A QUANTIFICATIONAL TREATMENT OF MODALITY

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I. Introduction.

The object of this paper is to present a (hitherto little explored) approach to the logic of modality from the direction of quantificational logic. This approach is based upon the idea of *possible though non-existent* objects. While some philosophers view this idea with distaste (in recent times most notably W. V. Quine), it has played a prominent role in the thought of others (notably Leibniz). In any case, this conception is basic to the ensuing considerations.

Our proposed quantificational construction of modality has the advantage — from the standpoint of the formal logician, at any rate — of reducing a (for him) relatively «strange» discipline, viz. modal logic, to a relatively familiar one, viz. the logic of quantification. All of the formal machinery developed in the context of the latter branch of logic — proofs of completeness, consistency, etc. — can be brought to bear upon the former.

II. The Basic Idea.

The root idea of our proposed construction of modality lies in considering the domain *A* of *actually existing objects* to be a (proper) subdomain of the domain *P* or *possible objects*. For the sake of convenience and exactness let us introduce some notational machinery. Let «(a) (...a...)» and «(∃a) (...a...)» represent universal and existential quantification over the (usual) domain *A* of actuals, and let «(Aa) (...a...)» and «(Ea) (...a...)» represent universal and existential quantification over the wider domain *P* of possibles [1]. We shall re-

[1] So that we of course have the bridging-rules:

$(Aa)(...a...) \rightarrow (a)(...a...)$

$(Ea)(...a...) \rightarrow (\exists a)(...a...)$

The resort to several styles of quantification can of course be circumvented by adopting only one (the widest) styles of quantifier, and defining the others by means of it, imposing a suitable restrictive condition. See Section IX of N. Rescher, On the Logic of Existence and Denotation, *The Philosophical Review*, vol. 68 (1959), pp. 157-180, and *idem*, Many-sorted Quantification, *Proceedings of the 12th (Venice, 1958) International Congress of Philosophy*, Firenze, 1960, pp. 447-453.

turn below to a further consideration of the idea of a «domain of possibles».

Given this machinery, we can now present compactly the basic idea, the «guiding intuition» for the proposed quantificational construction of modal propositions involving qualifiers. We shall construe the statement that it is necessary that everything φ's as amounting to the thesis that every possible-object φ's; and analogously we shall construe the statement that it is possible that something φ's as amounting to the thesis that some possible-object φ's. We thus propose that

$$\Box\,(x)\,\varphi x$$

be regarded as amounting to

$$(Ax)\,\varphi x$$

and that

$$\Diamond\,(\exists x)\,\varphi x$$

be regarded as amounting to

$$(Ex)\,\varphi x$$

It is at once clear that the usual duality relationships are preserved by this interpretation; so that, e.g., «$\sim\Box\,(x)\,\varphi x$» is equivalent with

«$\Diamond\,(Ex)\,\sim\varphi x$

III. Modality in Quantificational Systems.

Among the formal requirements that a logical theory of modality-with-quantification is usually held to satisfy are the following:

(T1) $\vdash (x)\Box\varphi x \leftrightarrow \Box(x)\varphi x$
(T2) $\vdash (\exists x)\Box\varphi x \rightarrow \Box(\exists x)\varphi x$
(R1) $\dashv \Box(\exists x)\varphi x \rightarrow (\exists x)\Box\varphi x$
(T3) $\vdash (x)\Diamond\varphi x \leftrightarrow \Diamond(\exists x)\varphi x$
(T4) $\vdash \Diamond(x)\varphi x \rightarrow (x)\Diamond\varphi x$
(R2) $\dashv (x)\Diamond\varphi x \rightarrow \Diamond(x)\varphi x$

Here '$\rightarrow$' represents entailment (or strict implication), '$\leftrightarrow$' represents mutual entailment (or strict equivalence), '$\vdash$' signalizes a thesis as acceptable (asserted), and '$\dashv$' signalizes a thesis as unacceptable (rejected).

Now if an adequate treatment of modality is to be given in purely quantification terms it must provide a quantificational construction of the modalities in such a way that these six requirements are met. It must thus provide a purely quantificational version of the following eight expressions:

(1) $\Box(x)\varphi x$
(2) $(x)\Box\varphi x$
(3) $(\exists x)\Box\varphi x$
(4) $\Box(\exists x)\varphi x$
(5) $\Diamond(x)\varphi x$
(6) $(x)\Diamond\varphi x$
(7) $\Diamond(\exists x)\varphi x$
(8) $(\exists x)\Diamond\varphi x$

And furthermore the purely quantificational rendition of these statement-forms must be such that the aforementioned requirements are satisfied, so that (1)$\leftrightarrow$(2), (3)$\rightarrow$(4) but not conversely, (5)$\rightarrow$(6) but not conversely, and (7)$\leftrightarrow$(8).

IV. The Quantificational Construction of Modality: I.

Returning now to the basic idea of Section II, we see at once that all of the requirements of the preceding section are satisfied by the following purely quantificational construction of quantified modal statements:

Modal Statement	*Quantificational Construction*
(1) $\Box(x)\varphi x$	$(Ax)\ \varphi\ x$
(2) $(x)\Box\varphi x$	$(Ax)\ \varphi\ x$
(3) $(\exists x)\Box\varphi x$	$(Ax)\ \varphi\ x$
(4) $\Box(\exists x)\varphi x$	$(x)\ \varphi\ x$
(5) $\Diamond(x)\varphi x$	$(\exists x)\ \varphi\ x$
(6) $(x)\Diamond\varphi x$	$(Ex)\ \varphi\ x$
(7) $\Diamond(\exists x)\varphi x$	$(Ex)\ \varphi\ x$
(8) $(\exists x)\Diamond\varphi x$	$(Ex)\ \varphi\ x$

The reader can readily satisfy himself that each of the six requirements of Section III is at once met by this purely quantificational construction of quantified modal statements.

V. A Shortcoming of this Construction.

But although the proposal of the preceding section is adequate to all requirements laid down so far, it has shortcoming that there are certain plausible *additional* requirements that it fails to satisfy. For it countenances two theses — viz. that (3) is no weaker than (1)-(2), and that (6) is no stronger than (7)-(8) — which, it could reasonably be argued, are unacceptable and must be rejected. It would be most plausible to add to the requirements of Section III two others:

(R3) $\dashv (\exists x)\Diamond\varphi x \rightarrow (x)\Diamond\varphi x$
(R4) $\dashv (\exists x)\Box\varphi x \rightarrow (x)\Box\varphi x$

And if these two additional requirements are accepted, the purely quantificational construction of modal statements presented in Section IV at once collapses, because it fails to satisfy R3 and R4.

VI. A Two-Layer View of Possible Objects.

In the effort to extend our fundamentally quantificational construction of modality to a fully adequate theory capable of avoiding R3 and R4, let us take the somewhat bold — and yet to be justified — step of splitting the domain P of possible objects into two subdomains P_1 and P_2, the latter, P_2, including those elements of P which one is willing to regard as only *remotely* possible; the former, P_1, including the rest, i.e., those elements of P which one is willing to regard as *proximately* possible. We now obtain *three* groups of quantifiers:

(i) $(\exists a)$ and (a) over the domain A
(ii) (E_1a) and (A_1a) over the domain $A \cup P_1$
(iii) (E_2a) and (A_2a) over the domain $A \cup P_1 \cup P_2$

Here, then, two extraordinary modes of quantification are introduced, E_2 and A_2 corresponding to the old E and A; and E_1 and A_1 representing a new, intermediate mode of quantification, ranging not over *all* possible objects, but only over the *proximately* or *plausibly* possible objects.

VII. Proximate and Remote Possibility.

The idea of *possible* as opposed to *actual* objects is anathema to some philosophers. And no doubt the idea of several types of possible objects, some more possible than others, would be viewed by them as weird nonsense. Let me try to motivate this idea.

I seems, first of all, that the idea of possible but not actual objects can most simply be introduced in terms of the factor of time. Thus in (i)-(iii) of the preceding section we could let A be the set of *all presently actual* (i.e., currently existing) objects, P_1 the set of *all proximately actual* objects (i.e., objects in existence within some specified time T of the present), and P_2 be the set of *all actual* objects (i.e., objects in existence at some time or other). This temporal approach has the merit of articulating the idea of *possible objects* without overstepping the bounds of that which is actual (at some time or other).

A second, perhaps no less plausible way of differentiating between the proximately and the remotely possible is in terms of the frequently-discussed distinction between physical and logical possibility. Thus we could here regard as proximately possible an object whose description is compatible with the laws of nature (as golden goose eggs, for example, are not), and as remotely possible an object whose description is compatible with the laws of logic (as round squares, for example are not, but golden goose eggs are).

In either of these ways, and no doubt in others as well, sense can be made not only of the distinction between the actual and the possible, but even of that between the remotely and the proximately possible.

VIII. The Basic Idea Re-Applied.

On the basis of this idea of a two-layer view of possible objects, we now re-apply the basic idea of Section II as follows: We propose that

$$\Box (x)\ \varphi x$$

be regarded as amounting to

$$(A_2 x)\ \varphi x$$

and that

$$\Diamond (\exists x)\ \varphi x$$

be regarded as amounting to

$$(E_2 x)\ \varphi x$$

Once again, then, we construe the statement that *necessarily* everything φ's as equivalent to the thesis that *every* possible-object φ's; and the statement that *possibly* something φ's as equivalent to the thesis that *some* possible-object φ's.

IX. The Quantificational Construction of Modality: II.

But we are now in a position to extend the purely quantificational construction of modal propositions in a more adequate way, as follows:

Modal Statement	*Quantificational Construction*
(1) $\Box (x)\, \varphi x$	$(A_2x)\, \varphi x$
(2) $(x) \Box\, \varphi x$	$(A_2x)\, \varphi x$
(3) $(\exists x) \Box\, \varphi x$	$(A_1x)\, \varphi x$
(4) $\Box (\exists x)\, \varphi x$	$(x)\, \varphi x$
(5) $\Diamond (x)\, \varphi x$	$(\exists x)\, \varphi x$
(6) $(x) \Diamond\, \varphi x$	$(E_1x)\, \varphi x$
(7) $\Diamond (\exists x)\, \varphi x$	$(E_2x)\, \varphi x$
(8) $(\exists x) \Diamond\, \varphi x$	$(E_2x)\, \varphi x$

It is at once apparent that this mode of construction of quantified modal propositions satisfies not only requirements T1 - T4 and R1 -R2 of Section III above, but also the added conditions R3 - R4 of Section V.[2]

X. Plausibility Considerations.

It is not difficult to give a rationale of plausibility to the foregoing construction-scheme. Lines (1) - (2) and lines (7) and (8) are supported immediately by the considerations of Section II above. Assuming for the moment that lines (4) and (5) have already been justified, lines (3) and (6) are readily supported. For (3) must be weaker than (1) - (2) but yet stronger than (4); and similarly (6) must be stronger than (5) but yet weaker than (7) - (8). It thus remains to consider the justification of lines (4) and (5).

Let us first consider the justification of line (5), i.e., the treatment of «$\Diamond (x)\, \varphi x$» as equivalent with «$(\exists x)\, \varphi x$». Assume first that: $(\exists x)\, \varphi x$. Then some things actually have φ. But then there is a «possible world» — viz. that in which these things are the only things — in which all things have φ, so that: $\Diamond (x)\, \varphi x$. Conversely assume that: $\Diamond (x)\, \varphi x$. Now there is surely a sense of possibility (though perhaps a more than minimally strong one — as will be discussed below) such

(2) Note that on the proposed construction iterated modalities are automatically defined for four cases, with the following results:

$\Box \Box (\exists x) \varphi x \leftrightarrow \Box (x) \varphi x$

$\Diamond \Box (\exists x) \varphi x \leftrightarrow \Diamond (x) \varphi x \leftrightarrow (\exists x) \varphi x$

$\Box \Diamond (x) \varphi x \leftrightarrow \Box (\exists x) \varphi x \leftrightarrow (x) \varphi x$

$\Diamond \Diamond (x) \varphi x \leftrightarrow \Diamond (\exists x) \varphi x$

In view of the first equivalence it is clear that we do *not* have

$\Box \Box p \leftrightarrow \Box p$

so that the modal system at issue is weaker than Lewis' systems S4 or S5.

that we can only maintain that it is possible (in this sense) that all things have the property φ when there is at least one thing that actually as φ.

Given this justification of (5), (4) at once becomes justified in terms of (5) by negation-duality.

XI. Time and Modality (3).

Let us return for a moment to the temporal construction of «possible objects» suggested in Section VII. Let us introduce temporalized quantifiers over individuals, $(E_t a)$ and $(A_t a)$, such that

«$(A_t x)\ \varphi x$» means «All x's existing at time t φ»

«$(E_t x)\ \varphi x$» means «Some x existing at time t φ's»

We shall also need (1) the temporal «constant» N for «now», (2) the constant T delimiting «proximate» time, and (3) (At) and (Et) as achronological quantifiers over times. We now have that:

«$(x)\ \varphi x$»	becomes	$(A_N x)\ \varphi x$
«$(\exists x)\ \varphi x$»	,,	$(E_N x)\ \varphi x$
«$(A_1 x)\ \varphi x$»	,,	$(At)\ [(/N\text{-}t/ \leqslant T) \rightarrow (A_t x)\ \varphi x]$
«$(E_1 x)\ \varphi x$»	,,	$(Et)\ [/N\text{-}t/ \leqslant T)\ \&\ (E_t x)\ \varphi x]$
«$(A_2 x)\ \varphi x$»	,,	$(At)\ (A_t x)\ \varphi x$
«$(E_2 x)\ \varphi x$»	,,	$(Et)\ (E_t x)\ \varphi x$

In the light of these constructions, we can re-examine the tabulation of Section IX to throw further light on the temporal construction of modality. Line (7) of that tabulation, for example, now tells us that only that is possible for some now-existing x which has been actual for some x at some time or other. And similarly, line (5) has it that only that is possible for all now-existing x's which is actual for some now-existing x.

XII. Remarks on Necessity and Possibility.

Just as the sense of «possibility» at issue is a strong one (because «$\Diamond (x)\ \varphi x$» is true only when «$(\exists x)\ \varphi x$» is), so the corresponding sense of necessity is also strong. For in view of line (4) we have

$$\Box (\exists x)\ \varphi x \leftrightarrow (x)\ \varphi x$$

(3) I owe the fundamental idea of this section to A. N. Prior's most stimulating book on *Time and Modality* (Oxford, 1957).

and so *a fortiori*

$(\exists x)\ \varphi x \rightarrow \Diamond(x)\ \varphi x$

Our sense of «necessity» is thus such that it can be *necessary* that *some* individual has a property only if *all* individuals *do in fact* have this property. *There are thus no «differentially necessary properties»,* i.e., properties necessarily belonging to one individual but not to another. On the concept of necessity at issue, «$\Box(\exists x)\ \varphi x\ \&\ \Box(\exists x) \sim \varphi x$» is self-inconsistent. So long as even a single individual fails to have a given property, it is not necessary — in the strong sense of «necessity» here at issue — that there be any individual that has this property. On the approach at issue here, necessity is a mode of universality (viz. actuality in *all* instances), and possibility is a mode of particularity (viz. actuality in *some* instance). That this type of «necessity» is not the only one which must of course be granted.

In the case of *abstract* entities, of numbers, for example, there are of course differentially necessary properties, so that the concept of necessity here at issue does not apply. (It is a patently necessary property of 2 — but not of 3 — to be a square root of 4.) And even in a *heterogeneous* class of concrete particulars this can be the case: it is a necessary porperty of Socrates (*qua* man) to be human, but not canine; and a necessary property of Fido (*qua* dog) to be canine, but not human. *But if we take as our framework of discussion a* homogeneous *group of concrete particulars — e.g., men — the thesis that there are no differentially necessary properties becomes now an eminently plausible one.* For whatever could reasonably be regarded as a necessary property of the man Socrates — being rational, mortal, animal, or the like — is patently also to be regarded as a necessary property of the man Caesar.

The point is that within a homogeneous group of concrete particulars, individuals cannot (by us, at any rate) be differentiated through *essential* characteristics (*pace* Leibniz with a concept of the «complete individual notion» of concrete particulars, which, however, is known only to God). The «strong» concepts of (particular) necessity as «that which is always exemplified» (in the sense that we have $\Box(\exists x)\ \varphi x \rightarrow (x)\ \varphi x$) and of (universal) possibility as «that which is sometimes exemplified » (in the sense that we have $(\exists x)\ \varphi x \rightarrow \Diamond(x)\ \varphi x$) will thus be applicable in any such homogeneous group of concrete particulars.

XIII. Conclusion.

In sum, then, it seems that among the many senses of necessity/possibility there is one (particularly strong) sense which conforms to the requirements of our present discussion. There is, indeed, some reason to think that it was a concept of necessity/possibility of just this kind that was in the mind of Aristotle when he launched the discipline of modal logic on its long and evenful history [4]. But that is quite another story, and a long one at that.

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(4) See Jaakko Hintikka, Necessity, Universality and Time in Aristotle, *Ajatus*, vol. 20 (1957), pp. 65-90, for some illuminating remarks about the relations between universality and necessity and between particularity and possibility in Aristotle.