. But this is wholly unacceptable as a criterion for identity of sense because it is precisely the premiss required in order to make the puzzle of the equivalent propositions count as a valid objection. Alternative (1), on the other hand, has the effect of identifying the senses of two names when the names are lambda-convertible, i.e., if 'N_n' λ-conv 'M_n', then $N_{n+1} = M_{n+1}$. An objection very similar to that against Alternative (2) may also be raised against Alternative (1). Let, e.g., '$(2 + 3)_n$' and '$(4 + 1)_n$' be lambda-convertible names of the same number, and let 'A_{n+1}' and 'B_{n+1}', respectively, denote the senses expressed by these names. Then according to Alternative (1), $A_{n+1} = B_{n+1}$. Next, let 'N_{n+1}' be the name of the sense expressed by '$(2 + 3 = 5)_n$', and 'M_{n+1}' be the name of the sense expressed by '$(4 + 1 = 5)_n$'. Given these conditions, it follows from *R3* that $N_{n+1} = M_{n+1}$, and by *R4* that $(2 + 3 = 5)_n = (4 + 1 = 5)_n$. Further ,by *R1* we may infer that if A believes $(2 + 3 = 5)_n$, then A believes $(4 + 1 = 5)_n$. But it is conceivable, surely, that the former may be true and the latter false.

Church's Alternative (0) is of a slightly different character. According to this criterion, any two names will be said to express different senses unless it can be proved that they express the same sense. The difficulty with Alternative (0) is that it does not pay sufficient respect to the vagaries and semantic irregularities of ordinary language. In short, it would preclude certain very simple and obviously valid inferences. However, since the same objection applies to Carnap's criterion of intensional isomorphism, it may prove helpful to discuss that criterion first.

Carnap formulates his criterion essentially as follows: Two expressions are intensionally isomorphic if and only if the complete expressions are logically equivalent and each part of one expression is logically equivalent to the corresponding part of the other. If we attempt to translate this into the Chruch-Frege conceptual framework, we get something like the following:

RC. If 'N_n' and 'M_n' are two name-forming expressions whose name-forming parts are 'A_n, B_n, C_n, ...' and 'A_n', B_n', C_n', ...' respectively, and for any given position in 'N_n' the name-forming part 'x' occurs if and only if 'x'' occurs in a similar position in 'M_n', and, furthermore, 'N_{n+1}' and 'M_{n+1}' are names of the senses expressed by 'N_n' and 'M_n' respectively, then $N_{n+1} = M_{n+1}$ if and only if both 'N_n' and 'M_n' are L-equivalent, and for every 'x', 'x' and 'x'' are L-equivalent.

One significant difference between Carnap's formulation and the present one is that Carnap's does not allow higher-level intensions. Thus there is no apparent way of dealing with higher-level occurrences of Frege's puzzle. *RC* appears to have a sort of *ad hoc* character. It is somehow hoped that if two sentences are logically equivalent in the ways specified, they really will express the same proposition, and if not equivalent, they will express different propositions. But suspicion of this kind cannot be pressed too far, and more concrete reasons must be sought out. I think that such reasons can be found.

(17) Horatio carved a round excision,

(18) Horatio carved a circular hole.

We would surely want to hold that (17) and (18) express the same proposition. But according to *RC*, this would be true only if 'round excision' and 'circular hole' were L-equivalent, which is simply not the case. Moreover, any common dictionary of synonyms would provide as many additional instances as we might wish for. Carnap considers such instances to be trivial, but trivial or not, they are still contrary instances and are too numerous to be ignored. Carnap's rule might be legitimate provided that our language exhibited a measure of regularity that it does not in fact exhibit, i.e., if it were always the case that when we wished to express the same sense, we adopted two expressions that happened to be logically equivalent. This same point constitutes an equally valid objection against Church's Alternative (0). The solution of the general problem of synonymy would be relatively simple if semantic irregularities were ruled out of consideration.

The most plausible way to deal with semantic irregularities is to adopt a device similar to Carnap's «meaning postulates». Whenever two names are used to express the same sense, but are not logically equivalent, we can state this fact by means of an appropriate postulate. Both *RC* and Alternative (0) can be modified to accomodate such postulates.

I think that we are now in a position to sketch out a theory along Church-Frege lines that will lend itself both to the solution of Frege's puzzle and to a satisfactory formulation of a logic of belief-sentences. We want first to specify the conditions under which a sentence of the form '$N_n = M_n$' will be true.

(a) If 'N_n' and 'M_n' are λ-convertible, then $N_n = M_n$.

(b) If $N_{n+1} = M_{n+1}$, then $N_n = M_n$.

(c) If N_n' and 'M_n' have the same denotation, then $N_n = M_n$.

(d) If '$N_n = M_n$' logically follows from true premisses, then $N_n = M_n$.

(e) If '$N_n = M_n$' is a meaning postulate or theorem, then $N_n = M_n$.

(f) If 'N_n' and 'M_n' are names whose name-forming parts are 'A_n, B_n, C_n, ...' and 'A_n', B_n', C_n', ...' respectively, and for any name-forming part 'x', 'x' occurs at a position in 'N_n' if and only if 'x'' occurs at a similar position in 'M_n', then if for every x, $x = x'$, then $N_n = M_n$.

(g) If 'N_n' and 'M_n' are names that do not satisfy any condition (a) — (f), then $N_n \neq M_n$.

Some brief comments are called for. Condition (a) does not assert that λ-convertible expressions express the same sense, but only that they have the same denotation. Thus when (a) is combined with (f), we do not encounter the objection raised against Church's Alternative (1). The conditions for the truth of the antecedent of (b) are found by raising all subscripts of (a) — (f) from 'n' to 'n + 1'. Condition (c) is primarily intented to apply to the case in which $n = 0$, but is also applicable to any other level. Condition (d) is designed to permit instances such as (14) which can be inferred from (12) and (13). The whole set of conditions would, of course, replace *R2* — *R4* and *RC*.

Condition (c) needs to be elaborated upon. Simple instances of synonymy in natural languages would appear as postulates in our formal system. We would want to hold, e.g., that 'round = circular', 'fortnight = period of two weeks', and 'excision = hole'. But synonymous expressions do not have only denotation in common; they share also the same sense. This could be expressed by pairs of the following form:

(19a) $N_n = M_n$,

(19b) $N_{n+1} = M_{n+1}$.

However, if the language contains 'rules of sense' of the form «'N_n' expresses N_{n+1}», then (c) would make it unnecessary to include (19a) among the postulates.

R1 is a special instance of the more general inference rule,

R1' . $x_n = y_n$, φx_n, ∴ φy_n.

By means of the Schönfinkel device, any function containing more than one variable may be treated as a function of a single variable ([8]). Thus 'bel (x, y_n)', i.e., 'x believes proposition y_n', would be treated

([8]) Church, A., *The Calculi of Lambda-Conversion*, Princeton University Press, 1941 sec.3.

like any other function, assuming, of course, that the appropriate type restrictions are observed for variables. *R1'*, then, applies to both ordinary and oblique contexts, and the same holds for identity-conditions (a)-(g).

Iterated oblique contexts pose no additional problems. Suppose, e.g., that Iago believes (12).

(12) Iago believes that [(Othello believes that)$_1$ (Desdemona loves Cassio)$_2$].

Note that while '(Desdemona loves Cassio)$_1$' denotes a proposition '(Desdemona loves Cassio)$_2$' does not. On the other hand, '(Othello believes that)$_1$ (Desdemona loves Cassio)$_2$' does denote a proposition.

In summary, I have been concerned to show in a reasonably informal way how an adequate theory of belief-sentences can be formulated within the Church-Frege framework. It is contended that *R1* is a legitimate rule provided that a certain form of logical criterion of identity is not permitted. A criterion that would avoid this difficulty is then put forward. Moreover, although belief-sentences have been chosen for purposes of illustration, the general solution would appear to be applicable to all non-modal occurrences of oblique reference.

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