ENTAILMENT: 'E' AND ARISTOTLE

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«Let us use the term 'entails' to express the converse of the relation *q follows from* or *is deducible from p.*» (1) The forty or so years separating us from this remark of G. E. Moore has seen a plethora of writing on *entailment* — much of it devoted simply to staking and establishing its claim to a peculiar and proper usage in the face of its competing relatives: 'implies,' 'materially implies,' and 'strictly implies.' One may at least hope by now that using 'entails' will appropriately communicate our concern for logical consequence or deducibility, the sense in which the conditional 'if p then q' would convey some connection between p and q. For the non-polemical, logical task consists in making as explicit as possible this «inner connection» indigenous to ordinary conditional statements. This is the aim of Anderson and Belnap's system E, and we might be satisfied to say that «q depends on the logical content of p if and only if $p \rightarrow q$ *is* provable in E,» were it not that E so frankly invites neighboring considerations, conscious as its authors are of formulating but a part of what men would call «reasonable argumentation.» (2) We hope to throw some auxiliary light on the meaning of arrow expressions in E by calling attention to significant similarities between this system and Aristotle's syllogistic. This approach was suggested by the obvious similarities in preoccupation, and has allowed us — in the spirit of a perennial philosophical enterprise — to highlight certain features of the syllogistic in a rather novel fashion — novel at least in clarity over against medieval commentators, and quite novel in the face of more recent commentary, notably Łukasiewicz. (3)

(1) G. E. MOORE, «External and Internal Relations,» *Proceedings of the Aris. Society*, 1919-20; reprinted in *Philosophical Studies* (London, 1922), p. 291.

(2) Alan Ross ANDERSON, «Completeness theorems for the system E of entailment and EQ of entailment with quantification,» Technical Report No. 6, Office of Naval Research Contract no. SAR/Nonr-609(16), New Haven. (Reprinted in *Zeitschrift für Mathematische Logik,* 6 (1959), 201-216). Cf. also ANDERSON and Nuel BELNAP, Jr., «Tautological entailments», *Philosophical Studies* 13 (1962), 9-24.

(3) For an incisive medieval commentary on what the syllogism is reaching for and what its elements presuppose, cf. ALBERTUS MAGNUS, *Commentarium*

I

REVIEWING OPINIONS

Before launching into a detailed comparison, let us briefly review selected articles since Moore to pinpoint the sense of *entailment* operating here.

First of all, Moore's examples are quite explicit:

> We shall then be able to say truly that '*p* entails *q*,' when and only when we are able to say truly that '*q* follows from *p*' or 'is deducible from *p*,' in the sense in which the conclusion of Barbara follows from the two premisses, taken as one conjunctive proposition; or in which the proposition 'this is colored' follows from 'this is red.' ([4])

We are neatly outside of a merely truth-functional relation.

Paul Weiss, writing some ten years later (1930), is concerned to distinguish 'entailment' from 'implication' (*not* simply 'material implication'), and lays it down that while both fail on 'T implies/entails F,' that nevertheless the antecedent of an *entailment* always is contained in (or contains) the alternatives of the consequent, while this is never true of *implication*. ([5]) Similarly, *entailment* is always necessarily true, while *implication* never is. Entailment, meanwhile, divides irreducibly into *extensional* and *intensional*, where extensionally, a part entails the whole of which it is the part, and intensionally, the more specific in meaning entails the more generic. While Weiss will argue incisively that the *extensional* notion is conceptually dependent upon («logically secondary to»; Strawson) the *intensional*, what concerns us is his insistence that there is no common notion underlying these two. Entailment, then, is not a generic, but an analogous notion,

in Priorum Analyticorum (Paris, 1890), Lib. I, Tract. 1, 2; Lib. IV, Tract. 6. For a general appraisal of scholastic commentary on Aristotle, cf. I. M. Bochenski, «De consequentiis scholasticorum earumque origine,» *Angelicum* (Rome) 15 (1938), 106-9. The modern commentary on Aristotle, now standard, is Jan Łukasiewicz, *Aristotle's Syllogistic*, 2nd. ed. (Oxford, 1957), Ernest Moody, *Truth and Consequence in Medieval Logic* (Amsterdam, 1953), comments in a similar tenor on the scholastics.

([4]) *Phil. Studies*, p. 291.

([5]) Paul Weiss, «Entailment and the Future of Logic,» *Proceedings of Seventh Internat. Congress of Phil.* (Oxford, 1930), pp. 143-50. On the level of commentary, 'contains' and 'is contained in' manifest an ambiguity hard to eradicate.

realized in two distinct and related, yet irreducible ways. 'Material implication' of PM turns out to be ***extensional entailment***, where the tautological entailments ($q \supset p \vee -p$, $p.-p \supset q$) are valid only because there is no real deduction here, since PM's logical propositions merely stand for an exhaustive set of alternatives, which is certainly entailed by any proposition. We shall see how valuable many of Weiss's remarks will prove to be — notwithstanding the obscurity of the original 'implies' with which 'entails' is contrasted.

Norman Malcolm, writing ten years after Weiss, wants to sharpen Kant's criterion, namely: «*p* entails *q* when the consequence is contained in the antecedent,» which he formulates as follows: «*p* entails *q* when *q* is not a 'further fact in addition to *p*'» (6) He would doubtless accept Prior's subsequent translation of this criterion from fact talk to proposition talk, but the difficulty remains: presumably *q* *looked* at first like a distinct fact from *p*; otherwise analysis would have no job to do. Yet if we are told that entailment is used to bring to light what is *implicity* contained in *p*, this simply restates the problem. What does it mean for *q* not *really* to be a further fact in addition to *p*, when it *looks* like it is ? It means some astute philosopher can show *q* to be deducible from *p*.

Jonathan Bennet and A.N. Prior are willing to grant the «connection of meanings» indigenous to entailment systems, but rather than try to formally eliminate the paradoxical entailments (Strawson apparently failed and Smiley can only come up with severely limited systems in reward for his efforts), they would convince us in Hegelian fashion that there *really is* a «connection of meaning» in the paradoxes. (7) «Any proposition asserts (at least implicitly) something about *all* objects whatsoever» *(hence* $p.-p \supset q$), and «a necessary proposition is implied by all since it can always be introduced» (so $q \supset p \vee -p$). Now to *introduce* is not to *deduce* (as Weiss has noted), and one might question just *how* any proposition *asserts* something about everything ? (Even should he want to say that a *judgment* of its truth or falsity is «implicitly total,» it would not follow from this

(6) Norman MALCOLM, «Nature of Entailment,» *Mind* 49 (1940), 333-47, esp. 341; A. N. PRIOR, «Facts, Propositions, and Entailment,» *Mind* 57 (1948), 62-68.

(7) J. BENNET, «Meaning and Implication,» *Mind* 63 (1954), 451-63; esp. 462-63; A. N. PRIOR, «Facts...»; P. F. STRAWSON, «Necessary Propositions and Entailment Statements,» *Mind* 57 (1948), 184-200; T. J. SMILEY, «Entailment and Deducibility,» *Proc. Aris. Soc.* 59 (1958-59), 233-55.

operation that anything could thereby be deduced from it. Judgment in this sense is not understanding.)

Finally and most recently, von Wright and Geach have insisted that entailment is a frankly *intensional* notion: «p entails q if and only if it is possible by means of logic to come to know the truth of 'p entails q' without coming to know the falsehood of p or the truth of q.» In other words, entailment is *essentially* relational and cannot be understood truth-functionally. (8) (Von Wrights' remark that it is the '$\supset$' understood *intensionally*, only reflects a straining to communicate his thesis in a truth-functional atmosphere. The '$\supset$' has no meaning but an *extensional* one !) Von-Wright's insistence *inter alia* that the «problem of entailment... leads ultimately to the problem of the conditions and meaning of demonstrability» is yet another warrant for a detailed comparison with Aristotle. (9)

Now there is a way, of course, of showing that $p \rightarrow q$ independently of truth-functions, by treating it as set inclusion and use Euler's circles. It is perhaps worth recalling that this handy pedagogical device neatly begs the question of relevance by presupposing it. There is no concentric representation of the paradoxical implications. And the further fact that relevance becomes purely extensional, making deduction and proof a trivial affair of inspection ought to warn us that the solution is too easy. The thrust of this maneuver was to make the syllogism mechanical, so allowing one to move to more flexible systems. But for Aristotle, συλλογιξειν is not something mechanical, and if the syllogism be inflexible, the explanation is tied to reasoning itself, a native process which the 'syllogism' rests on and certain aspects of which it seeks to codify — not legitimize.

Codifying is fascinating and necessary. It reflects reason's aesthetic side and in the drive to simplicity may demand greater formal freedom than Aristotle's syllogistic. But simplicity and transparency are not the *only* regulative principles of reason, and the subsequent efforts of Procrustean codifiers to so persuade us strikes one as dangerously internecine, though the relations are too abstract to see. Indeed, to try to break logic off from its roots in reasoning is self-defeating, and that not for «metaphysical» or «phychological» reasons

(8) Georg von Wright, «Concept of Entailment,» in *Logical Studies* (London, 1957), 166-91, esp. 181-2, 175; Also Peter Geach, «Entailment,» *Proc. Aris. Soc. Suppl.* Vol. 32 (1958), 157-72, esp. 164.

(9) G. von Wright, *Logical...*, pp. 135, 165, 188; p. viii: «The concept of entailment is closely related to the concept of demonstrability. And the demonstrable is that which, by means of logic, we *may* come to *know*.»

merely (though these may well be relevant), but simply because an ideal of purely formal freedom coupled with an aesthetic preoccupation with simplicity can lead to greater *restrictions* on reason than Aristotle dared imagine. In short, there is another kind of flexibility that the «rigidity» of the syllogism was made to exhibit. This is the «argumentation whereby one proceeds from what is known to what is unknown,» reason as the tool of inquiry, of our «coming to know.» ([10]) How little of this process need be germane to an axiomatic system can be judged from Paul Weiss's observation that a «deductive» system with inference defined truth-functionally and allowing substitution and detachment, useful as it may be structurally, has in fact frozen out *deduction*. 'Follow from' has very little hold where there is no first or last, where what comes before or after is largely a question of aesthetics.

But what is *deduction* ? What are we saying when we assert that q can be deduced from p ? We are at least making explicit what was hitherto but implicit. And vague as this contention may be, it already suggests the inadequacy of Euler's circles as more than a preliminary tool. Everything there is already transparent. The fact that one circle is contained within another is something to be *seen,* not unpacked. Nothing remains to be shown, and most would agree that deduction is a process *showing* that something is implicitly contained in another, and in the process, exhibiting it *as* so contained by explicitly bringing out its «grounds.» This, at any rate, is what it means to *demonstrate,* and von Wright reminded us of its affinity with entailment.

Aristotle treated this aspect of the syllogism, its use as a tool in coming to know, in drawing out the implications of our common notions, in the *Posterior Analytics*. Here the stringent conditions which implicitly define demonstration are conditions on *predication*. But this is simply to remind us that Aristotle took the subject-predicate form as normal to assertion, and modelled on it his «logic of terms.» Assertion itself is complex, molecular, and yet is also an element in reasoning, where it is called a 'premise.' ([11]) These 'premises' fit the mold of the syllogism, itself shaped with a view to handling the premises. We want to explore the analogies between the Aristotelian subject-predicate model and a propositional one which preserves the essentials of his predicative relation, and manifests even more per-

([10]) Albertus Magnus, *De predicabilibus* (Paris, 1890), Tract. I, c. 1, Geach, «Entailment» p. 164.

([11]) Cf. Albertus Magnus, *In Pr. An.* I, 3.

spicuously the role of assertions as premises and premises in argument. We offer the following reconstruction of Barbara:

I. *If* if for all *x*, *p*, then for all *x*, *q*
and
if for all *x*, *q*, then for all *x*, *r*,
then if for all *x*, *p*, then for all *x*, *r*.

(where 'for all *x*, *p*' may be read as 'all *x* is *p*' or 'all *x* exemplifies *p*' and so on.) That is:

II. *If* $(x)\ (Px \rightarrow Qx)$ and $(x)\ (Qx \rightarrow Rx)$ *then* $(x)\ (Px \rightarrow Rx)$, which is :

III. $(x)\ (((Px \rightarrow Qx)\ \&\ (Qx \rightarrow Rx)) \rightarrow (Px \rightarrow Rx))$, which abbreviated is:

IV. $((p \rightarrow q)\ \&\ (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Interpreting the arrow in the sense of Anderson and Belnap's E, we want to examine the syllogism in the light of E, and E in the light of Barbara, for what mutual illumination this might gain for the meaning of entailment.

II

ANALYSIS OF ARISTOTLE

Our thesis is multiple but converging. It is invited by Alan Anderson's concluding remark: «...but for expressions involving the arrow we know no suitable definition of 'true'» [12] and encouraged by Moore's choice of examples for entailment: the 'if...then...' of Barbara, and a statement of enveloping generality: 'this is red' entails 'this is colored.' Together, these point to the medieval contention that Aristotle's theory of the syllogism rests firmly on a doctrine of predication: that what permits one to draw necessary conclusions is precisely the order of enveloping generality among the premises. (If what is red is colored and what is colored is extended, then what is red is extended.) The position of the premisses is irrelevant, their relative universality is decisive. These two: position of premises and relative universality were distinguished by medievals as *artificial* and

(12) ANDERSON, «Completeness...» in *Zeit. für Math. Logik*, p. 216.

logical form, respectively. ([13]) Barbara is perfect, because the two forms coincide: the external or artificial form lucidly *manifests* the logical relationships involved, which license deduction. (For the scholastics, the *perfection* was that of art perfectly «imitating» nature.) The fundamental logical relation, that underlying all the others, is *predication,* and Barbara is paradigmatic since all of its premises are universal.

What is important to notice here is that one may conclude logically without Barbara, for it is not Barbara but the order of enveloping generality which licenses deduction. ([14]) But Barbara is not completely superfluous for it *shows* us that this is the case. ([15]) The heart of deduction is interlocking universality; the anatomical model designed to display its functioning in cut-away fashion is called 'Barbara.' Hence it is significant that Moore's examples are exactly the model and the kind of inclusion the model was constructed to manifest. And similarly, we shall find in medieval writings a continual shifting from the model to what it is meant to show, as Plato hit upon the state and its functions to manifest the inner workings of its citizen, man. We shall see also how Barbara precisely manifests the predicative relationship, carrying over to the conclusion the kind of predication found in the premises.

PREDICATION

But first, what is this 'predication' ? There is little doubt that it is fundamental to entailment — from Aristotle's defining it at the outset of his theory to recent controversy on the definition of '$\supset$' as set-inclusion. Aristotle himself defines it in terms of *inclusion:* «That one term should be included in another as in a whole is the same as for the other to be predicated of all of the first. And we say that one term is predicated of all of another whenever no instance of the subject can be found of which the other term cannot be asserted.» ([16])

([13]) Cf. ALBERTUS MAGNUS, *In Pr. An.*, I, 2.

([14]) Cf. AQUINAS, *In II Post. Anal.* 8 (486-8), where he speaks of a certain kind of definition validly *concluding* to what a thing is, and only differing from a (demonstrative) syllogism *incidentally,* i.e. in the order and position of terms.

([15]) ALBERTUS MAGNUS, *In Pr. An.* I, 2. On this as a logical ideal, cf. WITTGENSTEIN, *Tractatus,* 4.022.

([16]) *Prior Analytics* I, 1 (24b 27-29), trans. A. J. JENKINSON.

The definition is of universal affirmative predication; but «to be predicated of none,» he says, «must be understood in the same way.» It is significant that 'to be predicated of' in the sense of 'be included in' is defined here to cover indifferently what the *Posterior Analytics* will speak of as accidental and essential predication, where *essential* bespeaks an intrinsic tie between subject and predicate ('all men are rational'), and *accidental*, the explicit absence of (knowledge of) such a tie, where one must have recourse to enumeration ('all swans are white'). The 'all (no) ...is P' form, then, is indifferently extensional or intensional on Aristotle's definition.

But is there a notion of *predication* still more basic that is at work here, the sense of 'predicate' which applies to the ordinary declarative sentence ? This question is suggested by the fact that syllogistic premises are not ordinary sentences — if only because singular terms are prohibited and the same term must be able at times to serve as both subject and predicate. [17] Now Aristotle insists in the *De Interpretatione* that there is something natural about the way subjects and predicates fall out in ordinary speech, and we certainly use sentences to speak about individuals. [18] What is the difference between the ordinary sentence and a syllogistic premise, and how can we express it ?

The difference, we suggest, is as follows. The '*s* is *p*' of an ordinary declarative sentence *asserts* something of a subject. Analyzed, there is something said ('is *p*') and something of which it is said (*s*). This relationship, we would want to hold with Aristotle, is primitive. Aquinas likens it to matter-form, the «predicate being as it were the formal part; the subject, the material.» [19] The 'is *p*' is like form; it will give the assertion a direction or «sense.» But it *needs* the subject to complete itself; it is, as Frege would have it, «unsaturated,» while '*s*' alone *says* nothing.

The 'all *s* is *p*' or the syllogistic premise, however, does not so much assert something of another as it asserts a relation holding between two things that can be said of another — a relation of greater or less generality. The subject is, as it were, suppressed — or rather underlies *both* terms. [20] Aristotle has extended the '*s* is *p*' model of or-

(17) For Aristotle's syllogistic and singular terms, cf. ŁUKASIEWICZ, pp. 5-7.

(18) *De Interpretatione*, ch. 4, 5.

(19) *In I Peri Hermeneias* 5 (54), 8 (95-96); *In I Post. Anal. 36* (309-13), where paragraphs numbers in parentheses are to the Marietti manual edition (Torino, 1955). Cf. also *In II Peri Herm.* 2 (215).

(20) Cf. MOORE, *Phil. Studies*, pp. 296-7.

dinary speech to a technical use, and instead of reflecting that extension in the *form* of the premise, chose to introduce restrictions on the kinds of terms permitted. This required twisting the predicate into noun-like form, and forced the medievals to fabricate a theory of transformations to assimilate both insights: the radical difference between subject and predicate in ordinary speech, and the fact that in a logical premise the same term may be used as subject or predicate.

III

FROM STATEMENT TO PREMISE: AN HYPOTHESIS

What Aristotle is really saying by 'all *s* is *p*' is, we suggest: 'if anything is (has) *s*, then that thing is (has) *p*.' A model syllogism would look like the following:

IF if x is rational then x is risible
and if x is a man then x is rational,
THEN if x is a man then x is risible.

We have written the syllogism as a principle of inference, making explicit that the line in the form of argumentation answers to 'if... then...'. We have emphasised that the *form* of argumentation that is Barbara was constructed with an eye to manifesting the relation of enveloping universality holding among the terms of the premises. This way of recasting the premises shows the strictly parallel use of 'if...then...' — as the schema of each premise and of the entire paradigm. (As an abbreviation of this interweaving use of 'if... then...' we may now introduce the arrow (→), freighting it along the way with subsequent remarks and waiting until the end to discuss its relation to the arrow of E.)

This form has the added advantage of retaining Aristotle's insight into the irreducible character of the subject-predicate relation by allowing the terms whose relative generality will carry the deduction always to *function* as what they are, predicates. ([21]) The relation

([21]) Lest our concern for the *predicate* look archaic, we may refer to Wilfrid SELLARS, «Grammar and Existence: A Preface to Ontology,» *Mind* 69 (1960), 499-533, esp. 501, 512-13, 517. «The fundamental difference between 'triangularity' and '*that* x is triangular' would be that the latter makes explicit a *gappiness* or *incompleteness* which is perhaps implicit in the

'x is rational' or simply 'x rational' remains irreducible even if it be not the same in all cases, as in 'x rational' and 'x man', where the categories of the predicates differ. A set theoretic interpretation may help illustrate the point. Whereas one can, if he wishes, consider the relative universality expressed by 'if...then...' as set *inclusion* ('the set of all rational items is included within the set of all risible ones'), it is hardly illuminating to try thoroughgoing set-theoretic reconstruction by reading the 'x rational' relation as set *membership*. For the statement: 'if anything is a member of the set of rational items, then it is a member of the set of risible items' only makes us wonder what it means — what one has to be or do — to belong to the set of rational (risible) things. When reformulation only forces the issue to change its verbal apparel, one suspects that something fundamental is at stake. What remains true — and what the reformulation exhibits — is that whatever «category» the predicate may exemplify, what is essential is that it is said or asserted of a subject. To say that this relation cannot be reduced to set-membership (without investing 'ε' with the same properties, which belies the reduction) merely recalls in this context what has been frequently noted elsewhere: the inherent difficulty in carrying through a radically extensional interpretation of logical inference.

CONNECTION : ACCIDENTAL AND ESSENTIAL

There is a sense of 'extensional,' however, corresponding not to the truth-functional, but to the Aristotelian *accidental* 'all swans are white.' This usage is applicable, then, not to the relation 'x swan' or 'x white' but to the 'if...then...', and it says in effect that being a swan has nothing to do with being white, so far as we can tell, but merely that every specimen we have come across happens to be so. What is interesting here is that the overarching 'if...then...' of the syllogistic form — the principal arrow in the abbreviated '$(p \rightarrow q)$ & $(q \rightarrow r) \rightarrow (p \rightarrow r)$' — transmits faithfully from premises to conclusion the *character* of the relative generalities holding in the premises — denoted by the «secondary arrows.» Whatever the premises may exemplify, *accidental* or *essential* (necessary) connection, is carried over into the conclusion.

former.» Cf. also P. T. GEACH, *Reference and Generality* (Ithaca, 1962), pp. 31-34.

III

PERFECT PARADIGM

But of course these questions do not arise until one essays an interpretation or application of the logic. Aristotle speaks of them in the *Posterior Analytics*. Whatever be the connection between the propositional variables of a premise, one presumes that the connection between premises and conclusion in Barbara is an intrinsic or necessary one. Indeed it was meant to be, and the interlocking generality (be it accidental or essential) is so arranged that it will be. This of course suggests a further parallelism of «principal» and «secondary» arrows: a perfect form of Barbara intimated by Moore's example 'if anything is red then it is colored,' where the premises reflect the necessity of Barbara so that *every* arrow denotes an intrinsic or necessary connection.

This is Aristotle's notion of «scientific demonstration» or simply «demonstration,» which allows us not only to construct necessary chains of reasoning, but to reach necessary conclusions when we can come to know the (necessary) premises. The conditions for demonstration are what the scholastics called the two «modes of perseity,» ([22]) and on our reconstruction they stipulate for each premise that either

(1) the predicate of the consequent of a premise be contained in the predicate of the antecedent of the same premise, or

(2) the predicate of the antecedent be contained in that of the consequent. Thus

(1') 'If anything is a number, then it is divisible' (xNumber $\rightarrow$ xDivisible),

(2') 'If anything is a number, then it is odd or even.

There is little doubt that these stipulations, added to Barbara, allow us to construct entailments with every arrow denoting a necessary connection of meanings, but the result is a theory of demonstration so tight that Aristotle sees that nothing but geometry — and that Euclidean geometry — can conform to it. ([23]) If we take it as a model for natural science, then the connections must be «for the most part,»

([22]) For the meanings of *per se*, cf. Aquinas, *In I Post. Anal.* 10.

([23]) At least one can say that Aristotle usually proffers only mathematical examples of his demonstration; Aquinas saw that only mathematics (à la Euclid) fit: *In I Post Anal* 4 (43bis), *In II Post Anal* 12 (525).

and necessary conclusions give way to probabilities. But in spite of these grave inconveniences, Aristote never ceased to regard the «strict demonstration» of the *Posterior Analytics* as the very paradigm of scientific reasoning. Besides the necessity it promised, we suggest that a strong motivation was that it allowed him to show Barbara off to perfection, achieving what our reconstruction manifests: a perfect parallel or symmetry beween principal and secondary arrows, between the necessity of concluding from such interlocking generalities and the necessity of the premises so interlocked — which together, yield the same necessity in the conclusion. This is the crowning success of a paradigm: it can be made to *produce* the necessity it was constructed to lucidly illustrate. It is as though Michaelangelo's David were able to come to life.

ITS MEAGER UTILITY

But logicians are traditionally content with bare bones and lifeless models. In scholastic terms, they will settle for the *necessity of the consequence,* and do not demand to be handed over the *necessary consequent.* So whether the connection internal to the premises be accidental or essential, the enveloping universality of Barbara is enough to assure the conclusion's following of necessity. So the construction of the *Posterior Analytics,* useless as it is for working scientific methodology, seems also redundant to logic, except that it illustrates dramatically the kind of contrapuntal relation between form of premises and form of inference that Barbara actually is, the relation holding on our analysis between principal and secondary arrows.

IV

WHAT THE SYLLOGISM SHOWS

By employing the arrow as an abbreviation for the 'if...then...' that Barbara contains and manifests, we have meant to give an interpretation to the primitive relation of E. This is *prima facie* quite unexceptionable since E means to axiomatize that same common notion of *entailment* which most have no difficulty seeing as perfectly exemplified in Barbara. But is the interpretation useful ? Does it throw any light on a meaning of 'true' for expressions involving the arrow ? Aristotle would of course reply that any such statement is true if it can be reduced to Barbara (or Celarent), reduced that is to a finite series of premises each of whose antecedents is «less universal» than its consequent, and the consequent of the kth premise is the antecedent of the $(k+1)$th *seriatim*. But what is this *reduction* ? As a part of the process of logical proof, it will itself be subject to formulation — and what is to validate the ensuing «laws of reduction ?» Here we have, according to Łukasiewicz, the «fundamental flaw in the Aristotelian theory of proof.» What Aristotle apparently did not realize was that there was another system of logic operating here, a system more fundamental than the theory of the syllogism: the logic of propositions. Or if he did realize it — as certain expressions suggest — it had to wait for another half century to be formulated by the Stoics. Such at least is the contention of Łukasiewicz, exposed as well by Bochenski in his CS (formalization of the categorical syllogism). ([24])

But recent work by Strawson has made us wary of accepting any one logic as «more fundamental» than another, and to say that Aristotle's arguments «rest on» the logic of propositions sounds suspiciously as though one believes *this* logic *legitimizes* and not merely codifies rational inference. ([25]) But however one may react to the thrust of Łukasiewicz' remarks, it is interesting that the logic of propositions he claims Aristotle had to employ as an «auxiliary theory» contains not a few theses which are invalid in E. ([26]) Because of the affinity between entailment as envisaged by E and demonstration for Aristotle, one might suspect that something more (more *fundamental* ?) than the «logic of propositions» is operative in the Aristotelian «reduction» — perhaps indeed the same sense of «universality» which Barbara exploits and arranges in an enveloping series to present a paradigm of logical inference.

But let us examine our suspicion. Both Łukasiewicz and Bochenski,

([24]) Łukasiewicz, pp. 44, 47-48, 88-94; Bochenski, «On the Categorical Syllogism,» *Dominican Studies* (Oxford) 1 (1948), 35-57, reprinted in A. Menne (ed.), *Logica-Philosophical Studies* (Dordrecht, 1962). Cf. also Moody, *Truth...*, p. 78.

([25]) Cf. especially «A Reply to Mr Sellars,» *Phil. Review* 63 (1954), 216-231, but also *Introduction to Logical Theory* (London, 1952), chapters 1, 2, 8, and *passim*.

([26]) Łukasiewicz' «auxiliary theory» is listed on page 89. It includes the laws of simplification (CpCqp), exportation, and others invalid in E. We shall deal expressly with the corresponding list of Bochenski in *Dom. Studies*, pp. 41-2.

accepting the «logic of propositions» as *fundamental,* interpret each premise of the syllogism as a proposition. Hence

IF all *s* is *m*
and
all *m* is *p*,
THEN all *s* is *p*.

becomes 'If *p* & *q* then r' (CK*pqr*). Then the «law of exportation,» which allows us to pass from categorical to hypothetical syllogism, looks like

(3) CCK*pqr*C*p*C*qr* (Łuk. VII, Boch. 10).

Now if 'C' be material implication, the law is valid, but if 'C' is the arrow of E, it is invalid, since by substitution and *modus ponens,* we may derive C*p*C*qp*, where *p* is quite irrelevant to C*qp*. Guided by our premonition that Aristotle would not be «relying upon» logical laws invalid in E (a system with preoccupations so cognate to his), we might ask what permits this discrepancy. It seems clear that the interpretation of each syllogistic premise as itself a proposition, by neatly suppressing the dove-tailing structure of the syllogism, opens the door to a kind of substitution in which Aristotle simply would not have indulged. For example

(4) CCK*pqr*C*p*C*qr*, when for *r*, we put *p*, yields CCK*pqp*C*p*C*qp*, and by *modus ponens* (since CK*pqp* is logically true)

(4') C*p*C*qp*.

But Aristotle would not have lingered over the corresponding

(4") IF all *s* is *m*
and
all *m* is *p*,
THEN all *s* is *m*,

which is the detached CK*pqp*, since there is no concluding, no συλλογιξειν here.

Now here we notice a move common to Aristotelian and medieval logic: an appeal at critical moments to the rational ground of the whole enterprise, a ground *presupposed* to the systematic development and not a formal part of it. Hence the appeal must be entered in the form of extra- or meta-logical commentary, (27) This allowed the scholastics to develop two logical systems side by side — that

(27) So ABELARD may say «the perfect inference of a syllogism... does not depend on any connection between the terms, ...if it has the structure of the syllogism, it stands unshaken,» (*Dialectica,* p. 328) but MOODY (who cites this text) goes on to note that the syllogism was conceived *in use,* presupposing a natural language communicating something. (19-20) Hence the

of the categorical syllogism and a theory of «consequences» quite unrelated to it. Where these other «consequences» lacked the logical following that characterized the syllogism, this was accounted for by commentary, so that the notion of *consequence* became encrusted with distinctions. No one who knows medieval logic can find a trace there of the «anti-formalist» polemic of some neo-medievalists, but he will certainly notice an unresolved tension between a basic commitment to Barbara as the paradigm case of «logical following» and the drive to a symbolic calculus. [28] It would be more faithful to their insights into Aristotle to try to resolve the tension somewhat by incorporating into our symbolism as much of the commentary as we can. [29]

This we have tried to do in interpreting the syllogistic premise as itself two propositions, united by the same relationship which Babara is constructed to manifest. Historians of logic will recognize this tack as that of Theophrastus, transmitted by Alexander as «analogical» or «totally hypothetical syllogism.» Theophrastus, however, was not clear on the distinction between term and propositional variables, and his formulation does not *manifest* the parallelism between the form of Barbara and the form of each premise. As Bochenski notes, this would have forced him to an «intolerable» Greek construction, beginning each rule with a double 'ει' ('if'). Yet on our interpretation, his rules, reflected into formulae, constitute more of a *link* than a «curious half-way house between the logic of Aristotle and that of the Stoics.» Theophrastus' «inaugurating a logic of propositions and yet thinking all the while that he was continuing the Aristotelian syllogistic» becomes more plausible if the two turn out to be inextricably related. [30] By this reconstruction, we had hoped to avoid a

«connection» Abelard renounces is merely a «material» one of content, not a logical following. (15, note).

[28] As early as Albertus Magnus, the need for «transcendental terms» or *symbols* was clearly recognized (*In Pr. An.* I, 9), yet Moody tells us that «perhaps principally, the syllogism is a *formal* consequence» (73), and at least for Buridan, «material consequences are only evident, logically, insofar as they are «reduced» to formal consequences» — and here 'material' includes 'simply material' which is our «material implication.» (76)

[29] Cf. BOCHENSKI, «De consequentiis...» for an appraisal of scholastics on Aristotle.

[30] I. M. BOCHENSKI, *La Logique de Théophraste* (Fribourg en Suisse, 1947), 111-116. Also in his *History of Formal Logic* (Notre Dame, 1961), pp. 103-4. Theorems 37-41, reconstructed from Theophrastus' rules and listed on page 114 of *La Logique...* are valid in E.

prior commitment to the propositional calculus as «more fundamental,» taking rather as «fundamental» the relationship ($\rightarrow$) which Barbara both contains and reflects, exhibiting by our use of arrows this contrapuntal construction. Applied to the «law of exportation» we have

$$(3')\quad (((p\rightarrow q)\,\&\,(q\rightarrow r) \rightarrow (p\rightarrow r)) \rightarrow ((p\rightarrow q) \rightarrow ((q\rightarrow r) \rightarrow (p\rightarrow r))).$$

On a substitution similar to the previous, this form clearly shows us why Aristotle need not have bothered. For replacing r by q:

$$(3'')\quad ((p\rightarrow q)\,\&\,(q\rightarrow q) \rightarrow (p\rightarrow q)) \rightarrow ((p\rightarrow q) \rightarrow ((q\rightarrow q) \rightarrow (p\rightarrow q))).$$

Since the antecedent is detachable, we have the consequent as a theorem of E, where the very form of '$q\rightarrow q$' manifests why it is superfluous (in a way which '$CpCqp$' cannot).

But a closer look at (3') reveals some interesting logical facts. Together with similar reconstructions of the remaining theorems of Bochenski's CS which were originally invalid in E (the antilogism and what he calls «direct reduction»), in every case antecedent and consequent are singly theorems of E, but the entailment is not. But the antecedent in each case is Barbara $((p\rightarrow q)\,\&\,(q\rightarrow r) \rightarrow (p\rightarrow r))$, so that formulae such as

$$(5)\quad (p\rightarrow q)\ \&\ \overline{(p\rightarrow r)} \rightarrow \overline{(q\rightarrow r)}\,,$$
$$(6)\quad ((s\rightarrow t) \rightarrow (p\rightarrow q)) \rightarrow (((s\rightarrow t)\ \&\ (q\rightarrow r)) \rightarrow (p\rightarrow r))\,,$$

while themselves valid in E, cannot be shown to *follow from* the syllogism. Yet we arrived at the true consequents by reconstructing the theorems of a propositional theory of deduction on the model of Barbara. This encourages our suspicion that the syllogism Barbara is not to be construed as a *principle* (in the formal sense) of an entailment system, but is rather more *presupposed* to the whole enterprise, as manifesting most clearly the basic relationships at work. ([31])

(Our reconstruction also accounts for Aristotle's marked preference for Barbara over other forms of the first figure: Celarent or Ferio. Neither is deducible from Barbara, yet either one taken together

([31]) For the use of «presupposes» in this quasi-technical fashion, see P. F. Strawson, «On Referring», *Mind* 59 (1950), reprinted in A. Flew, *Essays in Conceptual Analysis* (London, 1956), 21-52; and *Introduction*, p. 175.

with Barbara yields the other. Their «imperfection» lies rather in the structure of their negative and particular premises. So long as these are treated in the fashion of a logic of terms, however, it seems that Aristotle was simply showing partiality for 'all' over 'no' and 'some', for 'all *p* is *q*', 'no *p* is *q*', and 'some *p* is *q*' are hardly that diverse. Interpreted propositionally, though, they come out:

(7) $(x)(Px \rightarrow Qx)$,

(8) $(x)\overline{(Px \,\&\, Qx)}$,

(9) $(\exists x)(Px \,\&\, Qx), = \overline{(x)\overline{(Px \,\&\, Qx)}}$.

'No *p* is *q*' and its negation, 'some *p* is *q*' do not speak of the irreducible relationship modelled on predication which Barbara manifests, for their propositional expression contains no arrow. In spite of Aristotle's original definition, then, there *is* a difference between universal predication and its contrary. ([32]) The fact that 'no *p* is *q*' and 'some *p* is *q*' can be converted, while 'all *p* is *q*' and its negation 'some *p* is *q*' cannot — a fact Aristotle knew very well — calls attention to a difference which the general similarity of the sentential form left obscure. That difference is clearly manifested by our propositional interpretation: whereas 'all *p* is *q*' goes into an arrow statement and may be fruitfully interpreted as «logical following,» 'no *p* is *q*' sidesteps the issue, 'no *p*' merely denying that anything exemplifies both *p* and *q*. Since this premise (and its negation), then, do not *internally* reflect the basic structure where by the syllogism concludes, they are «inferior» in logical form to the universal affirmative premise, and the same may be said for the syllogistic figure containing them.

CONCLUDING REMARKS

What may we conclude from these converging indications of our thesis ? That whatever the «other logic» may be that is operative in Aristotle's systematic reduction of syllogistic forms to the first figure, it cannot be said to be simply and without qualification the «logic of propositions.» Common logical relationships are no doubt

([32]) Cf. note 16. Aristotle speaks of the priority of the first figure in *Pr. An.* I, 23 (40b13), and *Post. an.* I, 6.

at work in Aristotle and *Principia Mathematica*, and Aristotle can speak of the premises of the syllogism *in globo* as single propositions, (33) but something else is controlling Aristotle's reductions as well. We have suggested that it is the paradigm of Barbara and the kind of *universality* the first figure is made to manifest. This is not to say that «the *dictum de omni et nullo*... is the axiom on which all syllogistic inference is based,» a proposition J.N. Keynes attributes to Aristotle and Łukasiewicz roundly criticizes. The notion of *universality* is not a «principle of the Aristotelian syllogistic» in the sense of a thesis entailing all the axioms. It is rather presupposed to the entire endeavor — a meaning of 'principle' that Łukasiewicz leaves room for, but confesses (in advance) not to understand. (35) Our propositional reconstruction of Aristotle's first figure has meant to bring into the open as much as possible exactly what is being presupposed. By using the arrow in each premise in a fashion strictly analogous to the line separating and uniting the two premises from (and with) their conclusion, we have meant to show at one and the same time how Barbara concludes in virtue of interlocking universality and that one's most accurate guide to the *nature* of this universality is the manner in which Barbara concludes. If the relative universality is radically *in* the premises — which themselves reflect the absolutely irreducible subject-predicate relation that suggested *universality* in the first place — it is in the kind of «logical following» which the syllogism is constructed to yield that we are *shown* what this universality *is* by seeing what it can *do*. As Paul Weiss has remarked, and this relative universality allows of both extensional and intensional interpretations: of a part entailing the whole of which it is a part, or of the more specific entailing the more general. And what is more, there seems to be no common denominator to the two. Each represents a «logical consequence» and *not* in virtue of some more fundamental law. (Whether the *extensional* is conceptually derivative from the *intensional* is a further question and beyond our scope. One can only say that it *seems* as though it is, Weiss argued that it is,

(33) *Prior Analytics* II, 4 (57bl)-*modus tollens*; II, 8 (59b 3), the «antilogism.»

(34) J.N. KEYNES, *Formal Logic* (London, 1906), p. 301; ŁUKASIEWICZ, pp. 46-7, 73-4.

(35) ŁUKASIEWICZ, p. 47: «It is a vain attempt to look for the principle of the Aristotelian logic, if 'principle' means the same as 'axiom.' If it has another meaning, I do not understand the problem at all.»

and the latest work in analysis is taking on the burden of *showing* that it is.)

So the truth of expressions involving the arrow, whether they be interpreted extensionally or intensionally, may be said to depend on the *relative universality* of their propositional variables, so arranged as to conclude after the fashion of Aristotle's first figure, even if not so manifestly as Barbara. (The second Belnap property: that every propositional variable of a tautological entailment must be both an antecedent and consequent part of the formula, where these parts are rigorously and recursively defined, assures such an «arrangement.» (36)) Whether this actually throws more light on what we mean by the truth of an arrow statement than «provable in E» does, is probably one of those matters of epistemic preference. Perhaps it would not be remiss to add that using Aristotle to «throw light on E» is not to *reduce* E to the theory of the syllogism. Quite the contrary, it has doubtless been evident how much this reading of Aristotle has profited from the researches embodied in E.

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(36) Cf. ANDERSON and BELNAP, «The pure calculus of entailment,» *Journal of Symbolic Logic*, 27 (1962), 19-52.