A NERVE NET SYSTEM IN MODAL LOGIC*

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1. Introduction

The application of mathematical models to social science, especially psychology, social psychology, and sociology, has found two kinds of pitfalls. Either the mathematical method is so specific that any application has to be confined to very restricted or trivial situations or they use mathematics primarily for illustrative purposes. The acceptance of many new mathematical and logical techniques in social science follows frequently a coresponding pattern, of first acceptance of the new concepts widely without particular concern for the exact procedure involved, then application of the technique by itself, and finally restriction to problems which have only marginal relation to the problem under consideration with possible further expansion by analogy. This seems to have been the fate of such new techniques as information theory, game theory, Markoff chains, or network theory.

A fundamental reason for this dilemma is in the state of the knowledge of social phenomena and, possibly, in the nature of the data themselves. The multiplicity of factors to be considered and the uncertainties still present in the measurement of human beings force excessive constraints to apply rigorous mathematical methods. Instead of forcing events into a structure to which these methods fit, it may be preferable to loosen up the logical procedure to make it more congenial to the subject matter. If this is done systematically, the procedures used can still have referents in the more exact rules of logic.

One such modification is given in the theory of modal logic. Addition of possibility and necessity to the logical operators makes logic more applicable to the study of actual human behavior. We shall examine the theory of nerve nets which — besides its intrinsic importance — is an explicit analysis of formal logic in its relation to life sciences. By showing the difficulties in actual application of formal logic, we can see the needs of a logic adapted to interpersonal

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behavior and the manner in which modal logic meets some of these requirements. It will also be shown how modal logic can be represented in networks so that the grounding in strict procedure is not lost.

1.2 Nerve Net Theory and Logic

Nerve net theory as developed by McCulloch and Pitts (1), von Neumann (2), Shannon and Moore (3) and others can represent different input-output relationships, thus simulating some behavior of living organisms. The nerve net works essentially as a binary formal logical system, representing the operations of Boolean algebra, including counting, and is thus capable of performing ordinary arithmetic. It can detect or correct errors, achieving any degree of desired accuracy. However, this is paid for at the cost of bulk and complexities. This mode of representation, for all its advantages, has some obvious deficiences in the representing of human behavior. Some of these are connected with the nature of deductive logic. This logic ignores the content of a proposition; the only measure of a proposition assessed is its truth value. Hence, any true or any false propositions are equivalent to each other, and any combination of propositions is either true or false. In consequence, in this system any proposition is relevant to any other.

This indifference to content contributes to the mathematical elegance of the theory but makes for difficulties in application. It even results in needless complexities in the establishment of accuracy: errors in the transmission of a proposition such as «Two and two are four» must be checked in the same way as those for a proposition like «It is raining today.» It becomes an even greater obstacle in the understanding of psychological processes, as in actual practice the truth and falsity of any proposition do not have implications for the truth or falsity of all other propositions as required in Boolean al-

(1) McCulloch, W. S. and Pitts, W., «A Logical Calculus of the Ideas Imminent in Nervous Activity», *Bulletin of Mathematical Biophysics*, 1943, 115-133.

(2) von Neumann, J., «Probabilistic Logics and the Synthesis of Reliable Organisms from Unreliable Components», in *Automata Studies*, Princeton Univ. Press, 1956.

(3) Shannon, E. E. and Moore, E. F., «Reliable Circuits Using Less Reliable Relays», *Journal of the Franklin Institute*, Sept. and Oct., 1956, 191-208, 281-297.

gebra. There it is a consequence of the definition of material implication; that is, that a true proposition is implied by any proposition and a false proposition implies any proposition. Attention to content requires a differentiation in the meaning of truth, which is not equivalent to a simple probability measure but to a distinction between analytic and synthetic truth. It also leads to the notion of relevance.

A functioning nervous system operates with continuous input and output. It has been surmised that it typically operates in an analogue way and that digital analysis constitutes a late and refined procedure (4) (5). Nerve net theory, analyses its input and output in a digital manner because the nerve net consists of binary units, neurons and synapses. The value of this theory will be increased if features of the analogue operation of the nervous system can be translated into essentially binary calculus of nerve net analysis. One such approach is the transition from the classical logical algebra to modal logic.

2. Modal Logic

Modal logic includes all the principles of Boolean algebra and in addition the operations of necessity and possibility. It is also at least a three-valued logic admitting a value of «indeterminate». These features lead to the inclusion of strict implication, which is defined as «It is not possible that a is true and b is not true.» If this relationship holds between two propositions, b can be deduced from a because of some necessary connection between the two propositions. If they both only happen to be true, strict implication is indeterminate. Deduction is thus used in its ordinary usage, and the meaning of a proposition is recognized, besides its truth value. The additions made by modal logic permit a closer approximation of networks to actual processes of organisms.

Since renewal of interest in modal logic through C. I. Lewis' paper in 1918 (6) (7) several different systems have been proposed. For the application to nerve net theory a system is required which has ex-

(4) von Neumann, J., *The Computer and the Brain,* Yale Univ. Press, 1958.

(5) Sebeok, T. E., «Coding in the Evolution of Signalling Behavior», *Behavioral Science,* Oct., 1962, 430-442.

(6) Lewis, C. I., *A Survey of Symbolic Logic,* Univ. of California Press, 1918.

(7) Lewis, C. I. and Langford, C. H., *Symbolic Logic,* (2nd. edition), Dover, 1959.

plicit transformation rules, a minimum of primitive concepts, and no differentiation of impulses by content. The best example of such a system was found in Prior's system Q (8).

Prior defines the modal operators as applying to time: necessity means «true at all times», possibility «true at some time», true and false refer only to the present time. As knowledge about other times may be imperfect, we have to admit a third value — indeterminate — for all times except the present one. This is a three-valued logic; although Prior assigns numerical values to the three states, he does not use them in any ordinal sense but only as a nominal distinction. We shall identify them, therefore, by symbols +, —, or 0. Each proposition is then characerized by a string of these symbols, theoretically infinite, each symbol representing the truth of the proposition during a particular time interval. The first symbol must be + or —. The logic is based on four pimitive operations: negation, conjunction, necessity, and possibility. Rules are given for the transformation of each symbol under each operation. This, then, is the required development of modal logic. It can be represented in a nerve net or a computer if it is possible to use the three-valued strings as input and to define the operations in a way in which the nerve net system can handle them.

3. Rules for Prior's System Q

Let us first define the four operations. The definitions given here are equivalent to those used by Prior (9) but modified to bring out certain points useful for later application.

3.1 Notation

Propositions are designated by small letters, a, b, ...; operations by capital letters N, K, L, or M. Operation symbols are put before the propositions they refer to. Propositions consist of a string of symbols +, —, or 0, but the first symbol cannot be 0. Three of the operations have only one argument: Na (negation of a), La (a is necessary), and Ma (a is possible). The fourth operation has two arguments: Kab (conjunction of a and b). Operations can be cumulated and are performed

(8) Prior, A. N., *Time and Modality*, Oxford, 1957.
(9) Prior, *op.cit.*, 43-44.

from right to left; e.g. NMNa: it is not possible that not a, or NKaNB: it is negated that a and not b, i.e. material implication.

3.2 Transformation Rule of Indeterminacy (0)

Any operation which includes a 0 will result in a 0. In a single-valued operation 0 is invariant. In a two-valued operation 0 in either of the two arguments will result in 0. Thus none of the operations will make indeterminate knowledge determinate. Information cannot be gained through formal logic of any kind.

3.3 Transformation Rules for N and K

Both of these operators work on single symbols without regard to the rest of the structure. N changes plus to minus and minus to plus. K operates on a pair of coresponding symbols in the sequences in the same way as ordinary conjunction does, i.e. two plusses make a plus, all other combinations are minus. Zeros are dealt with according to rule 3.2. These two operations and their combinations make all operations of Boolean logic possible.

3.4 Transformation Rules for L and M

The operators change the symbols according to the type of whole sequence. We distinguish three types according to the presence of plusses in the sequence. Type one consists of sequences which contain only plusses (propositions which are always true); type two of sequences which contain plusses and other symbols (propositions which are sometimes true); type three of those which contain no plusses (propositions of which it is not known if they are ever true). Type one and type three stay invariant under both L and M transformation. In type two operator L changes both plus and minus to minus, and operator M changes both to plus. Again, zero stays invariant. In other words, these rules mean that a proposition is necessary at any time if it was true at all times, and it will be possible at all times if it is true at any time, except, of course, at those times at which nothing is known about the truth of the proposition.

3.5 Meaning and Degrees of Truth

These rules allow all the operations of modal logic, in particular strict implication, which is defined as NMKaNb, it is not possible that a and not-b occur jointly. For two propositions to be connected in that way, b must be true whenever a is true. If the truth of b is not known for any time period when a is known to be true, there is no necessary relationship between the two propositions, although they are not inconsistent with each other. Thus two propositions can be indifferent toward each other. Meaning is thus defined as the pattern of times in which a propositions is true.

This system thus permits discussion of several degrees of truth, can differentiate between different propositions beyond their present truth value, admits unconnected propositions and does this in a way which consists of simple transformations of one another. We shall now construct a scheme of a nerve net which can perform according to this system.

4. The Network System

4.1 The Type of Input

In order to adapt this modal system to use in nerve net theory, we have to specify an input in such a way that a three-valued signal can be transmitted through an off-on circuit and make the operations conform to the basic operations of the nerve net. This means essentially that we can specify an output pattern for each combination of inputs. A system specified in this manner can perform the basic operations of Boolean algebra but cannot distinguish between two inputs to an organ beyond an on or off state. If these restrictions are observed, it has been shown that nerve nets can be drawn by simple rules from the input-output conditions ([10]). Consequently it is sufficient to present here only the in-out matrices.

As stated above, Prior conceives of each proposition as a string of three-valued symbols. Our nerve net will consist of a set of parallel organs, one of each symbol. Although the set is theoretically infinite, it corresponds in practive to a finite number, say n. Each of these organs consists of two parts, the determinacy net (D) and the falsity net (F). The inputs in the D net are firing if the proposition was indeterminate at the time to which the organ refers, and does not fire otherwise. The inputs in the F net are firing if the proposition is false at the time and do not fire otherwise. Outputs are read in the same

([10]) Culbertson, J. T., «Nerve Net Theory», in *Computer Applications in the Behavioral Sciences*, Prentice-Hall, 1962.

way: If the D-effector fires, the value corresponds to indeterminate (zero) no matter what the F-effector does; if the D-effector does not fire, firing in the F-effector means «false», no firing means «true». The whole network has thus (2n) receptors and (2n) effectors.

The rules or the operations divide into two kinds: the two Boolean operations, K and N, and the modal operations, L and M. For the former each organ can function separately, and only the three-valued nature of the logic provides any differences from ordinary nerve nets. For the latter the whole network has to be analyzed and classified into one of the three type before proceeding on each organ.

4.2 Input-Output Conditions for K and N

For the operator N the rules are as follows. In the D-network the output equals the input. In the F-network the output is the opposite of the input, dead on a firing input and vice versa.

K operates with the inputs from two propositions, a and b; the inputs for network are combined separately according to the following table:

Input	11	10	01	00
Output	1	1	1	0

Applying this scheme to the D-network makes the effector fire whenever one of the propositions is indeterminate; applying it to the F-network makes the corresponding effector fire only when neither proposition is false. As the F-effector is irrelevant whenever the D-network fires, it does fire only when both propositions are true.

4.3 Input-Output Conditions for L and M

Before applying the operators the whole input pattern has to be analyzed and must therefore be stored. The analysis for determination of the type (see above) proceeds as follows: The D and F inputs are combined according to the following table:

Input	11	10	01	00
Output	0	0	0	1

This results in only the «true» symbols firing. After this the number of firing fibers is counted. The determination of class is as follows:

X (number of outputs of previous table firing)	Type
n	I
$O<X<n$	II
O	III

After this each organ is treated separately, but there are two kinds of procedures, one for types I and III and one for II.

For types I and III the process is the same as for L and M: the D-network transmits the input unchanged. Except for indeterminate points in type III, the output of the D-network is always zero. For the F-network the inputs are reversed to obtain the meaning of firing as «false».

For type II the D-network transmits the input unchanged. In the L-organ all outputs in the F-network fire, and in the M-organ no output fires. This type corresponds to a proposition which is true at least once, but not at all times. It is therefore necessary at no time and possible at all times at which any information is available.

With a combination of these organs all operations of modal logic are possible. By giving the activation negative meaning (in the two circuits *on* means indeterminate and false respectively), the required organs include the NAND and NOR circuits and thus are convenient for transistor circuits ([11]).

5. Applications

The use of modal logic gives flexibility to the use of networks and makes it possible to use them in new contexts. To show two of its most interesting features, two kinds of applications will be sketched.

5.1 Reliability

The question of possible failure of either the inputs of the system has been attacked in different ways. Here we can use the differentiation between analytic and synthetic statements. If we have admitted the necessary truth of statement (La) at any time, it must be true at all times. Thus, for any statement of this kind an error which

([11]) Maley, G. A. and Earle, J., *The Logic Design of Transistoral Digital Computers,* Prentice-Hall, 1963.

make the statement true at some times and not at others is self-detecting and can then be checked. On the other hand, synthetic propositions which are possible but not logically necessary allow some errors which would keep them synthetic, and only their truth at different times would be stated in error. It is likely that this division leads to economy in actual thinking. In propositions whose universal truth is accepted, any error is automatically expurgated. Other statements depend on correct input; that is, exact empirical observation .In this connection it is instructive to note that Cowan ([12]) has shown he equivalence of error-correcting codes to the Lewis multivalued logic. It is proposed here that starting with a modal logic such as Lewis' and Prior's system will lead to different and simpler looks at the problem of reliability.

5.2 Consistency

Instead of different symbols referring to different time units we may also conceive that they refer to the relation to different conditions. We can then analyze different propositions on whether they are consistent, inconsistent, and indifferent under different conditions and under which conditions a person entertaining these propositions is consistent or inconsistent and what he can do to achieve consistency. This would then be an approach to «Psycho-Logic» ([13]). The symbols can also be interpreted as affective relationships, like, dislike, and neutral, and consistency of preferences can be analyzed. A number of current psychological theories postulate a drive toward balance or consistency ([14, 15, 16, 17]). Davis ([18]) has proposed a formalized system for these theories. Derivations from this system show clearly the limi-

([12]) COWAN, J. D., «Toward a Proper Logic for Parallel Computation in the Presence of Noise», in *Bionics Symposium*, Wright Air Development Division, 1960.

([13]) ABELSON, R. P. and ROSENBERG, M. J., «Symbolic Psycho-Logic: A Model of Attitudinal Cognition», *Behavioral Science*, Jan., 1958, 1-13.

([14]) FESTINGER, L., *A Theory of Cognitive Dissonance*, Row-Peterson, 1957.

([15]) HEIDER, F., *The Psychology of Interpersonal Relations*, Wiley, 1958.

([16]) NEWCOMB, T., «An Approach to the Study of Communicative Acts», *Psychological Review*, Nov., 1953, 393-404.

([17]) OSGOOD, C. E., SUCI, G. J. and TANNENBAUM, P. H., *The Measurement of Meaning*, Univ. of Illinois Press, 1957.

([18]) DAVIS, J. A., «Structural Balance, Mechanical Solidarity and Interpersonal Relations», *American Journal of Sociology*, Jan., 1963, 444-462.

ting point of the principle of similarity and balance and the points where a theory of dissimilarity would have to enter. Balance theories can be taken as describing limiting conditions [19]. The procedure shown here introduces new concepts which will help in investigating these different sets of processes. There is a difference between saying that individuals tend toward situations which are compatible and that they tend to situations in which all points are necessarily connected. It is not clear which of these suppositions are basic to balance theories. Dealing with ideas which are possible but not necessary or not even true at the present time is not incompatible with organization of one's cognitions and may represent much actual human behavior. Understanding of imbalance through the logic of possibility and necessity will then give a more lifelike model of human behavior.

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(19) BACK, K. W., «Can Subjects be People and People be Subjects ?» in *Mathematical Methods in Small Group Processes*, Stanford Univ. Press, 1962.