"NEXT" AND "OUGHT"
ALTERNATIVE FOUNDATIONS FOR VON WRIGHT'S TENSE-LOGIC, WITH AN APPLICATION TO DEONTIC LOGIC

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1. *Introduction*

In his paper entitled entitled "And Next" ([13] in the bibliography) Georg Henrik von Wright studies a new tense-logical constant which he pertinently characterizes as a kind of *asymmetrical conjunction.*
The axiomatic system set forth in [13] — the T-calculus — is intended to capture the formal properties of this constant. A first attempt to study it was made in von Wright [11], where he also emphasized the importance of the present type of tense-logic to a satisfactory development of *deontic logic*; later, in [10], E. J. Lemmon made a similar point in connection with the logic of imperatives.

In the present paper, we first give a new *axiomatic* basis for the T-calculus by defining an equivalent system DT, which, we think, is in several respects more easily handled than the former. Next, we present a *semantical*, or *model-theoretic*, treatment of the system DT (and indirectly of the T-calculus) which is essentially based on the techniques of Kripke [8] and Hanson [6]. The system is then proved sound and complete by means of the method of semantic tableaux, which is also seen to provide a decision procedure for it that appears to be much smoother than the normal-form method suggested in [13] for the T-calculus (1). Finally, we consider a combination DDT of our tense-logic DT with deontic logic as well as some philosophical applications of it — *inter alia*, a way out of what may be called "Chisholm's Puzzle of Contrary-to-Duty Imperatives" (see [5]).

(1) An interesting further object of investigation would certainly be that of relating the present kind of tense-logic to the systems considered by A. N. Prior in his well-known work in the field — in particular, see [14] and [15]. We have to refrain from this here.

2. *von Wright's tense-logic T*

The tense-logic T is based on a denumerably infinite list of propositional variables, two primitive truth-functional connectives $\sim$ and $\wedge$ (in terms of which $\vee$, $\supset$, and $\equiv$ are defined in the usual way), and a primitive *binary* tense-logical connective T ("and next"). The wffs (well-formed formulae) of the calculus are given as usual except that we have an additional stipulation: if A and B are wffs, then (ATB) is a wff. Moreover, in T we define a *unary* tense-logical connective $\Box$ ("next") by

Def $\Box$. $\Box A =_{df} A^{+}TA$, where A^{+} is an arbitrary fixep tautology, say, the wff '$\sim(p \wedge \sim p)$'.

Brackets may *inter alia* be omitted under the convention that T makes a greater break than $\equiv$, $\equiv$ than $\supset$, $\supset$ than $\vee$, $\vee$ than $\wedge$, and $\wedge$ than $\sim$.

The axiom schemes and rules of inference for von Wright's T-calculus (2) are as follows:

PC. A set of axiom schemes adequate for the classical propositional calculus, with Modus Ponens.

A1. $(A \vee BTC \vee D) \equiv (ATC) \vee (ATD) \vee (BTC) \vee (BTD)$

A2. $(ATB) \wedge (CTD) \equiv (A \wedge C\,T\,B \wedge D)$

A3. $A \equiv (A\,T\,B \vee \sim B)$

A4. $\sim(A\,T\,B \wedge \sim B)$

Substitutability of provable equivalents (Subs $\equiv$): If $\vdash A \equiv B$, and D is the result of substituting B for one or more occurrences of A in C, then $\vdash C \equiv D$.

3. *The "quasi-deontic" tense-logic DT*

The vocabulary of the tense-logic DT now to be considered is like that of the T-calculus except that the unary connective $\Box$ replaces T as a primitive logical constant, the latter being defined by

Def T. $ATB =_{df} A \wedge \Box B$.

(2) Inessential deviations from von Wright's own formulation of the T-calculus consist in our not taking v, $\supset$, and $\equiv$ as primitives, and in dispensing with the rule of variable-substitution in favour of axiom-schemes. On the other hand, our addition of Def $\Box$ constitutes a somewhat more substantive deviation.

DT has the following axiom schemas and rules of inference:

PC.

a1. $\Box A \supset \sim\Box\sim A$

a2. $\sim\Box\sim A \supset \Box A$

a3. $\Box(A \supset B) \supset (\Box A \supset \Box B)$

Necessitation (Nec): If $\vdash A$, then $\vdash \Box A$.

The reason for speaking of DT as a "quasi-deontic" system is this. On one hand, DT contains such deontic logics as F = {PC, a3, Nec} and D = {PC, a1, a3, Nec}, given by Hanson [6] p. 178, as well as the systems D1 and D2, given by Lemmon [9] p. 184 (although it does not contain any of the stronger deontic systems considered by these authors); in addition, DT meets the familiar requirement that neither $\Box A \supset A$ nor $A \supset \Box A$ should be derivable in a deontic logic (cf. e.g. [1] p. 101). On the other hand, a deontic interpretation of DT is effectively precluded by the presence of a2 (which would then assert that permissibility entails obligatoriness); moreover, the presence of a2 yields the result that DT is not contained in the modal system S5, nor in any of the deontic logics considered by Hanson and Lemmon, since these are all weaker than S5. Finally, it may be noticed that if in DT we replace a1 by $\Box A \supset A$, or a2 by $A \supset \Box A$, we obtain a very strong "degenerate" modal system which collapses into PC.

4. *The inferential equivalence of T and DT*

In this section we show that T contains DT and, conversely, that DT contains T. To establish the first result we derive in T the schemata a1-a3, the rule Nec, and the definition Def T in the form of an equivalence.

To derive a1 in T:

(1)	$(A^{+}TA) \wedge (A^{+}T\sim A) \supset (A^{+} \wedge A^{+}TA \wedge \sim A)$	A2, PC
(2)	$\sim(A^{+} \wedge A^{+}TA \wedge \sim A)$	A4
(3)	$(A^{+}TA) \supset \sim(A^{+}T\sim A)$	(1), (2), PC

(3) = a1 by Def $\Box$.

To derive a2:

(1) $\sim(A^+T \sim A) \supset ((A^+T \sim \sim A) \vee (\sim A^+T \sim A) \vee$
$(\sim A^+T \sim \sim A))$ PC, von Wright's T1 ([13] p. 295 f.)

(2) $\sim(A \wedge \sim A\, T\, B)$ von Wright's T3 ([13] p. 296)

(3) $\sim A^+ \equiv A \wedge \sim A$ PC

(4) $\sim(\sim A^+T \sim A)$ (2), (3), Subs $\equiv$,PC

(5) $\sim(\sim A^+T \sim \sim A)$ Similarly

(6) $\sim(A^+T \sim A) \supset (A^+TA)$ (1), (4), (5), PC, Subs $\equiv$

(6) = a2 by Def $\Box$.

To derive a3:

(1) $(A^+T\, A \supset B) \wedge (A^+TA) \wedge \sim(A^+TB) \supset$
$(A^+ \wedge A^+T(A \supset B) \wedge A) \wedge (A^+T \sim B)$
A2, (6) in the proof of a2,
PC, Subs $\equiv$

(2) $(A^+ \wedge A^+T(A \supset B) \wedge A) \wedge (A^+T \sim B) \supset$
$(A^+ \wedge A^+ \wedge A^+T(A \supset B) \wedge A \wedge \sim B)$ A2

(3) $\sim(A^+ \wedge A^+ \wedge A^+\, T\, (A \supset B) \wedge A \wedge \sim B)$
A4, PC, Subs $\equiv$

(4) $(A^+T\, A \supset B) \supset ((A^+TA) \supset (A^+TB))$ (1), (2), (3), PC

(4) = a3 by Def $\Box$.

To derive Nec in T:

(1) $\vdash A$ hyp.

(2) $\vdash A^+$ PC

(3) $\vdash A^+TB \vee \sim B$ (2), A3, PC

(4) $\vdash A \equiv B \vee \sim B$ (1), PC

(5) $\vdash A^+TA$ (3), (4), Subs $\equiv$,PC

where (5) = $\vdash \Box A$ by Def $\Box$.

Finally, we give a T-derivation of Def T in the form of

(1) $(ATB) \equiv A \wedge (A^+TB)$.

To prove the left-to-right implication in (1):

(2) $(ATB) \supset A$ von Wright's T4

(3) $(ATB) \equiv (A \wedge A^+\, T\, B \wedge B)$ PC, Subs $\equiv$

(4) $(ATB) \wedge (A^+TB) \equiv (A \wedge A^+\, T\, B \wedge B)$ A2

(5) $(ATB) \equiv (ATB) \wedge (A^+TB)$ (3), (4), PC

(6) $(ATB) \supset (A^{+}TB)$ (5), PC
(7) $(ATB) \supset A \wedge (A^{+}TB)$ (2), (6), PC
(7) = Q.E.D.

To prove the right-to left implication in (1):
(8) $A \wedge (A^{+}TB) \supset (A \wedge A^{+}T\,B)$ von Wright's T5
(9) $A \wedge (A^{+}TB) \supset (ATB)$ (8), Subs $\equiv$, PC
(9) = Q.E.D.

Thus, T contains DT. To establish the converse result we derive in DT the schemata A1-A4, the rule Subs $\equiv$, and Def $\Box$ in the form of an equivalence.

To derive A1 in DT:
(1) $(A \vee B) \wedge (\Box C \vee \Box D) \equiv (A \wedge \Box C) \vee (A \wedge \Box D) \vee (B \wedge \Box C) \vee (B \wedge \Box D)$ PC
(2) $\sim\Box\sim(C \vee D) \equiv (\sim\Box\sim C \vee \sim\Box\sim D)$ Familiar already in {PC, a3, Nec}
(3) $\Box A \equiv \sim\Box\sim A$ a1, a2, PC
(4) $\Box(C \vee D) \equiv \Box C \vee \Box D$ (2), (3), PC
(5) $(A \vee B) \wedge \Box(C \vee D) \equiv (A \wedge \Box C) \vee (A \wedge \Box D) \vee (B \wedge \Box C) \vee (B \wedge \Box D)$ (1), (4), PC
(5) = A1 by Def T.

To derive A2 in DT:
(1) $A \wedge \Box B \wedge C \wedge \Box D \equiv A \wedge C \wedge \Box B \wedge \Box D$ PC
(2) $\Box(B \wedge D) \equiv \Box B \wedge \Box D$ Familiar already in {PC, a3, Nec}
(3) $(A \wedge \Box B) \wedge (C \wedge \Box D) \equiv (A \wedge C) \wedge \Box(B \wedge D)$ (1), (2), PC
(3) = A2 by Def T.

To derive A3:
(1) $A \equiv A \wedge \Box(B \vee \sim B)$ PC, Nec
(1) = A3 by Def T.

To derive A4:

(1) $\sim \Box (B \wedge \sim B)$ — familiar already in {PC, a1, a3, Nec}

(2) $\sim(A \wedge \Box(B \wedge \sim B))$ — (1), PC

(2) = A4 by Def T.

In order to derive the rule Subs $\equiv$ in DT it is again sufficient to appeal to {PC, a3, Nec}, which admits these well-know rules: (i) if $\vdash A \equiv B$, then $\vdash \sim A \equiv \sim B$, (ii) if $\vdash A \equiv B$, then $\vdash A \wedge C \equiv B \wedge C$, (iii) if $\vdash A \equiv B$, then $\vdash C \wedge A \equiv C \wedge B$, and (iv) if $\vdash A \equiv B$, then $\vdash \Box A \equiv \Box B$.
From these four rules Subs $\equiv$ follows at once.

Finally, we note that the DT-derivation of Def $\Box$ in the form of

(1) $\Box A \equiv A^{+} \wedge \Box A$

is immediate by virtue of PC (A^{+} being a tautology).

The proof that T and DT are inferentially equivalent is complete.

5. *A modelling for the tense-logic DT*

A *model sequence* is either a non-empty finite sequence $\langle H_1, H_2, \ldots, H_k \rangle$ $(k \geq 1)$ of arbitrary objects or a denumerably infinite sequence $\langle H_1, H_2, \ldots \ldots \rangle$ of arbitrary objects. Given any H_i in a denumerable model sequence $(i = 1, 2, \ldots)$ we say that H_{i+1} is the unique *successor* of H_i in the sequence; similarly for any H_i, H_{i+1} in a finite k-termed model sequence where $1 \leq i < k$. Moreover, we stipulate that in a finite k-termed model sequence $\langle H_1, \ldots, H_k \rangle$ $(k \geq 1)$ the last term H_k is to be its own successor in the sequence. Let 'RH' denote the successor of H in a model sequence. According to the above stipulations, then, our successor-function R is taken to satisfy the condition: if H belongs to a model sequence — whether finite or denumerable — then RH always belongs to it as well.

By a *primary valuation* for DT we mean any binary operation φ associated with a model sequence $\mathscr{S}$ which, for each member H of $\mathscr{S}$, assigns a truth-value to each atomic wff (propositional variable) of DT. Thus, if P is atomic and H is any member of $\mathscr{S}$, then $\varphi(P, H) = 1$ (truth) or $\varphi(P, H) = 0$ (falsity).

Next, we define a certain ternary operation w which, given φ and H (H any member of a model sequence $\mathscr{S}$), assigns a truth-value to each wff of DT. w is called the *secondary valuation* for DT and is inductively defined as follows:

(i) $w(P, \varphi, H) = \varphi(P, H)$, when P is an atomic wff.
(ii) $w(\sim A, \varphi, H) = 1$, iff $w(A, \varphi, H) = 0$.
(iii) $w(A \wedge B, \varphi, H) = 1$, iff $w(A, \varphi, H) = 1$ and $w(B, \varphi, H) = 1$.
(iv) $w(\Box A, \varphi, H) = 1$, iff $w(A, \varphi, RH) = 1$.

By a DT-*system* we understand an ordered quadruple $(\varphi, H_1, \mathscr{S}, R)$, where φ is a primary valuation, and where $\mathscr{S}$ is a model sequence, R its successor-function, and H_1 its first term. A wff A is said to be *true in* a DT-system $(\varphi, H_1, \mathscr{S}, R)$ if $w(A, \varphi, H_1) = 1$; *false in* $(\varphi, H_1, \mathscr{S}, R)$ if $w(A, \varphi, H_1) = 0$. A is *valid* in DT iff A is true in every DT-system. Finally, by a DT *model of* (*countermodel to*) a wff A we shall understand a DT-system $(\varphi, H_1, \mathscr{S}, R)$ such that $w(A, \varphi, H_1) = 1(0)$. Evidently, A is valid iff there is no countermodel to A.

We show below that our calculus DT is *sound and complete* in the sense that a wff is valid if and only if it is provable in DT. Since DT is equivalent to von Wright's T-calculus, the soundness and completeness of the latter is then indirectly established as well.

Informal explanation. The members of a model sequence are only known to us *via* their position in the sequence; what they are in other respects is left unspecified. For our intuitive tense-logical purposes we may think of them as states of the world at successive moments of time; the first term H_1 of a model sequence is singled out as the initial state taken up for consideration, RH_1 (= H_2 if the sequence is at least 2-termed) is the state that comes next to H_1 in time, and so on. We do not assume that two successive world-states represented by H_i and H_{i+1} in a model sequence have to be temporally contiguous in any strict sense of the word. It is also plain that we suppose time to be *discretized* into units whose length is left unspecified. Finally, by paraphrase of the valuation clause (iv) in the definition of w above, a wff 'NextA' is taken to be true in a state H under the "interpretation" φ, iff, A is true under φ in the state RH that comes next to H (where H occurs as a term of a sequence of world-states succeeding each other in time).

The modellings given by Kripke [8] for alethic modal logics (M, B, S4, S5) and by Hanson [6] for deontic logics (D, DM, DB, DS4, DS5) are based on appropriate notions of a *model structure*, i.e. an ordered triple (G, K, R) such that K is a non-empty set, $G \in K$, and R is a binary relation defined on K. A similar notion is available in our model-theory for DT: a DT *model structure* is a triple (H_1, $\mathscr{S}$, R), where $\mathscr{S}$ is a model sequence, H_1 its first term, and R our successor-relation over $\mathscr{S}$. This notion differs most conspicuously from those employed by Kripke and Hanson in that it assumes R to be a *functional*, or *many-one*, relation; obviously, a DT model structure will always have the property: if H is a term of $\mathscr{S}$, then there is a *unique* H′ in $\mathscr{S}$ such that HRH′ ($H' = RH$). Kripke's and Hanson's model structures, on the other hand, are all taken to satisfy the weaker condition: if $H \in K$, then there is a (not necessarily unique, by any means) $H' \in K$ such that HRH′ (3). If in addition R is assumed to be reflexive (reflexive and symmetric, reflexive and transitive, an equivalence relation) over K, we get Kripke's notion of an M- (B-, S4-, S5-) model structure. To obtain the corresponding deontic notions, one must first of all drop the assumption that R is reflexive over the whole of K; further appropriate restrictions on R closely parallelling Kripke's then yield Hanson's concepts of a DM-, DB-,DS4-, and DS5-model structure.

6. *Semantic tableaux*

A neat device for testing whether or not a given wff is valid (in DT) is afforded by the method of semantic tableaux, originally developed by Beth [4] in the field of quantification theory, further elaborated by Kripke [7], [8] in that of alethic modal logic, and quite recently extended to deontic logic by Hanson [6]. Clearly, a necessary and sufficient condition that a wff A should *not* be valid is that there should exist a countermodel to A, i.e. a system (φ, H_1, $\mathscr{S}$, R) such that $w(A, \varphi, H_1) = 0$. We represent this situation by putting A in the *right* column of a tableau. Further tableaux will be introduced later as a result of the rules Yl(a) and Yr(a) given below;

(3) With just this restriction on R, the result is Hanson's modelling for the deontic logic D = {PC, a1, a3, Nec}.

these are said to be *auxiliary*, while the initially introduced tableau is said to be the *main* one. As we shall see, we are in general not dealing with a single tableau but with a *finite sequence* of tableaux $\mathcal{T} = \langle t_1, t_2, \ldots, t_k \rangle$ where for each i such that $1 \leq i < k$, $t_{i+1} = St_i$, and $t_k = St_k$, so that S is a successor-function among tableaux parallel to the function R in the modelling for DT, and where the first term t_1 is singled out as the main tableau. Moreover, we shall see from the rule $\wedge$r below that a tableaux-construction may introduce a system of *alternative* sequences of the kind mentioned. Given, then, a main tableau with A in the right column, we continue the construction by the following rules (which apply to any tableau, main or auxiliary):

Nr. If $\sim$B appears in the right column of a tableau, put B in the left column of that tableau.

Nl. If $\sim$B appears in the left column of a tableau, put B in the right column of that tableau.

$\wedge$r. If B $\wedge$C appears in the right column of a tableau t, there are two alternatives: Extend t either by putting B in the right column or by putting C in the right column. More precisely, if t is the *i*-th ($i = 1, 2, \ldots$) tableau in a sequence $\mathcal{T}$ and t has B $\wedge$C on the right, replace $\mathcal{T}$ by two alternative sequences $\mathcal{T}'$ and $\mathcal{T}''$ which are like $\mathcal{T}$ except for having as their *i*-th term tableaux t′ and t″, respectively, where t′ (t″) is like t except that in addition it contains B (C) on the right.

$\wedge$l. If B $\wedge$C appears in the left column of a tableau, put B and C in the left column of that tableau.

Yr(a). Suppose that $\Box$B appears on the right of the *i*-th tableau t_i ($1 \leq i$) in a sequence $\mathcal{T}$ and that there is no tableau t_{i+1} in $\mathcal{T}$. Then start out a new auxiliary tableau $t_{i+1} = St_i$ with B on the right of t_{i+1}.

Yr(b). Suppose that $\Box$B appears on the right of t_i in $\mathcal{T}$ but that there is already in $\mathcal{T}$ a tableau $t_{i+1} = St_i$ (by virtue, originally, of an application of Yr(a), or of Yl(a) below). Then put B on the right of t_{i+1}.

Yl(a). Suppose that $\Box$B appears on the left of t_i in $\mathcal{T}$ and that there is no t_{i+1} in $\mathcal{T}$. Then start out $t_{i+1} = St_i$ with B on the left.

Yl(b). Suppose that $\Box$B appears on the left of t_i in $\mathcal{T}$ but that $t_{i+1} = St_i$ is already in $\mathcal{T}$. Then put B on the left of t_{i+1}.

Following Kripke we define a tableau as *closed* iff some wff A appears on both sides of the tableau, a sequence of tableaux as closed iff some term in it is closed, a system of alternative tableaux-sequences as closed iff each of the alternative sequences is closed. Furthermore, a construction is closed if at some stage of the construction a closed system of alternative tableaux-sequences appears. Finally, to facilitate termination of a construction, we adopt Kripke's restriction that a rule is not to be applied to a wff occurring in a closed tableaux-sequence, nor is it to be applied if it is "superfluous" (e.g., Yr(b) — Yl(b) — is superfluous if B already appears on the right — left —of t_{i+1}; and so on).

7. *Decidability, equivalence of tableaux to models*

By means of arguments analoguous to those of Kripke in [8], we can show that the method of semantic tableaux yields a decision procedure for the calculus DT. It is enough to establish these two lemmata:

L1. For any wff A, the tableaux-construction for A terminates in finitely many steps.

L2. For any wff A, the construction for A is closed if and only if A is valid.

L1 is obvious already in view of the fact that each of our tableau rules eliminates a connective. A rigorous proof may proceed on Kripkeesque lines (see [8], p. 87 f.) by an induction on the *degree* of A, where this notion is defined inductively as follows: Deg (A) = 1, if A is atomic; Deg ($\sim$A) = Deg (A); Deg (A $\wedge$ B) = Max (Deg (A), Deg (B)); and Deg ($\Box$A) = Deg (A) + 1. We note incidentally that, by virtue of the DT-definition of von Wright's connective T, the present notion of degree extends his concept of the degree of a s.c. *history* — a special kind of wff in the T-calculus — to all wffs of that calculus; the extension is effected by adding to the above inductive definition the clause: Deg (ATB) = Deg (B) + 1, if Deg (A) $\leq$ Deg (B); otherwise, if Deg (A) > Deg (B), Deg (ATB) = Deg (A). We also observe that the Kripkeesque argument for L1 guarantees an analogue to the stronger result for M and B stated in [8] (p. 88 at the bottom): Let A be a wff of DT such that Deg (A) $= m$; then the DT-construction for A terminates in a closed or non-closed system of alternative tableaux-sequences, each of which contains at most m terms.

Again, L2 can be established in essential analogy with Kripke's proofs of Lemma 1 and Lemma 2 in [8] (pp. 76 ff.). In the 'if'-part of L2, only the finite case needs to be taken into account, because of L1.

8. *Soundness and completeness of DT*

To verify that all provable wffs of DT are valid we just check that the tableaux-construction for each DT-axiom is closed, and that the rules, i.e. Modus Ponens and Nec, preserve validity (cf. [8], p. 82, and [7], p. 11).

To establish the completeness of DT, i.e. that every valid wff is provable, we have recourse to the notion of the *characteristic formula* (chf) of a system of alternative tableaux-sequences at a given stage of a construction. First, define the chf of a tableau t at a stage as $A_1 \wedge \ldots \wedge A_m \wedge \sim B_1 \wedge \ldots \wedge \sim B_n$, where the A_i are the wffs occurring on the left of t at the given stage and the B_j are the wffs occurring on the right of t at that stage. Further, let $\langle t_1, \ldots, t_k \rangle$ be any one of the alternative tableaux-sequences present at a given stage; define the chf of such a sequence as $C_1 \wedge \Box^1 C_2 \wedge \ldots \wedge \Box^{k-2} C_{k-1} \wedge \Box^{k-1} C_k$, where C_i is the chf of t_i ($1 \leq i \leq k$), and where we write '$\Box^{k-j}$' for a string of k-j $\Box$'s ($j = 1, \ldots, k$-1) and nothing for the empty string of $\Box$'s. Finally, let $D_1, \ldots, D_n$ be the chf's of the alternative sequences in a system of sequences at a stage; then the chf of (the system at) that stage is defined as $D_1 \vee \ldots \vee D_n$.

The completeness of DT is a consequence of the following lemma: if A is the chf of the initial stage of a construction, and B is the chf of any stage, then $\vdash_{DT} A \supset B$; and one obtains this lemma by showing, along Kripkeesque lines, that the chf of the m-th stage implies the chf of the ($m + 1$)st stage (cf. sect. 4.2 of [8]). The following theorem-schemata and derived rules are needed for the proof:

d1. $\vdash \sim A \supset \sim A$

d2. $\vdash \sim\sim A \supset A$

d3. $\vdash A \wedge B \supset A \wedge B$

d4. $\vdash \sim(A \wedge B) \supset (\sim(A \wedge B) \wedge \sim A) \vee (\sim(A \wedge B) \wedge \sim B)$

d5. $\vdash A \wedge (B \vee C) \supset (A \wedge B) \vee (A \wedge C)$

d6. If $\vdash A \supset B$, then $\vdash C \wedge A \supset C \wedge B$

d7. If $\vdash A \supset B$, then $\vdash \Box^m A \supset \Box^m B$ $(1 \leq m)$

Familiar in {PC, a3, Nec}

d8. $\vdash \Box^m(A \vee B) \supset \Box^m A \vee \Box^m B$

Proof, by an easy induction on m, using d7 and $\Box^1(A \vee B) \supset \Box^1 A \vee \Box^1 B$ (see sect. 4 above).

d9. $\vdash \Box^m A \wedge \Box^m B \supset \Box^m(A \wedge B)$

Proof, by an induction on m, using d7 and $\Box^1 A \wedge \Box^1 B \supset \Box^1(A \wedge B)$.

The proof of our analogue of Kripke's lemma is then broken down into cases, depending on the rule applied to obtain the $(m + 1)$st stage from the m-th. Cases Nl, Nr, and $\wedge$1 are justified respectively by d1-d6-d7, d2-d6-d7, and d3-d6-d7 — plus some obvious PC-applications.

Case $\wedge$r.

Before: [4] $A_1 \wedge \ldots \wedge \Box_i^{-1} (\sim(A \wedge B) \wedge X) \wedge \ldots \wedge \Box^{k-1} A_k$; $1 \leq i \leq k$, $(\sim(A \wedge B) \wedge X) = A_i$.

After: [4] $(A_1 \wedge \ldots \wedge \Box^{i-1}(\sim(A \wedge B) \wedge \sim A \wedge X) \wedge \ldots \wedge \Box^{k-1} A_k) \vee (A_1 \wedge \ldots \wedge \Box^{i-1} (\sim(A \wedge B) \wedge \sim B \wedge X) \wedge \ldots \wedge \Box^{k-1} A_k)$.

Justified by

(1) $\sim(A \wedge B) \wedge X \supset (\sim(A \wedge B) \wedge \sim A \wedge X) \vee (\sim(A \wedge B) \wedge \sim B \wedge X)$ — d4, d5, d6, PC

(2) $\Box^{i-1}(\sim(A \wedge B) \wedge X) \supset \Box^{i-1}(\sim(A \wedge B) \wedge \sim A \wedge X) \vee \Box^{i-1}(\sim(A \wedge B) \wedge \sim B \wedge X)$ — (1), d7, d8, PC

(3) (Before) $\supset$ (After) — (2), d6, d5, PC

[4] In the spirit of Hanson, we mean by "before" and "after" the chf of the relevant alternative tableaux-sequence, before and after the application of the rule that obtains the $(m + 1)$ st stage from the mth one.

Case Y1(a).

Before: $A_1 \wedge \Box^1 A_2 \wedge \ldots \wedge \Box^{k-1}(\Box B \wedge X)$; $1 \leq k$,
$(\Box B \wedge X) = A_k$.

After: $A_1 \wedge \Box^1 A_2 \wedge \ldots \wedge \Box^{k-1}(\Box B \wedge X) \wedge \Box^k B$; $B = A_{k+1}$.

Justified as follows:

(1) $\Box B \wedge X \supset \Box B$ PC

(2) $\Box^{k-1}(\Box B \wedge X) \supset \Box^{k-1}\Box B$ (1), d7

(3) (Before) $\supset$ (After) (2), d6, PC

Case Yr(a).

Before: $A_1 \wedge \ldots . \wedge \Box^{k-1}(\sim\Box B \wedge X)$; $1 \leq k$,
$(\sim\Box B \wedge X) = A_k$.

After: $A_1 \wedge \ldots \wedge \Box^{k-1}(\sim\Box B \wedge X) \wedge \Box^k \sim B$; $\sim B = A_{k+1}$.

Justified by

(1) $\sim\Box B \wedge X \supset \Box\sim B$ a2, d7, PC

(2) $\Box^{k-1}(\sim\Box B \wedge X) \supset \Box^{k-1}\Box\sim B$ (1), d7

(3) (Before) $\supset$ (After) (2), d6, PC

Case Y1(b).

Before: $A_1 \wedge \ldots \wedge \Box^{i-1}(\Box B \wedge X) \wedge \Box^i A_{i+1} \wedge \ldots \wedge \Box^{k-1}A_k$;
$1 \leq i < k$.

After: $A_1 \wedge \ldots \wedge \Box^{i-1}(\Box B \wedge X) \wedge \Box^i(B \wedge A_{i+1}) \wedge$
$\ldots \wedge \Box^{k-1}A_k$.

Justified by

(1) $\Box B \wedge X \wedge \Box A_{i+1} \supset \Box(B \wedge A_{i+1})$ d9, PC

(2) $\Box^{i-1}(\Box B \wedge X \wedge \Box A_{i+1}) \supset \Box_{i-1}\Box(B \wedge A^{i+1})$ (1), d7

(3) $\Box^{i-1}(\Box B \wedge X) \wedge \Box^{i-1}\Box A_{i+1} \supset$
$\Box^{i-1}(\Box B \wedge X \wedge \Box A_{i+1})$ d9

(4) $\Box^{i-1}(\Box B \wedge X) \wedge \Box^i A_{i+1} \supset \Box^i(B \wedge A_{i+1})$ (2), (3), PC

(5) (Before) $\supset$ (After) (4), d6, PC

Case Yr(b).

Before: $A_1 \wedge \ldots \wedge \Box^{i-1}(\sim\Box B \wedge X) \wedge \Box^i A_{i+1} \wedge$
$\ldots \wedge \Box^{k-1}A_k$; $1 \leq i < k$.

After: $A_1 \wedge \ldots \wedge \Box^{i-1}(\sim\Box B \wedge X) \wedge \Box^i(\sim B \wedge A_{i+1}) \wedge$
$\ldots \wedge \Box^{k-1}A_k$.

Justified as follows:

(1) $\sim\Box B \wedge X \wedge \Box A_{i+1} \supset \Box(\sim B \wedge A_{i+1})$ a2, d7, PC, d9

Then the argument proceeds just as in Case Y1(b). This completes the proof of the desired lemma.

The completeness result for DT is then obtained from the lemma by an obvious adaptation of Kripke's corresponding proof: since, by hypothesis, A is valid, the construction for A is closed, i.e. there is a stage, say the m-th, when every alternative tableaux-sequence is closed, and with the chf $D_1 \vee \ldots \vee D_n$; concentrate on any D_1. By hypothesis, $D_j = A_1 \wedge \ldots \wedge \Box^{i-1}(C \wedge \sim C \wedge X) \wedge \ldots \wedge \Box^{k-i}A_k$, for some i $(1 \leq i \leq k)$ and for some wff C. Of course $\vdash \sim(C \wedge \sim C \wedge X)$ by PC, hence $\vdash \sim \Box^{i-1}(C \wedge \sim C \wedge X)$ by Nec, a1, PC, d7, so $\vdash \sim D_j$ by PC. Since j was arbitrary, $\vdash \sim(D_1 \vee \ldots \vee D_n)$. By the lemma, $\vdash \sim A \supset D_1 \vee \ldots \vee D_n$ ($\sim A$ being the chf of the initial stage of the construction, $D_1 \vee \ldots \vee D_n$ that of the m-th one). Since $\vdash \sim(D_1 \vee \ldots \vee D_n)$, $\vdash$ A. Q.E.D.

9. *Combining DT with deontic logic: the calculus DDT*

We close this paper by a discussion of some possible ways of textending DT into a deontic logic proper. To that purpose we add ao the vocabulary of DT a unary obligation-connective **O** (read as 'it ought to be the case that' or 'it is obligatory that'), which we suppose to satisfy the principles of Hanson's system D, i.e.

PC,

O1. $\mathbf{O}A \supset \sim\mathbf{O}\sim A$,

O3. $\mathbf{O}(A \supset B) \supset (\mathbf{O}A \supset \mathbf{O}B)$,

O-Nec: If $\vdash$ A, then $\vdash$ **O**A.

We shall not consider any combinations of DT with any of Hanson's stronger systems (DM, DS4, etc.). The deontic-tense-logical calculus that results from addition of **O**1, **O**3, and **O**-Nec to DT will be called "DDT". Our present problem is: How are we to characterize DDT model-theoretically?

Well, as was intimated in section 5 above, Hanson's modelling for D is based on the notion of a D model structure, i.e. an ordered triple (G, K, R) with K a non-empty set, G $\in$ K, and R a relation defined on K satisfying the condition: If H $\in$ K, then there is an H′ $\in$ K such that HRH′. Intuitively, 'HRH″ might be read as

'H′ is permissible with respect to H'; moreover, a formula **O**A is evaluated as true in a world H just in case A is true in every world permissible with respect to H, i.e. in every H′ such that HRH′. Given these rough preliminaries, we suggest a modelling for DDT as follows.

A DDT *model structure* is an ordered triple ($\mathscr{G}$, $\mathscr{K}$, $\mathscr{R}$), where $\mathscr{K}$ is a non-empty set *of model sequences*, $\mathscr{G} \in \mathscr{K}$, and $\mathscr{R}$ is a binary relation whose domain consists of *terms of* members of $\mathscr{K}$, and whose counterdomain consists of members of $\mathscr{K}$. Furthermore, the following condition is placed on $\mathscr{R}$ by a DDT model structure: For each $\mathscr{S} \in \mathscr{K}$ and for each term H_i of $\mathscr{S}(i = 1, 2, \ldots)$ there is an $\mathscr{S}^i \in \mathscr{K}$ such that $H_i \mathscr{R} \mathscr{S}^i$.

Primary valuations (for DDT) are now understood as assigning a truth-value to each atomic wff, in each term of every member of $\mathscr{K}$ in a DDT model structure. The secondary valuation for DDT results from the clauses (i)-(iv) of section 5 by addition of

(v) $w(\mathbf{O}A, \varphi, H_i) = 1$, iff $w(A, \varphi, \mathscr{S}^i) = 1$ for each $\mathscr{S}^i$ such that $H_i \mathscr{R} \mathscr{S}^i$.

Here, H_i is the *i*-th term of an arbitrary $\mathscr{S} \in \mathscr{K}$; moreover, we stipulate that $w(A, \varphi, \mathscr{S}) = 1$ iff $w(A, \varphi, 1 : \mathscr{S}) = 1$, where '$1 : \mathscr{S}$' simply denotes the first term of $\mathscr{S}$. A DDT-*system* can then be defined as a pair (φ, ($\mathscr{G}$, $\mathscr{K}$, $\mathscr{R}$)), a wff A being *true in* (φ, ($\mathscr{G}$, $\mathscr{K}$, $\mathscr{R}$)) just in case $w(A, \varphi, \mathscr{G}) = w(A, \varphi, 1 : \mathscr{G}) = 1$. The definitions of DDT-validity, DDT-model, and DDT-countermodel, with respect to a wff A, are then obvious enough.

We are fairly convinced that DDT is sound and complete with respect to the modelling just given. However, all details have not been checked yet, and we refrain from going into them here. Instead, we wish to consider some philosophical applications of DDT that appear to be of a certain interest.

10. *Applications of DDT*

A striking and important feature of the combined deontic-tense-logic DDT is that none of the following "commutation"-schemata are valid (derivable):

(1) $\mathbf{O}\Box A \supset \Box{\sim}\mathbf{O}{\sim}A$

(2) $\mathbf{O}\Box A \supset \Box\mathbf{O}A$

(3) $\Box\mathbf{O}A \supset \mathbf{O}\Box A$

(4) $\mathbf{O}\Box A \equiv \Box\mathbf{O}A$

Of these, (4) is obviously stronger than (2) as well as (3), both of which are in turn stronger than (1): moreover, (2) and (3) are independent (given DDT, of course). To see that none of (1)-(4) are DDT-valid, it is thus sufficient to establish the non-validity of (1). To the latter purpose we define a DDT countermodel to (1), i.e. a DDT model of $\mathbf{O}\Box A \wedge \Box\mathbf{O}{\sim}A$, as follows. (Without loss of generality we can assume A to be atomic.) Well, let

$\mathscr{G} = \langle G_1, G_2 \rangle$ $\qquad \mathscr{S}^{11} = \mathscr{S}^1$

$\mathscr{S}^1 = \langle H_1, H_2 \rangle$ $\qquad \mathscr{S}^{12} = \mathscr{S}^1$

$\mathscr{S}^2 = \langle J_1 \rangle$ $\qquad \mathscr{S}^{21} = \mathscr{S}^2$

$\mathscr{K} = \{\mathscr{G}, \mathscr{S}^1, \mathscr{S}^2\}$

$\mathscr{R} = \{(G_1, \mathscr{S}^1), (G_2, \mathscr{S}^2), (H_1, \mathscr{S}^{11}), (H_2, \mathscr{S}^{12}), (J_1, \mathscr{S}^{21})\}$.

Furthermore, let $w(A, \varphi, H_2) = \varphi(A, H_2) = 1$ and $w(A, \varphi, J_1) = \varphi(A, J_1) = 0$. (The value of $w(A, \varphi, H)$ for $H = G_1, G_2, H_1$ can be defined arbitrarily; similarly for the value of $w(B, \varphi, H)$, where B is any atomic wff other than A and $H = G_1, G_2, H_1, H_2, J_1$.).

On this definition, then, it is obvious that $(\mathscr{G}, \mathscr{K}, \mathscr{R})$ is a DDT model structure and that the system $(\varphi, (\mathscr{G}, \mathscr{K}, \mathscr{R}))$ is a DDT model of $\mathbf{O}\Box A \wedge \Box\mathbf{O}{\sim}A$, so that (1) is not valid. It is also clear that one cannot make for the validity of (1) unless one is prepared to adopt some condition or other to the effect that, in the present example, J_1 (= the first (and only) term of $\mathscr{S}^2$) is identical wih H_2 (= the second term of $\mathscr{S}^1$). However, no such condition seems admissible — for our deontic-tense-logical purposes, at least — and I shall try to illustrate this point by discussing the present example more informally. Broadly speaking, acceptance of any of (1)-(4) as valid would deprive us of certain highly interesting and useful distinctions in deontic logic, so their failure of validity is only to be welcomed, after all.

An intuitive paraphrase of our example may run as follows. At a given stage of a particular round of chess, White argues thus: "if I don't take Black's subsequent acting into account, then it now certainly ought to be so that I have my queen on f7 at my next move, because then I am bound to win; however, since this is obvious to Black, he'll play in a way that makes it impossible for me to have my queen on f7 at my next move: so it will certainly be true at my next move that I ought not then to have my queen on f7, because, if I have, I am bound to loose." Let A be the statement that White's queen is on f7, and let G_1 be the situation in which White's argument takes place. In the first part of the latter he considers what would be the best continuation of the play from his point of view, *if* he were allowed completely to disregard the *actual* continuation of the play — in other words, he considers the sequence $\langle H_1, H_2 \rangle$ which he takes to be "ideal" relative to G_1, and just to G_1, and reaches the conclusion that $\mathbf{O}\Box A$ is true in G_1. In the second, more realistic, part of his argument, White takes the actual continuation of the play into account, i.e. Black's countermove, as well as the ensuing situation with respect to himself ($= G_2$); he then finds out that A is false in $\langle J_1 \rangle$ which is ideal relative to G_2, and concludes that $\Box\mathbf{O}\sim A$ must be true in G_1. In short, then the difference of $\mathbf{O}\Box$ from $\Box\mathbf{O}$, given a common "starting-point" G_1, amounts to this: $\mathbf{O}\Box$ speaks of what takes place at the *next* stage in a course of events supposed to be *ideal* relative to G_1 (i.e. at H_2 in our example); whereas $\Box\mathbf{O}$ speaks of what takes place at the *first* stage in a course of events which is ideal relative to the *next* stage G_2 in the *actual* course of events (i.e. at J_1 in our example).

Our present distinction between $\mathbf{O}\Box$ and $\Box\mathbf{O}$, moreover, appears to be helpful in connection with what might be called the "Puzzle of Contrary-to-Duty Imperatives" — a tricky situation in deontic logic with which, among others, Chisholm [5], von Wright [12], and I myself [2], [3] have been dealing. Consider four statements

(i) It ought to be that B — formally: $\mathbf{O}B$,

(ii) It ought to be that if B then A — formally: $\mathbf{O}(B \supset A)$,

(iii) If not B, then it ought to be that not A — formally: $\sim B \supset \mathbf{O}\sim A$,

(iv) Not B — formally: $\sim B$.

(iii) is here a contrary-to-duty imperative which tells us what ought to be the case if the duty expressed by (i) be neglected. The trouble with (i)-(iv) is that they jointly entail a contradiction, e.g. in the system D we have $\vdash$ $(\mathbf{O}B \wedge \mathbf{O}(B \supset A) \wedge (\sim B \supset \mathbf{O}\sim A) \wedge \sim B) \supset \mathbf{O}A \wedge \mathbf{O}\sim A$ as well as $\vdash \sim(\mathbf{O}A \wedge \mathbf{O}\sim A)$; on the other hand, the set (i)-(iv) appears to be perfectly consistent intuitively. Now, one aspect of the Puzzle of Contrary-to-Duty Imperatives concerns the problem of adequately formalizing the hypothetical norms (ii) and (iii); apparently, the difficulties encountered here are bound up with the peculiarities of material implication. However, it seems to me that the present puzzle can be fruitfully discussed quite apart from that aspect, viz. by bringing in the distinction of $\mathbf{O}\Box$ from $\Box\mathbf{O}$.

Let us restate the puzzle as follows. Consider

(i) It ought to be that B,
(v) It ought to be that A, *since* it ought to be that B,
(iv) Not B,
(vi) It ought to be that not A, *since* not B.

Evidently, the joint force of (i)-together-with-(ii) in the previous formulation of the puzzle is just that of (i)-together-with-(v) in the present one. Similarly, the joint force of (iv)-together-with-(iii) is just that of (iv)-together-with-(vi). Now, (v) patently contradicts (vi)–however, even the restated argument does seem to make good sense. How? Well, as *one* possible (and good, I think) explanation, I want to suggest that the argument be formalized in the following way, in disregard of the 'since'-clauses:

(i′) $\mathbf{O}B$,
(v′) $\mathbf{O}\Box A$,
(iv′) $\sim B$,
(vi′) $\Box\mathbf{O}\sim A$.

There is no difficulty in defining a DDT model of the conjunction of these four wffs: we just add to our previous stipulations concerning $(\varphi, (\mathscr{G}, \mathscr{K}, \mathscr{R}))$ that $w(B, \varphi, G_1) = \varphi(B, G_1) = 0$ — supposing B to be atomic — and that $w(B, \varphi, H_1) = \varphi(B, H_1) = 1$. To make another chess-paraphrase involving a supposed argument of White's, let B mean that White's bishop is on e6. Then the ideal course of events from White's present point of view would be, first to have his bishop on e6 (B true in H_1) and next to have his queen on f7 (A true in H_2), i.e. $\mathbf{O}B$ and $\mathbf{O}\Box A$ are both true in G_1. However, in view of the fact that White's bishop is *not* on e6 in the initial situation (and in view of Black's actual countermove, if you like to take one into account), White again reaches the conclusion that $\Box\mathbf{O}\sim A$ is true in G_1, $\mathbf{O}\sim A$ in G_2, and $\sim A$ in J_1.

Examples illustrating the present kind of reasoning may of course be multiplied. To give just one further illustration, let B mean that Jones refrains from stealing and let A mean that Jones is not punished for theft.

We now turn to a different matter. von Wright [11] and Lemmon [10] argue that a proper study of deontic statements and of imperatives, respectively, should be based on a tense-logic or 'change-logic' like the T-calculus. Disregarding the facts that in [11] von Wright also works with a logic of action and that in [10] Lemmon applies change-logic to imperatives rather than to deontic statements, they may both be taken to be concerned with the formalization of such locutions as

It ought to be that A changes to B,
and It is wrong (forbidden) that A changes to B,

where, perhaps, A and B are to be restricted to s.c. state-descriptions. Essentially, their suggestions could be paraphrased as follows within the framework of DDT. We are to introduce into the latter suitable formal counterparts of the above locutions, by defining a *binary* obligation-operator $\mathbf{O}_x(\ ,\ )$ and a *binary* forbiddance-operator $F_x(\ ,\ )$. It is, moreover, to be understood that the definition of these operators be framed in terms of $\mathbf{O}$ and $\Box$ (or T, if you prefer) in such a way that every occurrence of the tense-logical connective falls within the scope of some $\mathbf{O}$-occurrence — thus, e.g., we dismiss forms like $A \wedge \Box\mathbf{O}B$ and $A \wedge \Box\mathbf{O}\sim B$ as obviously failing to render the intended intuitive meaning of the above locutions. Let us then consider some $\mathbf{O}_x$-(F_x-) candidates:

$\mathbf{O}_1(A, B) =_{df} \mathbf{O}(A \wedge \Box B)$, i.e. $\mathbf{O}(A\,T\,B)$,
$F_1(A, B) =_{df} \mathbf{O}_1(\sim A, \sim B)$, i.e. $\mathbf{O}(\sim A \wedge \Box\sim B)$, i.e. $\mathbf{O}(\sim A\,T \sim B)$,

$\mathbf{O}_2(A, B) =_{df} A \wedge \mathbf{O}\Box B$, i.e. $A \wedge \mathbf{O}(A^+TB)$,
$\mathbf{F}_2(A, B) =_{df} \mathbf{O}_2(A, \sim B)$, i.e. $A \wedge \mathbf{O}\Box \sim B$, i.e.
$A \wedge \mathbf{O}(A^+T \sim B)$,
$\mathbf{F}_3(A, B) =_{df} \mathbf{O} \sim (A \wedge \Box B)$, i.e. $\mathbf{O} \sim (ATB)$, i.e.
$\mathbf{O}((AT \sim B) \vee (\sim ATB) \vee (\sim AT \sim B))$.

$\mathbf{F}_3$ could obviously be viewed as the negation of the following binary permission-operator:

$\mathbf{P}_1(A, B) =_{df} \sim\mathbf{O} \sim (A \wedge \Box B)$, i.e. $\sim \mathbf{O} \sim (ATB)$.

Another such operator would be

$\mathbf{P}_2(A, B) =_{df} A \wedge \sim\mathbf{O} \sim \Box B$, i.e. $A \wedge \sim \mathbf{O} \sim (A^+TB)$,

as the negation of which we could define the forbiddance-operator:

$\mathbf{F}_4(A, B) =_{df} \sim (A \wedge \sim\mathbf{O} \sim \Box B)$, equivalent to $(A \wedge \mathbf{O}\Box \sim B)$
$\vee (\sim A \wedge \sim\mathbf{O} \sim \Box B) \vee (\sim A \wedge \mathbf{O}\Box \sim B)$.

Comments We note that $\mathbf{O}_1$, $\mathbf{F}_1$, $\mathbf{P}_1$ fail to be 'normatively neutral with respect to the initial state of affairs" because

(1) $\mathbf{O}_1(A, B) \supset \mathbf{O}A$,
(2) $\mathbf{F}_1(A, B) \supset \mathbf{O} \sim A$,
(3) $\mathbf{P}_1(A, B) \supset \sim\mathbf{O} \sim A$

are easily seen to be DDT-valid. If $\mathbf{O}_1$, $\mathbf{F}_1$, $\mathbf{P}_1$ are replaced here by $\mathbf{O}_2$, $\mathbf{F}_2$, $\mathbf{P}_2$, respectively, the resulting schemata are no longer valid. In addition to being thus normatively neutral with respect to the initial state of affairs, the latter trio of operators obviously assert the (present) existence of that state of affairs. As for the operators $\mathbf{F}_3$ and $\mathbf{F}_4$, we observe that none of

(4) $\mathbf{F}_3(A, B) \supset \mathbf{O} \sim A$,
(5) $\mathbf{F}_3(A, B) \supset A$,
(6) $\mathbf{F}_4(A, B) \supset \mathbf{O} \sim A$,
(7) $\mathbf{F}_4(A, B) \supset A$

are valid, and that

(8) $\mathbf{F}_1(A, B) \supset \mathbf{F}_3(A, B)$,
(9) $\mathbf{F}_2(A, B) \supset \mathbf{F}_3(A, B)$,
(10) $\mathbf{F}_2(A, B) \supset \mathbf{F}_4(A, B)$

are valid, whereas the converse results all fail. Finally, we may note the validity of

(11) $(A \wedge \mathbf{F}_4(A, B)) \equiv \mathbf{F}_2(A, B)$; on the other hand,
(12) $(A \wedge \mathbf{F}_3(A, B)) \equiv F^2(A,B)$ fails to be valid.

From an intuitive viewpoint, the above considerations should certainly lead us to prefer $\mathbf{O}_2$, $\mathbf{F}_2$, $\mathbf{P}_2$ to $\mathbf{O}_1$, $\mathbf{F}_1$, $\mathbf{P}_1$ in connection with the formalization of the locutions at stake. We are convinced that von Wright and Lemmon would agree with this, in spite of the fact that their actual notation might suggest the contrary. Again, Lemmon ([10] p. 59) in effect suggests the distinction of $\mathbf{F}_4$ from $\mathbf{F}_2$ as well as the validity of (11); so, by virtue of the non-validity of (12), we conjecture that he would favour $\mathbf{F}_4$ rather than $\mathbf{F}_3$, in spite of his actual notation which precisely suggests $\mathbf{F}_3$.

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