THE PARADOXES OF STRICT IMPLICATION

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Philosophers and logicians trying to give accounts of various logical concepts such as analyticity, implication, consistency, etc., have been continually plagued by the recurrence of the so-called "paradoxes of strict implication":

P1 An analytic statement is implied by every statement.

P2 The denial of an analytic statement implies every statement.

P3 Every analytic statement is equivalent to every other analytic statement.

These are called the "paradoxes of strict implication" because they first came to prominence in modern philosophy as a consequence of C.I. Lewis's "calculus of strict implication" (1). Frequently, when an account of implication and analyticity which seems otherwise unassailable is found to entail the paradoxes of strict implication, this is taken as sufficient grounds for rejecting that account. But some philosophers and logicians (e.g., C.I. Lewis) have stubbornly insisted that there is nothing wrong with having the paradoxes of strict implication as consequences of an account of these logical concepts. They maintain that the "paradoxes" represent surprising but nevertheless true principles of logic.

This paper will examine what I think are the three most plausible of the reasons that have been given for thinking that the paradoxes of strict implication should not be consequences of a correct account of implication and analyticity. I shall argue that none of these reasons is sufficient to support that conclusion. My conclusion will be that the paradoxes are in fact true principles of logic, surprising though they may be.

(1) C. I. LEWIS and C. H. LANGFORD, *Symbolic Logic*, New York and London, 1932.

1. *Analyticity and Implication*

If we are going to investigate whether the paradoxes of strict implication are true of analyticity and implication, we must first be clear about just what we mean by "analyticity" and "implication". These concepts have been defined in a vast number of ways, and the paradoxes may well be false of them on some of these definitions, and true on others.

I shall adopt what seems to be the classical definition of implication: One statement, P, implies another statement, Q (henceforth, $P \rightarrow Q$), if and only if Q can be inferred from, or derived from, P, by means of some valid proof. And all this requires is that there be an *a priori* way of getting to know that Q if you are given that P. This is the way G. E. Moore (2) originally defined "entailment", and this is the way C. I. Lewis (3) defined "implication".

Let us also define "analytic equivalence" by saying that P is analytically equivalent to Q (henceforth, $P \leftrightarrow Q$) if and only if both $P \rightarrow Q$ and $Q \rightarrow P$.

By "analyticity" I mean simply *a priori* provability. In other words, P is analytic (henceforth, AP) if and only if P can be proven by means of some valid proof. This definition is parallel to the definition of implication, and given a few simple principles it can be shown to be equivalent to a number of other standard definitions of analyticity (4). In particular, arguments can be given to show that AP if and only if there is a Q such that $\sim P \rightarrow (Q \,\&\, \sim Q)$, i.e., if and only if the denial of P implies an explicit contradiction.

The paradoxes of strict implication can now be stated concisely as follows:

P1 If AP, then $(Q)(Q \rightarrow P)$.
P2 If AP, then $(Q)(\sim P \rightarrow Q)$.
P3 If AP and AQ, then $P \leftrightarrow Q$.

(2) G. E. Moore, "External and Internal Relations", *Proceedings of the Aristotelian Society*, 1919-1920, reprinted in *Philosophical Studies*, London and New York, 1922, pp. 276-309.

(3) *Symbolic Logic*.

(4) I have in mind here a theory of implication and analyticity along the general lines of my "Implication and Analyticity", *Journal of Philosophy*, vol. LXII (1965), pp. 150-157.

There is a fourth principle which is closely related to P1-P3 and which is generally considered to be just as paradoxical as they are:

P4 $(P \,\&\, {\sim}P) \rightarrow Q$.

Those who object to P1-P3 object with equal vehemence to this principle. And with good reason, because the general paradox P2 is easily obtained from P4 as follows:

(i) If AP then there is an R such that ${\sim}P \rightarrow (R \,\&\, {\sim}R)$;

(ii) $(R \,\&\, {\sim}R) \rightarrow Q$, by P4;

(iii) Therefore, if AP then ${\sim}P \rightarrow Q$.

This argument makes use of the paradox P4, the principle that a statement is analytic just in case its denial implies an explicit contradiction, and the transitivity of implication. As most authors accept the latter two principles, we can largely restrict our attention to P4 rather than discussing the general paradoxes directly.

In order to make it clear just what one is up against in denying the truth of the paradoxes, let me give two very simple and quite intuitive proofs of P4:

Argument 1

(1)	$(P \,\&\, {\sim}P)$	premise
(2)	P	from (1)
(3)	$(P \vee Q)$	from (2)
(4)	${\sim}P$	from (1)
(5)	$[(P \vee Q) \,\&\, {\sim}P]$	from (3) and (4)
(6)	Q	from (5)

Therefore, $(P \,\&\, {\sim}P) \rightarrow Q$.

Argument 2

(1) If $(P \,\&\, {\sim}Q) \rightarrow R$, then $(P \,\&\, {\sim}R) \rightarrow Q$.

(2) $(P \,\&\, {\sim}Q) \rightarrow P$.

(3) Therefore, $(P \,\&\, {\sim}P) \rightarrow Q$.

The first is a classical argument, known to the medieval logicians, and rediscovered by C. I. Lewis. The second makes use of what is essentially the principle of the antilogism. These are intuitively valid proofs.

It is good to note what the consequences would be of rejecting the paradoxes. If we are to say that the paradoxes are false, then we must somehow argue that the above two arguments which purport to prove P4 are invalid, and in order to do that we must reject at least some of the intuitively valid inferences made in those proofs. But all of those inferences seem just too obvious to be denied unless some very good reason can be given for their denial. Thus there is a very strong *prima facie* case for thinking that the paradoxes of strict implication are logical truths. With this in mind, let us now examine the grounds on which various philosophers have rejected the paradoxes.

2. *Intuitive Rejections of the Paradoxes*

The question is now whether the paradoxes of strict implication are really paradoxical, or whether they express actual, although possibly surprising, facts about valid inference. Some philosophers seem to have a direct intuitive reaction to the paradoxes. A. E. Duncan Jones (5), without any argument, says that they are simply "outrageous". And E. J. Nelson (6) takes the position, again on simply intuitive grounds, that "these paradoxes seem so utterly devoid of rationality that I consider them a *reductio ad absurdum* of any view which involves them".

Unfortunately, I find that I have no intuitive reaction to the paradoxes one way or the other, and I fail to see how anyone else can either. How can we possibly know on intuitive grounds, without argument, whether it might be the case that given any analytic statement, and any other statement, there is some possibly quite complex valid argument with the help of which we can infer the analytic statement from the other statement? I don't think it is possible to determine whether that is true on purely intuitive grounds. As we shall see, many of the *arguments* that have been given to show that the paradoxes are false confuse implication with some other relation, and I suspect that these intuitive rejections of the paradoxes stem from similar confusions. At any rate, those of us who have no intuitions about the paradoxes cannot be convinced by these purely intuitive rejections. A philosophical position cannot be

(5) A. E. Duncan-Jones, "Is Strict Implication the same as Entailment?", *Analysis*, vol. 2 (1934-35), pp. 70-78.

(6) E. J. Nelson, "Intensional Relations", *Mind, vol.* 39 (1930), pp. 440-453.

refuted by an intuition that is not generally shared. If something is to be found wrong with the steps of inference that led to the paradoxes, some stronger arguments must be given than these simply intuitive rejections.

3. *Implication and Valid Inferences*

There is one seemingly very powerful argument which has convinced many people that the paradoxes of strict implication are false. Let us consider the fact that an analytic statement is implied by every statement, and the fact that every statement is implied by the denial of an analytic statement. These are the first two paradoxes. A. F. Emch (7), N. D. Belnap (8), and G. H. von Wright (9) have all attacked these paradoxes for the same reason. For example, Belnap writes that if the paradoxes were true

> Geometry teachers could shorten their work by explaining that since the sides of an equilateral triangle are all equal, and necessarily so, hence that proposition follows from Euclid's axioms. A single axiom, 'Justice is, and is not, a virtue', would suffice for the Hegelian deduction of the world, and the literal truth (or falsity) of the *Iliad* would suffice to prove the binominal theorem. Peano need not have bothered to show that '7 + 5 = 12' follows from his postulates, for P.3 [the statement of the paradox] guarantees this antecedently (10).
>
> If Lewis and others are right in claiming that strict implication is the same thing as entailment [implication], then in virtue of the paradoxes, we would be unable to discriminate between valid and invalid arguments when the premise is necessarily false or the conclusion necessarily true. The theory would make it non-

(7) A. F. Emch, "Implication and Deducibility", *Journal of Symbolic Logic*, vol. 1 (1936), pg. 30.

(8) N. D. Belnap, Jr., *A Formal Analysis of Entailment*, Technical Report No. 7, Office of Naval Research, Group Psychology Branch, Contract SAR/Nonr — 609(16), New Haven, 1960, pg. 5.

(9) G. H. von Wright, *Logical Studies*, London, 1957, pg. 174.

(10) *ibid.*, pg. 5.

sense to say "Your conclusion is necessarily true but your argument is very bad", or "Your premise is self-contradictory, and furthermore, your argument is invalid". This is a serious charge ([11]).

I think that this type of objection to the paradoxes turns out to rest on a confusion between implication and valid inferences. Let us consider the following example of an inference:

$$\frac{2 + 2 = 4}{\text{Therefore, the sum of the angles of a triangle is 180 degrees.}}$$

Emch, Belnap, and von Wright seem to think that if we maintain the truth of P1-P4 we cannot say that this argument is invalid (which it clearly is), because the conclusion is analytically true, and so implied by the premise. Their argument implicitly assumes that if $P \rightarrow Q$, then if in the course of an argument we infer P, then on the next line we can infer Q. But that simply is not true. Suppose that $P \rightarrow Q$, but that we haven't yet established that fact. Then we can't *use* that fact in an argument and so infer Q from P. The argument only becomes valid once we have filled it out and supplied the proof that $P \rightarrow Q$. And then we would have an argument that anyone, including Emch, Belnap, and von Wright, would have to accept as valid.

The concept of a valid inference is an essentially historical concept. In a valid inference, we proceed from what we already know, to things we did not know before. We can only make use of the fact that one statement implies another if we have already established that fact. In giving a proof we make use of things we have already proven. The fact that we have already proven something may validate an inference that would otherwise be invalid. To say that P implies Q means merely that *there is* some valid argument by which we can infer Q from P — not that the inference from P to Q (in a single step) *is* a valid argument.

I think it is fairly evident, once the distinction has been made clear, that the type of objection to the paradoxes given by Emch, Belnap, and von Wright, turns out to rest on a confusion between

([11]) *ibid.*, pg. 5.

implication and valid inference. It is not the *fact* of an implication which validates an inference, but rather our previously having established that fact. We must not confuse implication with valid inference.

4. *Meaning and Implication*

We must now consider what has probably been the most influential of all of the objections to the paradoxes. It is frequently asserted that implication requires a "necessary connection between meanings" ([12]) and that such a connection is lacking in the paradoxes. Before considering this objection, let us recall how implication was defined. Implication, as that term is being used here, is the converse of the relation of deducibility. This is the way it was defined, and to say that it requires a connection of meanings, whether that is true or not, is to assert something new which is not part of the definition. This should be borne in mind, because a number of philosophers seem to have just turned things around, in effect defining implication in terms of a connection between meanings, and then objecting to the paradoxes on those grounds. This becomes particularly evident when we find some philosophers (e.g., Geach) going so far as to contrast implication with deducibility.

Whether it is justified or not, there is indeed a strong temptation to say that implication requires a connection between meanings. For example, Blanshard ([13]) writes, "what lies at the root of the common man's objection [to the paradoxes] is the stubborn feeling that implication has something to do with the *meaning* of propositions, and that any mode of connecting them which disregards this meaning and ties them together in despite of it is too artificial to satisfy the demand of thought." Similarly, E. J. Nelson ([14]) writes that implication "is a necessary connection between meanings". A. E. Duncan-Jones ([15]) proposed a theory of implica-

([12]) E. J. Nelson, "Intensional Relations".

([13]) Brand Blanshard, *The Nature of Thought*, London, 1939, vol. II, pg. 390.

([14]) "Intensional Relations".

([15]) "Is Strict Implication the same as Entailment?", pg. 71.

tion according to which *P* implies *Q* if and only if "*Q* arises out of the meaning of *P*". In the same vein, N. D. Belnap [16] discusses the "principle of relevance", which is that *A* cannot imply *B* "if *A* and *B* have nothing to do with one another, if, that is, *A* and *B* are totally disparate in meaning."

It cannot be denied that there is a strong temptation to identify implication with a relation between meanings. However, we must be more explicit about just what this relation is. Let us begin with the case of analytic equivalence. It is probably the predominant view that the statement that *p* (e.g., the statement that $2 + 2 = 4$) and the statement that *q* are analytically equivalent just in case to say that *p* means the same thing as to say that *q*. Similarly, we are apt to say that the statement that *p* implies the statement that *q* just in case part of what it means to say that *p* is that *q*. For example, the statement that John is a bachelor implies the statement that John is unmarried, because part of what it means to say that John is a bachelor is that John is unmarried. This is the predominant view, but I do not think it is the correct view.

Let us look at the paradoxes in relation to the above view of implication. According to it, any two analytic statements must have the same meaning. For example, it must mean the same thing to say that $2 + 2 = 4$ as to say that all bachelors are unmarried. But that is absurd. These statements do not mean the same thing. Thus if we took the above view of implication, the paradoxes would indeed be false. This would require that we reject the various proofs we have given for the paradoxes. Let us examine then those rules that philosophers have been led to reject by assimilating implication to this relation between meanings.

The first person to object to the paradoxes on the grounds that implication requires a relation between meanings seems to have been E. J. Nelson [17]. To formulate his alternative theory of implication, he employed certain intensional connectives — V, an intensional disjunction; and an intensional conjunction symbolized by juxtaposition. He then rejected such logical laws as that $p \rightarrow p\mathrm{V}q$, and that $pq \rightarrow p$. Whether he is right in this is difficult to tell, be-

(16) BELNAP, *A Formal Analysis of Entailment*, pg. 9.
(17) "Intensional Relations".

cause of the unclarity of his intensional connectives. If he is right this would at first appear to block the standard derivations of the paradoxes. But as has frequently been noted, "it cannot be denied that there *are* relations corresponding to the extensional conjunction and disjunction ..., and these are all that are required by the independent proofs of the paradoxes. That is, Nelson can make his point only by denying not just $pEpVq$ [$p \rightarrow pVq$], but also $pEpvq$ [$p \rightarrow pvq$]." [18] But when we remember that implication is just the converse of deducibility, this is absurd. If we know that P is true, then we can concude (simply by the truth-table for "v") that (PvQ) is also true. This is all that is necessary for us to be able to say that we can infer (PvQ) from P, and hence that $P \rightarrow (P\text{v}Q)$. The implication that Nelson is led to deny just cannot be denied. This manner of blocking the paradoxes cannot be successful.

Duncan-Jones [19] is not quite so radical in the selection of which rules to deny, but of course he must deny some of them if he is to reject the paradoxes. He denies, for example, that $P \rightarrow Q$ if and only if $\sim Q \rightarrow \sim P$. But once again, when we think of implication in terms of deducibility, this rejection cannot be maintained. If we have a proof of Q on the premise P, then we know that if P is true then Q is true. If we then take as a premise that Q is false, we know that P must be false. This is inferring $\sim P$ from $\sim Q$. Thus $\sim Q \rightarrow \sim P$. And conversely. Thus Duncan-Jones is led to deny principles of inference which cannot plausibly be denied.

Belnap's principle of relevance leads him to reject such seemingly obvious rules of inference as the rule of detachment for material implication, and the principle of the disjunctive syllogism, i.e., that $[(P \supset Q) \& P] \rightarrow Q$. and that $[(P \text{ v } Q) \& \sim Q] \rightarrow P$, on the grounds that these principles commit a Fallacy of Relevance. However, if we look at Belnap's proof that the rule of detachment commits a Fallacy of Relevance, it amounts to nothing more than showing that if we accept it together with all of our other rules of inference, we can prove the paradoxes of strict implication. This merely shows that if something is wrong with the paradoxes, then something is wrong with one of our rules, but not necessarily the rule of de-

(18) J. F. BENNETT, "Meaning and Implication", *Mind*, vol. 63 (1954), pp. 451-463.

(19) "Is Strict Implication the same as Entailment?".

tachment. Thus he has not succeeded in pinpointing an error in our proofs of the paradoxes.

On the grounds that implication requires a relation between meanings, a number of different positions have been taken as to what rules of inference must be rejected to avoid the paradoxes, but none of them seem at all plausible. We just cannot cogently reject any of them. This strongly suggests that implication and this meaning relation are in fact two quite different relations. I think if we now turn to a further proposal that has been made as to what is wrong with the paradoxes, we can give a conclusive argument to show that these two relations are distinct.

A number of different philosophers have independently suggested that implication is not unrestrictedly transitive ([20]). In examining their arguments it is good to keep in mind the question of whether what they are giving is a proof that implication is not transitive, or a proof that the relation, "part of what it means to say that ... is that —" is not transitive. Lewy presents us with two paradoxes, the second of which is the following:

> Consider the following three propositions :(D) "Caesar is dead if and only if Russell is a brother"; (E) "Russell is a brother if and only if Russell is a male sibling"; and (F) "Caesar is dead if and only if Russell is a male sibling." ... now, it seems to me that the following propositions are all true: (1) the conjunction of (D) and (E) entails (F); (2) (E) is necessary; (3) (D) is contingent; and (4) (F) is contingent. And from these four propositions it seems to follow that we can truly say that (D) entails (F). The conjunction of (D) and (F), however, entails (E); but if (D) entails (F), and the conjunction of (D) and (F) entails (E), it seems to follow that (D) entails (E). Yet it is quite clear that (D) does *not* entail (E) ([21]).

Lewy's other paradox has the same form as this one. In analysing

([20]) Casimir Lewy, "Entailment", *Aristotelian Society*, supplementary volume 32 (1958), pp. 123-142; P. T. Geach, "Entailment", *Aristotelian Society*, supplementary volume 32 (1958), pp. 157-172; T. J. Smiley, "Entailment and Deducibility", *Proceedings of the Aristotelian Society*, vol. 59 (1959), pp. 233-254.

([21]) "Entailment".

these paradoxes, both Geach and Lewy arrive at the conclusion that implication is not transitive.

Now let us ask just what these paradoxes show. If we think of implication in the sense of valid proof, I can see no reason at all to deny that the proposition that Caesar is dead if and only if Russell is a brother implies the proposition that Russell is a brother if and only if Russell is a male sibling. Geach and Lewy seem to think it is obvious that such an implication does not hold, but that is not at all obvious. Of course, it is not obvious without proof that this implication does hold, but in fact, Lewy's argument seems to provide just such a proof. When implication is understood as the existence of a valid proof, I can see no reason to say that Lewy's argument is paradoxical. If however we interprete implication as a relation between meanings, the conclusion of Lewy's argument does seem paradoxical. So interpreted, the argument goes as follows:

1. Part of what it means to say that Caesar is dead if and only if Russell is a brother, is that Caesar is dead if and only if Russell is a male sibling. [(D) $\rightarrow$ (F)]
2. Part of what it means to say that Caesar is dead if and only if Russell is a brother, is that Caesar is dead if and only if Russell is a brother. [(D) $\rightarrow$ (D)]
3. Therefore, part of what it means to say that Caesar is dead if and only if Russell is a brother, is that Caesar is dead if and only if Russell is a brother, and Caesar is dead if and only if Russell is a male sibling. [(D) $\rightarrow$ (D) & (F)]
4. Part of what is means to say that Caesar is dead if and only if Russell is a brother, and Caesar is dead if and only if Russell is a male sibling, is that Russell is a brother if and only if Russell is a male sibling. [(D) & (F) $\rightarrow$ (E)]
5. Therefore, part of what it means to say that Caesar is dead if and only if Russell is a brother, is that Russell is a brother if and only if Russell is a male sibling. [(D) $\rightarrow$ (E)]

It seems indisputable that the three premises of this argument, (1) (2), and (4), are all true. And it seems equally indisputable that the conclusion, (5), is false. Therefore, either adjunction or transitivity must fail for this meaning relation. If we admit (3) to be true, then it must be transitivity that fails. However, it isn't clear to me whether

(3) is true or not. I feel some inclination to say that it is true, but not a very strong inclination. But whichever it is that fails, adjunction or transitivity, this is enough to differentiate this meaning relation from implication, because implication, as the converse of deducibility, is certainly both adjunctive and transitive. After all, if when you are given that *P* is true you can get to know in some *a priori* way that *Q* is true, and also that *R* is true, then you can get to know in an *a priori* way that (*Q* & *R*) is true, and that is all that is involved in saying that implication is adjunctive. And similarly for transitivity.

Thus Lewy's argument shows that we have two different implicative relations to contend with — the relation we have called "implication", and the relation between meanings expressed by "part of what it means to say that ... is that —". They are not identical, the former being transitive and adjunctive (among other things) and the latter not. Furthermore, there seems to be every reason for thinking that the so-called "paradoxes of strict implication" are really truths about the former relation, but not the latter.

It is of interest to try to explain why philosophers have such a tendency to try to assimilate implication to a meaning relation. In fact, I think there is an important connection between these two relations. I have argued elsewhere ([22]) that the relation "part of what it means to say that ... is that —" gives us those "immediate" implications that can be known immediately, without proof. Then other implications which cannot be known immediately are generated by stringing these immediate implications together with the help of rules of inference such as adjunction and transitivity. Thus this meaning relation gives us those fundamental implications which are used as a basis for proving more complicated implications, but the meaning relation does not carry over to the complicated implications.

5. *The Geach-von Wright Proposal*

Finally, I want to consider an interesting proposal made by G. H. von Wright and P. T. Geach. Von Wright ([23]) suggests the following

([22]) "Implication and Analyticity".

([23]) *Logical Studies*, pg. 181.

definition of entailment [implication]: "p entails q, if and only if, by means of logic, it is possible to come to know the truth of $p \supset q$ without coming to know the falsehood of p or the truth of q," or as he restates it, "p entails q, if and only if, $p \supset q$ is demonstrable independently of demonstrating the falsehood of p or the truth of q". Similarly, Geach (24) writes, "we may state the truth-condition for 'p entails q' as above: 'there is an *a priori* way of getting to know that $p \supset q$ which is not a way of getting to know either that $\sim p$ or that q' " (Geach uses Polish notation, which I have transformed into the more customary notation). According to the Geach-von Wright proposal, we may well have implications holding between statements that are themselves demonstrably true or demonstrably false (analytically true or analytically false), but what they deny is that $P \rightarrow Q$ if it turns out that every proof of $(P \supset Q)$ contains in it, or somehow simultaneously constitutes, a proof of Q or of $\sim P$. This is a suggestion as to what is wrong with the direct proofs of the paradoxes of strict implication. For example, the proof of P2 proceeded by first showing that the denial of an analytic statement implies an explicit contradiction, and then that the contradiction implies an arbitrary statement. According to von Wright and Geach, this would not work, because that proof involves a proof that the antecedent of the implication is analytically false.

Similarly, von Wright uses his definition to argue that $(P \& \sim P)$ does not imply an arbitrary statement, Q. He argues as follows:

> $(P \& \sim P) \supset P$ and $(P \& \sim P) \supset Q$ are both demonstrable truths of logic. We may use truth-tables to show that they are tautologies. By the same method we may also show that $(P \& Q) \supset P$ is a tautology. ... Now, having in this way demonstrated that, for any propositions P and Q, the proposition $(P \& Q) \supset P$ is a truth of logic, we may use this insight to demonstrate that $(P \& \sim P) \supset P$ is a truth of logic too. ... This allows us to say that $(P \& \sim P) \supset P$ is demonstrable *independently* of demonstrating the falsehood of the antecedent or the truth of the consequent, and *a fortiori* to say that $(P \& \sim P)$ entails P. ...
> Consider how $(P \& \sim P) \supset Q$ may be proved. It can be ob-

(24) "Entailment", pg. 165.

tained neither by substitution nor by detachment from any formula which is provable in a truth-table without also disproving $(P \mathbin{\&} \sim P)$. And if a truth-table is used directly to prove the formula in question, this truth-table would also prove the falsehood of $(P \mathbin{\&} \sim P)$. (25)

Thus he concludes that $(P \mathbin{\&} \sim P)$ does not imply Q.

Although the Geach-von Wright definition of implication seems plausible, I do not think it is immediately obvious that it is a correct characterization of implication when "implication" is construed as we have defined it. But supposing for the moment that it is, let us ask whether it is sufficient to block the paradoxes. The Geach-von Wright definition does have the characteristic that it will make it very difficult to *prove* that the paradoxes hold in their full generality. For example, our proof of P2 proceeded as follows: If AP, then for some R, $\sim P \rightarrow (R \mathbin{\&} \sim R)$. But $(R \mathbin{\&} \sim R) \rightarrow Q$. Therefore, $\sim P \rightarrow Q$. This proof has the form of a *proof scheme* which can then be filled in with any particular statements we choose. But given the Geach-von Wright definition, it will no longer be possible to provide any such proof scheme. The reason is simply that if we give any general proof scheme which works for every analytic statement, then the simple fact that the scheme works for P in itself shows that P is true. Thus the proof that $\sim P \supset Q$ which results from the application of the scheme is also a way getting to know that $\sim P$ is false, and so violates the Geach-von Wright restriction. Although there may be another way of proving that the paradoxes hold, I don't know what it would be. Thus it seems that the Geach-von Wright definition blocks our *proofs* of the paradoxes, but in a rather strange manner, and it does not follow from this that it blocks the paradoxes themselves.

In fact, I think there is good reason for thinking that this proposal doesn't really block the paradoxes at all. Although for the reasons given above we can't prove the general paradox that if AP, then $\sim P \rightarrow Q$, those reasons do not apply to the specific case P4: $(P \mathbin{\&} \sim P) \rightarrow Q$. This specific case is generally thought to be just as paradoxical as the general case (at least by those who think that the

(25) *Logical Studies*, **pg. 187.**

paradoxes are paradoxical in the first place), and the Geach-von Wright definition is explicitly intended to avoid this particular case just as much as the general paradox. Consider von Wright's argument that we cannot deduce $(P \& \sim P) \supset Q$ with any argument which does not already prove that $(P \& \sim P)$ is false. A slight modification of the second argument in section two for P4 will do the job nicely. First note that for any statements, P, Q, and R, the following is a truth of logic: $[(P \& \sim Q) \supset R] \supset [(P \& \sim R) \supset Q]$. This can be verified by a truth-table. Furthermore, $(P \& \sim Q) \supset P$ is a tautology. Then, contrary to von Wright, by substitution and detachment we can conclude that $(P \& \sim P) \supset Q$, and we have proven this without proving the falsity of $(P \& \sim P)$.

Nor is it obvious that the suggestion of von Wright and Geach blocks the more general paradox that the denial of an analytic statement implies an arbitrary statement R. Suppose P is analytic. Then there is some contradiction, e.g., $(Q \& \sim Q)$, which can be derived from $\sim P$. But if we can derive $(Q \& \sim Q)$ from $\sim P$, this will be a proof that $\sim P$ is false, and so will prevent us from proving in this way that $\sim P \rightarrow R$. However, as a general rule, the conjuncts of the contradiction will be derived independently, and then conjoined. Furthermore, to arrive at a derivation of an arbitrary statement, R, there is no point in conjoining them. We can leave them unconjoined, and even in a much disguised form, and still get a derivation of R. Such a proof, without some additional steps, would not then be a way of getting to know that P, and Geach's or von Wright's suggestion would not block the proof of the paradox. Thus it is not at all clear that the suggestion of Geach and von Wright for blocking the paradoxes is successful.

Of course, it really doesn't make any difference whether the Geach-von Wrights definition avoids the paradoxes or not unless it can be shown to be correct. And this is something that Geach and von Wright have not even tried to do. They seem to have taken it as intuitively evident. But when it is recalled just how implication was defined, I don't think it is at all evident that the Geach-von Wright definition is correct.

How then might one support the Geach-von Wright proposal? There seem to be two ways. First, it might be suggested that any argument which was intended to prove that $P \rightarrow Q$ but violated their restrictions is intuitively invalid, and so if all arguments intended to prove that $P \rightarrow Q$ violate their restrictions, then it simply isn't provable that $P \rightarrow Q$, and so isn't true. But this doesn't seem to be the case. For example, look at the first argument for the principle that $(P \,\&\, \sim P) \rightarrow Q$, in section two. That argument violates the Geach-von Wright restrictions, because it deduces both P and $\sim P$ from $(P \,\&\, \sim P)$, and thus proves $\sim(P \,\&\, \sim P)$. But the argument looks quite valid. It simply is not the case that it is intuitively invalid. Thus the Geach-von Wright proposal cannot be supported in this way.

The other possibility is that the Geach-von Wright definition is simply an *ad hoc* attempt to avoid the paradoxes of strict implication (much as are the restrictions on the axiom of comprehension in set theory). It seems clear that it was largely motivated by a desire to avoid the paradoxes of strict implication. But I think that motivation has now been removed, because: (1) the Geach-von Wright definition does not avoid a number of specific implications (e.g., P4) which are supposed to be just as paradoxical as the general case; (2) it is not clear that the definition actually avoids *any* of the paradoxes; (3) the reasons for thinking there is something wrong with the paradoxes in the first place have already been discussed and have been shown to rest on confusions. This third point requires further elaboration: It is important to note that the Geach-von Wright definition, when taken as an *ad hoc* device, is not itself a reason for rejecting the paradoxes. Rather, it relies upon the existence of other reasons. Thus if those other reasons are shown to be spurious, we can reject the Geach-von Wright definition as unmotivated. And I think that this latter task has now been accomplished.

6. *Conclusions*

I have now considered and answered what I think are the three most important objections to the so-called "paradoxes of strict implication". There seems to be no reason to think that the paradoxes are not actually truths about implication. They only seem paradoxical when one confuses implication (as it has been defined here) with some other logical concept. The two arguments against the paradoxes that at first seemed the best were, on further examination, seen to turn around a confusion of the existence of an implication with the validity of an inference, and a confusion of implication with the relation "part of what it means to say that ... is that —". And the Geach-von Wright proposal was found to be unsupportable unless we already have some *other* grounds for rejecting the paradoxes.

There does not seem to be any good reason for thinking that the paradoxes are false. And the fact that they follow logically from the acceptance of rules of inference which seem intuitively valid is a good reason for thinking that they are actually true. The only justifiable conclusion one can draw from all of this is that the paradoxes of strict implication are interesting and surprising truths about implication.

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